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DAM-RESERVOIR INTERACTION FOR INCOMPRESSIBLE-UNBOUNDED FLUID DOMAINS USING AN EXACT TRUNCATION BOUNDARY CONDITION

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ABSTRACT

In this paper, dam-reservoir interaction for a vibrating structure in an unbounded and incompressible and inviscid fluid is analyzed by using finite element approach. An exact boundary condition is developed for truncating surface of unbounded fluid domain. In the derivation of boundary condition, it is assumed that vibration of dam is in the normal direction of dam-reservoir interface and this interface is vertical. Moreover, bottom of fluid is rigid and horizontal. The derived boundary condition is implemented in the finite element code and results are compared with by using Sommerfeld's and Sharan's boundary conditions. It is seen that the proposed boundary condition is efficient and gives better results than the previous published results.

Keywords: Dam reservoir interaction, finite element method, truncating boundary, hydrodynamic pressure.

INTRODUCTION

An important factor in the design of dams in seismic regions is the effect of hydrodynamic pressure exerted on the face of the dam as a result of earthquake ground motions. For an accurate analysis of hydrodynamic pressure on the dam having irregular geometries, the reservoir is generally treated as an assemblage of finite elements.

Zienkiewicz et al. (1965) presented the finite element formulation for analyzing the coupled response of submerged structures assuming water to be incompressible. Nath (1971) analyzed the problem using the method of finite differences but neglecting radiation damping. Chakrabarti and Chopra (1974) have formulated the reservoir as a continuum of infinite length. Two dimensional problem of the added-mass effect of horizontal acceleration of a rigid dam with an inclined upstream face of constant slope was solved analytically by Chwang and Housner (1978) using a momentum balance approach.

In the finite element formulation, unbounded domain of reservoir arise a problem in modeling. To achieve this difficulty, the unbounded domain should be truncated at a certain distance away from the structure. The most commonly used boundary condition along the truncation surface is the Sommerfeld radiation condition (1949). Since this boundary condition takes the form of that for a rigid stationary boundary, the behavior of the reservoir domain is not

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truly represented. An another boundary condition along the truncating surface for an unbounded and incompressible fluid domain is developed by Sharan (1985). Although this boundary condition is better than the Sommerfeld radiation condition, it does not represent the behavior well when truncation surface is very near to dam surface.

An exact boundary condition along the truncating surface of an unbounded reservoir domain is developed by approximating the analytical solution of the hydrodynamic pressure. A numerical study is done to compare the results of Sommerfeld's and Sharan's boundary conditions.

FORMULATION OF UNBOUNDED RESERVOIR DOMAIN

Analytical Formulation of the Hydrodynamic Pressure

For incompressible and inviscid fluid, the hydrodynamic pressure *p* resulting from the ground motion of a rigid dam (figure 1) satisfies the *Laplace* equation in the following form

$$\nabla^2 p = 0 \tag{1}$$

Following boundary conditions are defined by assuming effects of surface waves and viscosity of the fluid are neglected :

At fluid-solid interface (S1),

$$\frac{\partial p}{\partial n} = -\rho a_n \tag{2}$$

where a_n is the ground acceleration subjected on dam face.

