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NUMERICAL MODELING OF CEMENT GROUT INJECTION IN SATURATED POROUS MEDIA

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ABSTRACT

A theoretical model for numerical simulation of cement grout injection in saturated porous media is proposed. Grout mix and saturating water are assumed to be miscible such that a single fluid phase needs to be considered. The model is composed of two mass balance equations. The first one reflects the total mass transport of the fluid in the porous medium. The other one represents the grout transport by advection, diffusion and dispersion within the fluid phase. The final formulation of the problem yields to a non linear coupled system of equations because of the dependency of fluid properties with grout concentration. The above time dependant problem is discretized in time and space by finite differences using an implicit backward Euler scheme. Finally numerical results are compared with experimental injection tests performed under uniaxial conditions.

Keywords: saturated porous media, injection

INTRODUCTION

In foundation engineering field grouting is a technique aimed to reinforce soils or to reduce permeability (watertightness). It consists in injecting grout under pressure in the soil that needs to be treated. Grouting can be performed either at a constant injection pressure or at a constant pumping rate. Silicate gel used formerly for grouting were chemically unstable and did not respect environmental norms. They are now replaced by cement grouts which behavior during injection differs from grouts without particles (chemical grouts). Propagation mechanisms for cement grouts are not well-known and must be investigated for a more efficient treatment of soils. The purpose of this paper is to present a numerical model for the grout propagation. This model can predict the evolution, in time and space, of the grout concentration and the fluid pressure during grouting.

MATHEMATICAL MODEL

The grout permeation in the saturated porous media is modeled by a nonlinear coupled system of two partial differential equations. The first one represents the grout transport by advection, diffusion and dispersion within the fluid phase according to the macroscopic

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description of transport phenomena in porous media presented in (Bear J. and Bachmat Y., 1991). The concentration equation (or transport equation) is parabolic and convective-dominated. This approach has been also used by F.Bouchelaghem (2001). The second one reflects the total mass transport of the fluid phase that carries the grout component. This is an essential equation since the velocity distribution in the domain is needed as an input data for the transport problem. The pressure equation is elliptic. The two equations described above yield to a coupled problem because of the dependency of fluid properties with grout concentration.

Pressure equation

Darcy's law is assumed for the fluid phase. We consider an unidirectional flow of a homogeneous liquid through a homogeneous, fixed and nondeformable porous medium. The fluid phase mass conservation is combined with Darcy's law leading to the pressure equation (1)

$$n\beta_p \frac{dp}{dt} + n\beta_c \frac{dc}{dt} - \nabla \bullet \left(\frac{K}{\mu} [\nabla p \pm \rho g] \right) = 0 \tag{1}$$

where μ is the viscosity of the fluid mixture, ρ the density of the fluid mixture, K the permeability of the medium, n the porosity of the medium and g the gravity vector. *p(x,t)* denotes the pore fluid pressure. $d()/dt = \partial()/\partial t + \nabla().V$ represents the material derivative for the fluid phase with *V* the velocity of the fluid phase. The $\pm$ depends on whether the vertical axis is taken in the direction of gravity or not. Equation (1) is written considering the equation of state (2) for the fluid phase

$$\beta_p = \frac{1}{\rho} \frac{\partial \rho}{\partial p}\bigg|_c \quad \text{and} \quad \beta_c = \frac{1}{\rho} \frac{\partial p}{\partial c}\bigg|_p \tag{2}$$

β_p is the coefficient of compressibility of the fluid at constant concentration while β_c is the coefficient of concentration at constant pressure. The coefficient of concentration is equal to zero later.

Boundary conditions for the one dimensional problem

$$\frac{q_{imp}}{S} = nV = -\frac{K}{\mu}(\nabla p \pm \rho g)\bigg|_{x=0,t} \text{, for a constant pumping rate } q_{imp} \tag{3}$$

or $p(0,t) = p_{imp}$, for a constant injection pressure p_{imp}. (4)

$$p(L,t) = 0 \tag{5}$$

Initial condition

The hydrostatic pressure is assumed at the initial state.

$$p(x,0) = \rho g x \tag{6}$$

Transport equation

We assume a saturated porous medium. The total flux of the grout component transported within the fluid phase is made up of the sum of an advective, a diffusive and a dispersive flux. The transport equation is given below.

$$n\frac{\partial c}{\partial t}-\frac{K}{\mu}\left(\nabla p\pm\rho g\right)\nabla c-\nabla\bullet(nD_h\nabla c)=0 \tag{7}$$