At bottom of fluid domain, if bottom is rigid (S2), one can write the following

$$\frac{\partial p}{\partial n} = 0 \tag{3}$$

At far end (S3) where x coordinate is infinite,

$$p = 0 \tag{4}$$

At free surface (S4) when neglecting the surface waves,

$$p = 0 \tag{5}$$

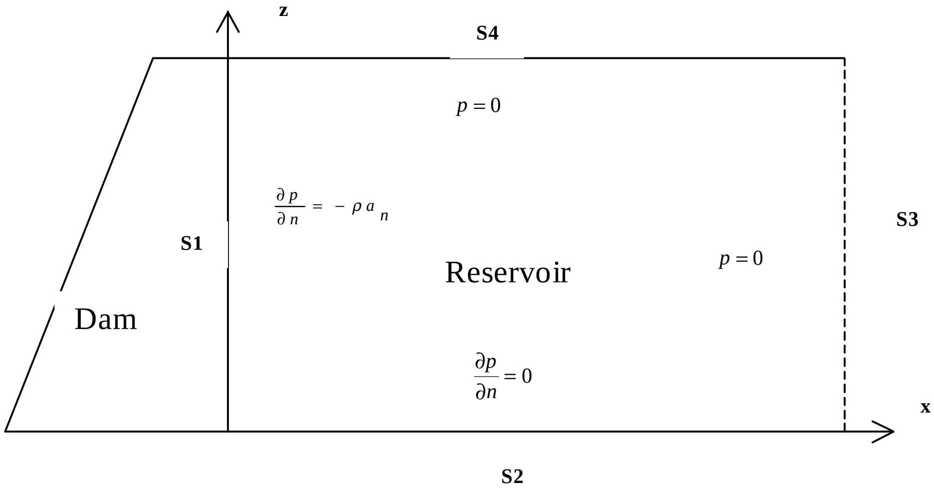

Figure 1. Rigid dam and fluid

Analytical solution of equation (1) due to above given boundary conditions is :

$$p(x,z) = 2a_n \rho H \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda_n^2} \exp(-\lambda_n \frac{x}{H}) \cos(\lambda_n \frac{z}{H}) \tag{6}$$

where $\lambda_n = \frac{(2n-1)\pi}{2}$, H is height of fluid, a_n is ground acceleration, and ρ is mass density of fluid.

Solution of equation (6) is obtained by assuming that: 1) the fluid domain extends to infinity and its motions is two dimensional, 2) fluid-structure interface is vertical, 3) the submerged structure is rigid, 4) the bottom of fluid domain is rigid and horizontal.

Proposed Boundary Condition for Truncated Surface

The proposed boundary condition along the truncated surface of reservoir is obtained by the expanding the analytical solution of hydrodynamic pressure (equation 6) for its gradient in the x direction.

By using the equation (6), one can write the following for the gradient on the truncated surface as

$$\frac{\partial p(x,z)}{\partial n} = -2a_n \rho \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda_n} \exp(-\lambda_n \frac{x}{H}) \cos(\lambda_n \frac{z}{H}) \tag{7}$$

where $\lambda_n = \frac{(2n-1)\pi}{2}$, H is height of fluid, a_n is ground acceleration, and ρ is mass density of fluid.

Sommerfeld radiation condition for the truncated surface is given by

$$\frac{\partial p(x,z)}{\partial n} = 0 \tag{8}$$

and Sharan's boundary condition is given by

$$\frac{\partial p(x,z)}{\partial n} = -\frac{\pi p}{2H} \tag{9}$$

Finite Element Formulation

Assuming hydrodynamic pressure to be the node unknown, the pressure at any point inside an element can be written by

$$p = [N]\{p^e\} \tag{10}$$

where $\{p^e\}$ is the vector of pressures at the element nodes and $[N]$ is matrix of interpolation functions.

By discretizing the equation (1) and applying the boundary conditions equation (2,3,5 and 7), one can write the following expression

$$[H]\{p\} = \{B\} \tag{11}$$

where

$$H_{ij} = \sum \int_{R_e} \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dR \tag{12}$$

$$B_i = \sum \int_{S_e} N_i \frac{\partial p}{\partial n} dS \tag{13}$$

In equation (12) and (13), R_e and S_e denote region and external boundary of an element respectively.

NUMERICAL EXAMPLE

A computer program was developed to test the accuracy of the proposed boundary condition along the truncated surface by using two-dimensional finite elements having 4 nodes rectangular elements. The geometry of rigid dam is shown in figure 2, in which dam is subjected to a horizontal uniform acceleration *a*. The infinite reservoir was analyzed for four different locations of the truncation boundary, each resulting in a different size *l* of an equivalent finite reservoir. Typical finite element model is shown in figure 2.

Results for hydrodynamic pressure distribution were obtained by using Sommerfeld, Sharan and proposed boundary conditions in table 1 in which results were compared with the exact value (Westergaard,1933) $C_0 = p_0 / \rho a H$ at the bottom of the dam face.

The truncated boundary taken very close the dam face is the interest of this study, because taking the truncation boundary at a far distance gives good results for the other boundary conditions, but we loose the efficiency in terms of number of unknowns. As one chose the truncation boundary near to dam face results of hydrodynamic pressures become better compared to Sommerfeld and Sharan boundary conditions.