K, μ, ρ, n and g have been defined for the pressure equation. $c(x,t)$ denotes the grout concentration. D_h is the hydrodynamic dispersion including diffusion and dispersion effects. Below is the expression of D_h for an one dimensional uniform flow.

$$D_h = a_L V + D^* \tag{8}$$

where a_L is the longitudinal dispersion coefficient and D^* is the diffusion coefficient

Boundary conditions for the one dimensional problem

$$c(0,t)=c_{imp} \quad and \quad \left.\frac{\partial c}{\partial x}\right|_{x=L,t}=0 \tag{9}$$

Initial condition

At the initial state the porous medium is saturated in water and the grout invades the medium only after the beginning of the injection.

$$c(x,0)=0 \tag{10}$$

The pressure equation and the concentration equation must be solved simultaneously since the fluid properties depend on the grout concentration. The fluid viscosity is assumed linear with respect to concentration.

$$\mu(c)=\left(\mu_g-\mu_w\right)\frac{c}{c_{imp}}+\mu_w \tag{11}$$

Quantities with subscripts g or w refer to grout or water viscosity.

FINITE DIFFERENCE FORMULATION IN ONE DIMENSION

The implicit backward Euler scheme is employed to discretize the problem described above. It consists in a backward difference in time and a centered difference in space. Combining these expressions leads to a tridiagonal matrix system. The diffusion-dispersion term of the transport equation is discretized as follow because of the variable coefficient D_h.

$$\frac{\partial}{\partial x}\left(nD_h^{q-1}\frac{\partial c^q}{\partial x}\right)_j^{t+1} \approx \frac{1}{\Delta x}\left\{ nD_h^{q-1}\frac{\partial c^q}{\partial x}\bigg|_{j+1/2} - nD_h^{q-1}\frac{\partial c^q}{\partial x}\bigg|_{j-1/2} \right\}^{t+1}$$

$$\text{with} \quad D_h^{q-1}\frac{\partial c^q}{\partial x}\bigg|_{j+1/2}^{t+1} \approx \frac{1}{2}\left\{ D_h^{q-1}\Big|_{j+1} + D_h^{q-1}\Big|_j \right\}\frac{1}{\Delta x}\left\{ c_{j+1}^q - c_j^q \right\}^{t+1} \tag{12}$$

$$\text{and} \quad D_h^{q-1}\frac{\partial c^q}{\partial x}\bigg|_{j-1/2}^{t+1} \approx \frac{1}{2}\left\{ D_h^{q-1}\Big|_j + D_h^{q-1}\Big|_{j-1} \right\}\frac{1}{\Delta x}\left\{ c_j^q - c_{j-1}^q \right\}^{t+1}$$

Discretization of the transport equation

The viscosity depends on the concentration. The following equation (13) is solved by successive substitutions. q is an iteration parameter needed by the Gauss-Seidel-type method.

$$\begin{aligned} c_j^{q,\,t} &= \left\{ \frac{K\Delta t}{2n\Delta x}\left[\frac{1}{\mu}\frac{\partial p}{\partial x} \pm \rho g\right]_j^{q-1} - \frac{\Delta t}{2\Delta x^2}\left[D_h\Big|_{j-1}^{q-1} + D_h\Big|_j^{q-1} \right] \right\} c_{j-1}^{q,\,t+1} \\ &- \left\{ \frac{K\Delta t}{2n\Delta x}\left[\frac{1}{\mu}\frac{\partial p}{\partial x} \pm \rho g\right]_j^{q-1} + \frac{\Delta t}{2\Delta x^2}\left[D_h\Big|_{j+1}^{q-1} + D_h\Big|_j^{q-1} \right] \right\} c_{j+1}^{q,\,t+1} \\ &+ \left\{ 1 + \frac{\Delta t}{2\Delta x^2}\left[D_h\Big|_{j+1}^{q-1} + 2\,D_h\Big|_j^{q-1} + D_h\Big|_j^{q-1} \right] \right\} c_j^{q,\,t+1} \end{aligned} \tag{13}$$

Discretization of the pressure equation

The velocity depends on the pressure. The following equation (14) is solved by successive substitutions.