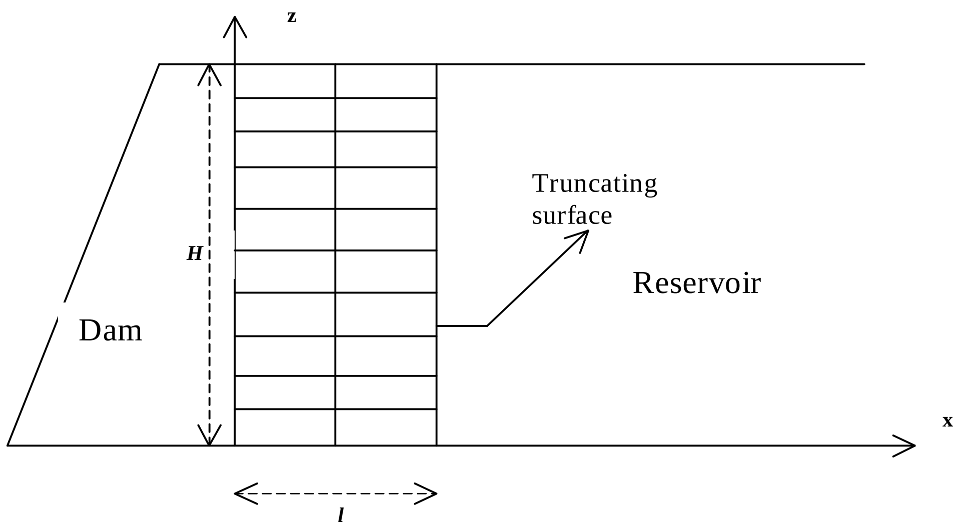

Figure 2. A typical finite element mesh for a 10 row in vertical direction and 2 column in horizontal direction.

Table 1: Comparison of bottom hydrodynamic pressure with exact solution (Exact value for C_o is *0.7425*)

l/H	Mesh Size	Sommerfeld		Sharan		Proposed	
		C_0	% Error	C_0	% Error	C_0	% Error
0.1	10x1	5.0184	576	0.5255	29	0.7408	0.22
0.1	20x2	5.0273	577	0.7072	4.75	0.7417	0.11
0.2	10x2	2.5615	245	0.7303	1.64	0.7414	0.15
0.2	20x4	2.5647	245	0.7295	1.75	0.7402	0.31
0.5	10x5	1.1646	57	0.7420	0.07	0.7419	0.08
0.5	20x10	1.1658	57	0.7417	0.11	0.7423	0.03
1.0	10x10	0.8152	10	0.7424	0.01	0.7423	0.03
1.0	20x20	0.8155	10	0.7424	0.01	0.7424	0.01

It can be seen in the table 1 that proposed boundary condition produced better results for truncated surface very near to dam face, i.e. *l/H=0.1*

In figures 3-6, distribution of hydrodynamic pressure on dam face is compared with Sommerfeld, Sharan and current boundary condition for different truncated lengths and for different mesh sizes.

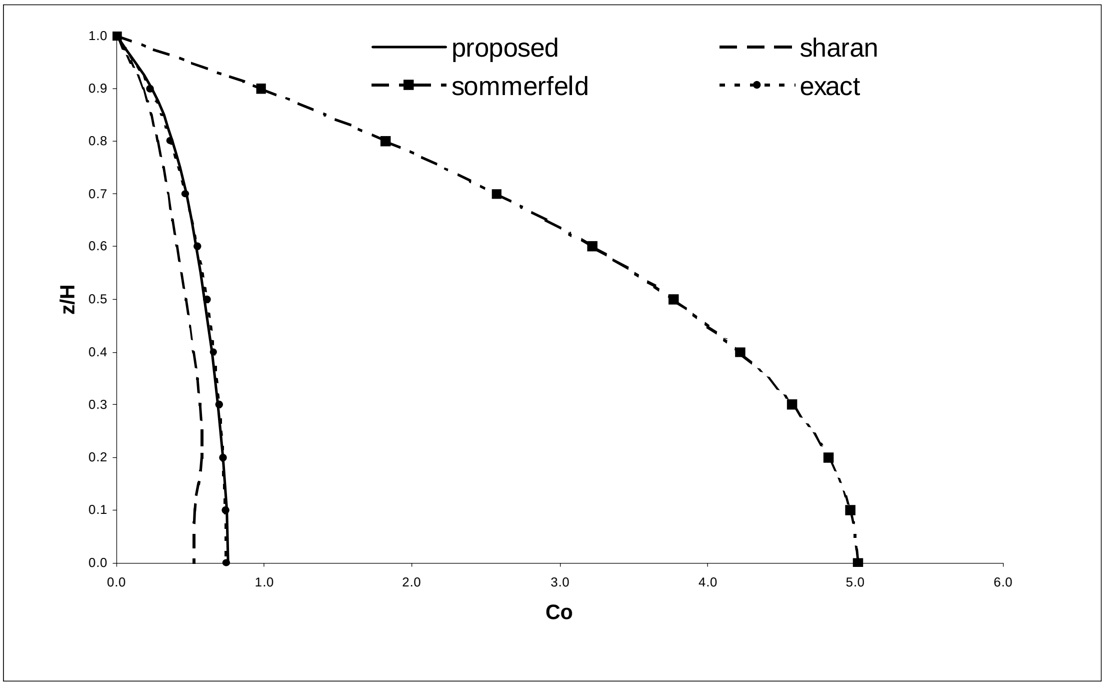

Figure 3. Comparison of proposed and Sommerfeld's and Sharan's boundary condition for hydrodynamic pressure on dam for *l/H=0.1* and mesh size *10x1*.

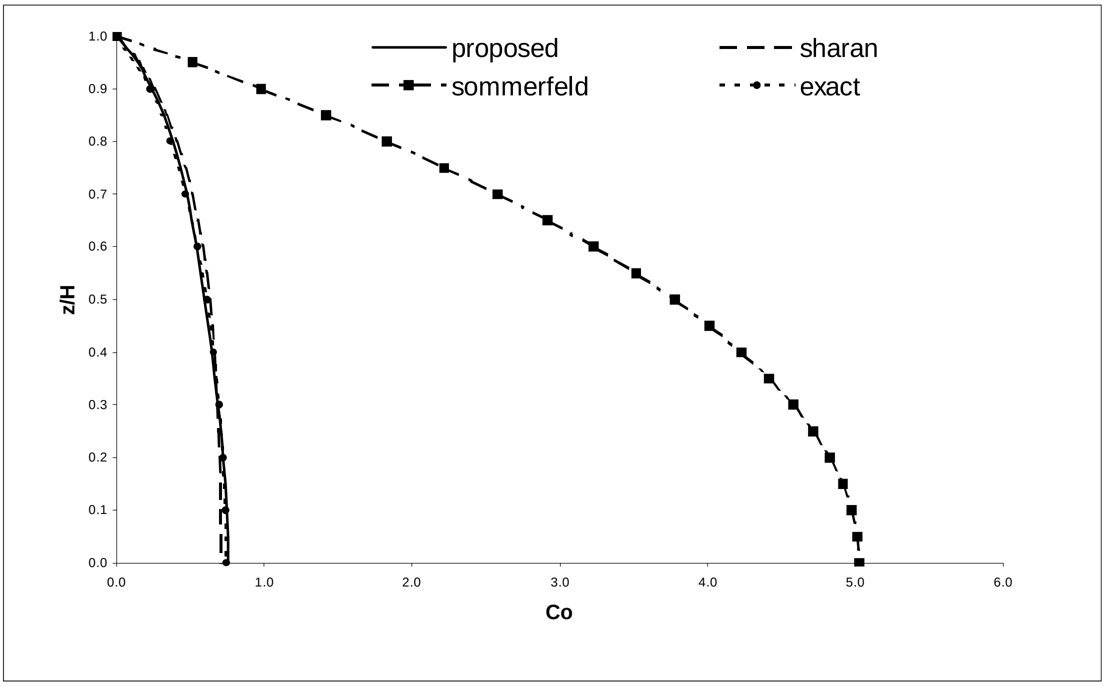

Figure 4. Comparison of proposed and Sommerfeld's and Sharan's boundary condition for hydrodynamic pressure on dam for *l/H=0.1* and mesh size *20x2*.

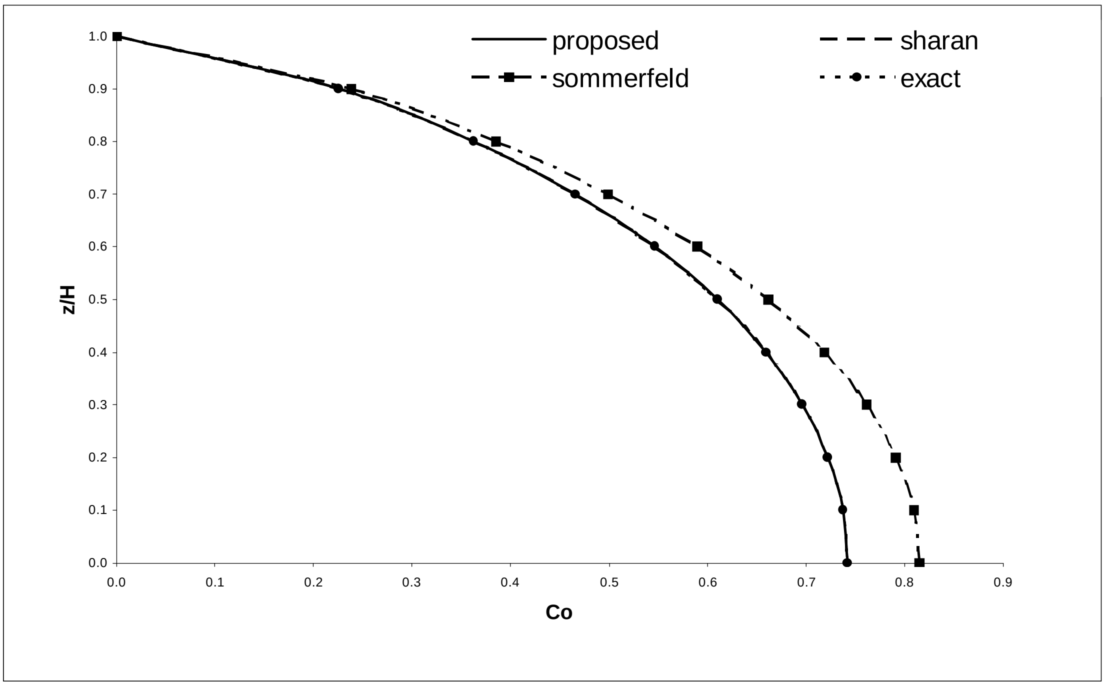

Figure 5. Comparison of proposed and Sommerfeld's and Sharan's boundary condition for hydrodynamic pressure on dam for *l/H=1.0* and mesh size *10x10*.

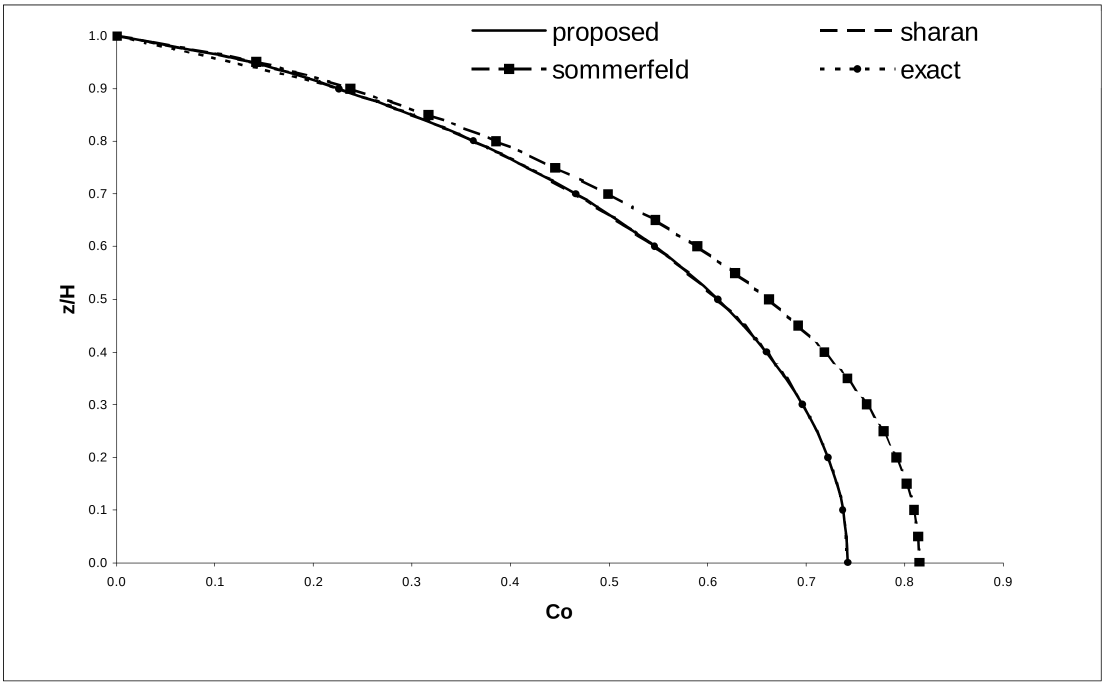

Figure 6. Comparison of proposed and Sommerfeld's and Sharan's boundary condition for hydrodynamic pressure on dam for *l/H=1.0* and mesh size *20x20*.

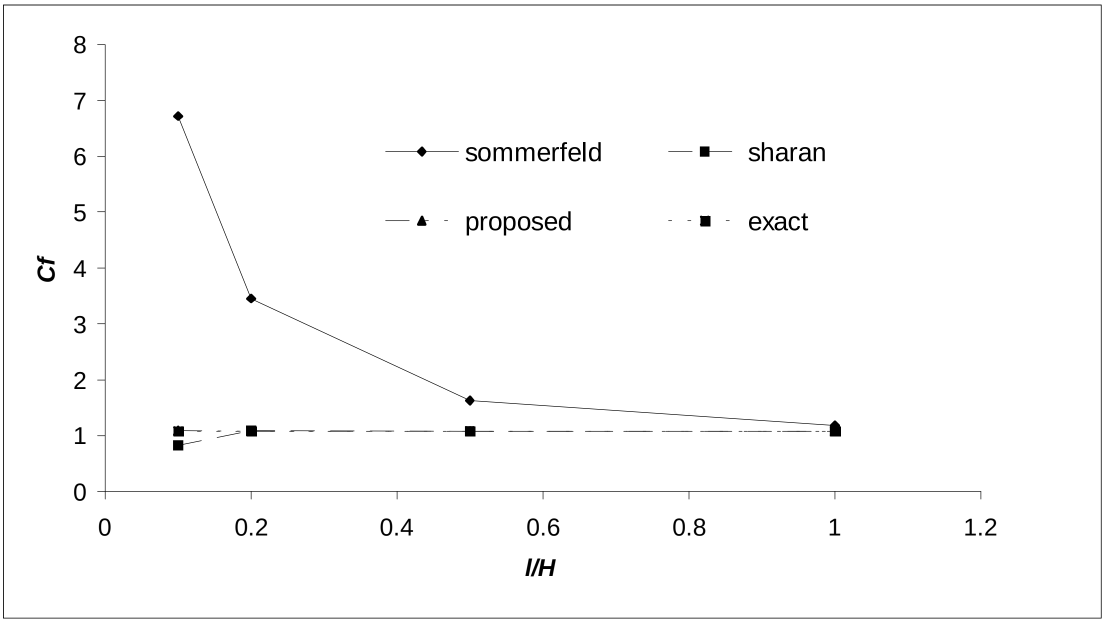

Figure 7. Comparison of total hydrodynamic pressure on dame face.

Comparison of total hydrodynamic pressure on dam face is done in figure 7. In that figure, C_f resembles the ratio of total hydrodynamic pressure to total hydrostatic pressure.

CONCLUSIONS

Dam-reservoir interaction for a vibrating structure in an unbounded and incompressible and inviscid fluid is analyzed by using finite element approach. An exact boundary condition is developed for truncating surface of unbounded fluid domain. The derived boundary condition is implemented in the finite element code and results are compared with by using Sommerfeld's and Sharan's boundary conditions. It is seen that the proposed boundary condition is efficient and gives better results than the previous published results and this new boundary condition is extendable for compressible fluid domains.

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