$$\begin{aligned} P_j^{q,t} &= \left\{ \frac{K\Delta t}{2n\Delta x}V^q - \frac{K\Delta t}{2n\beta_p(\Delta x)^2}\left(\frac{1}{\mu_j^q} + \frac{1}{\mu_{j-1}^q} \right) \right\} p_{j-1}^{q,t+1} \\ &- \left\{ \frac{K\Delta t}{2n\Delta x}V^q + \frac{K\Delta t}{2n\beta_p(\Delta x)^2}\left(\frac{1}{\mu_j^q} + \frac{1}{\mu_{j+1}^q} \right) \right\} p_{j+1}^{q,t+1} \\ &+ \left\{ 1 + \frac{K\Delta t}{2n\beta_p(\Delta x)^2}\left(\frac{1}{\mu_{j+1}^q} + \frac{2}{\mu_j^q} + \frac{1}{\mu_{j-1}^q} \right) \right\} p_{j+1}^{q,t+1} - \frac{K\rho g\Delta t}{2n\beta_p\Delta x}\left\{ \frac{1}{\mu_{j+1}} - \frac{1}{\mu_{j-1}} \right\} \end{aligned} \tag{14}$$

The advection-dispersion equation is often used to describe solute transport in porous media (Bear, 1979). However numerical methods (finite elements and finite differences methods) often exhibit oscillatory behavior and/or excessive numerical dispersion near relatively sharp concentration fronts (Huyakorn & Pinder, 1983). These problems become serious for advective-dominated transport characterized by small dispertivities (Diaw & al., 2001). To circumvent numerical oscillations the finite difference method will be subjected to Courant and Peclet number restrictions using a fine discretization.

RESOLUTION METHOD

The coupled problem is solved by a Gauss-Seidel-type method for nonlinear partial differential equations (PDE) systems as explained in (Langtangen, 1999). Equations of the problem are solved in sequence with an outer iteration. Let q be an iteration parameter. Quantities with superscript q denote approximations in the qth iteration. The Gauss-Seidel procedure can then be expressed as in Algorithm 1. The transport equation is first solved (by successive substitutions) with respect to c, using the most recently computed *p* values for the velocity in formulas. Thereafter the pressure equation is solved with respect to *p*, using the most recently computed *c* values. The pressure equation is still nonlinear and is solved by successive substitutions. The attractive feature of the Gauss-Seidel iteration approach to the nonlinear problem is that we only need to solve standard scalar PDEs.

Algorithm 1. Gauss-Seidel-type method for systems of nonlinear PDEs

Given a guess c^0 for the solution of $F^c_{concentration}(c, p) = 0$,

and a guess p^0 for the solution of $F^p_{pressure}(c, p) = 0$

1-update q

2-solve $F^c_{concentration} = 0$ for $c_i^q \; and \; p_i^{q-1}, \; i = 1,...,m$ with respect to *c* by successive substitutions

3-solve $F^p_{pressure} = 0$ for $c_i^q \; and \; p_i^q, \; i = 1,...,m$ with respect to *p* by successive substitutions

4-check convergence criterion

UNIAXIAL INJECTION TEST

The injection tests are performed in columns of length 0.75 m and 0.08 m in diameter filled with Loire sand (0.70 m). Loire sand characteristics are available in Dano & al.(2003). The sand is put in place in the column by layers and a compaction method is used to get a fixed density. The injection is performed at a constant pumping rate ($q_{imp} = 1.5E-6 \; m^3/s$) after the sand has been saturated in water. The material properties are given in table 1.

Table 1. Material properties

Porosity	0.335
Grout density	1370 kg/m^3
Grout viscosity	2.9*10^{-3} Pa.s
Intrinsic permeability	1.17*10^{-11} m^2
Diffusion coefficient	1.0*10^{-10} m^2/s
Longitudinal dispersion coefficient	2.0*10^{-2} m
Compressibility coefficient	3.0*10^{-8} Pa^{-1}

Instrumentation

To enable the comparison between numerical and experimental results the pore pressure and the grout concentration within the fluid phase are required. Hence three pressure transducers are positioned along the column and one at the bottom of the column (injection point). They are space out 0.2 m apart. The whole concentration field is not experimentally available and only the grout front position can be obtained. It is detected by acoustic emission (AE) using transducers coupled with three wave guides which cross the sand column. They are horizontally placed at the same height than the pressure transducers. An increase of the acoustic activity is observed when the grout reach the wave guide. Consequently the grout front position is able to be detected at different times and locations in the sand column. The technique of AE is validated by performing injection tests in transparent columns. In this case the grout propagation is visually followed to ensure that the AE detection really occurred when the grout arrived at the wave guide. During injection tests the volume and the weight of injected grout, the pumping pressure and the pumping rate are also recorded.

COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

The calculation of the grout concentration evolution with the distance from the injection point is represented in figure (1d).Figures (1a), (1b), and (1c) show the acoustic activity recorded by AE versus time for the three transducers c1, c2 and c3 associated with the wave guides.

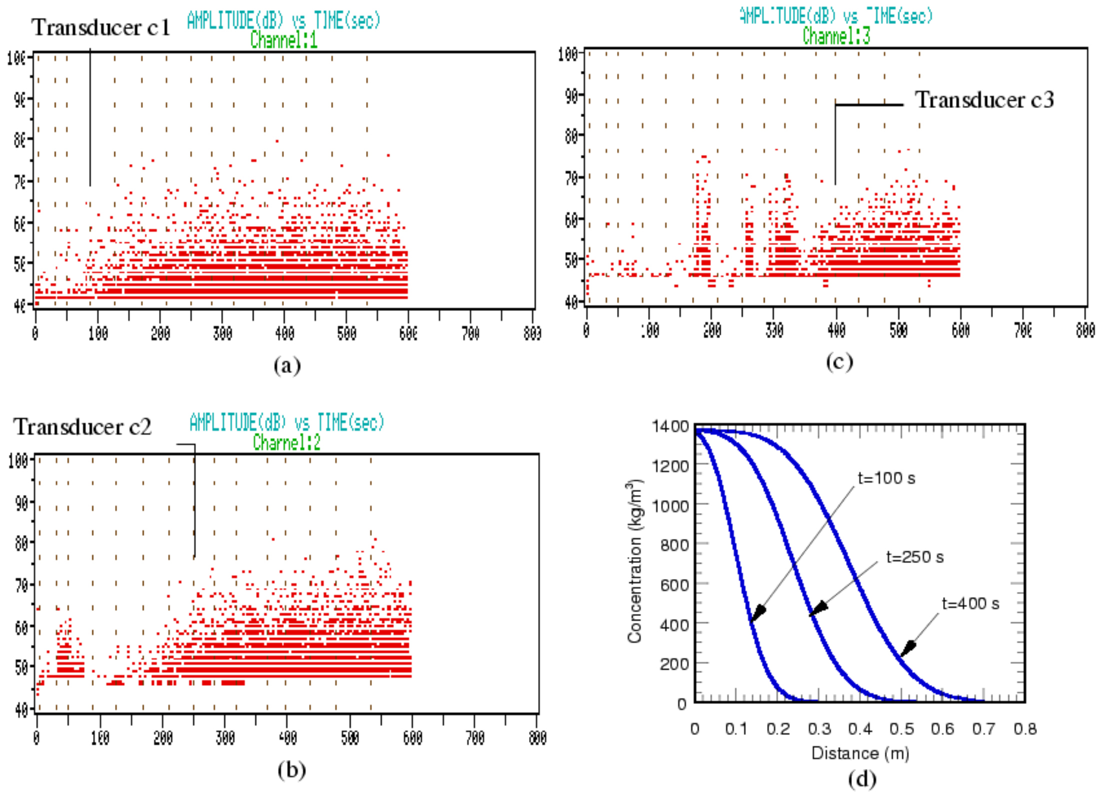

Figure 1.(a), (b), (c) Grout propagation follows by transducers c1, c2, c3. (d) Evolution of the numerical grout concentration with the distance from the injection point.

Transducers c1, c2 and c3 detect successively the grout front at times t=100 s, t=250 s and t=400 s. The numerical concentration is also plotted (figure (1d)) at these times to allow a direct comparison between the numerical results and the experimental ones. The whole concentration field can not be obtained by acoustic emission and only the grout front position is experimentally available.The latter is numerically located (figure (1d)) where the grout concentration becomes non zero. There is a good agreement between the numerical concentration computed with the developed model and the grout front detection obtained by AE. Indeed the numerical front at time t=100 s is approximately located at z=0.2 m corresponding to the c1 transducer location. At times t=250 s and t=400 s the numerical front is successively positioned at z=0.4 m and z=0.6 m corresponding to the c2 and c3 transducer positions. In figure (1c) a non explained increase of the acoustic activity is observed before the grout reach the wave guide. It may be caused by external sound or by air trapped in the porous medium.

Figure (2) shows the evolution of the injection pressure with time. Experimental pressure exhibit oscillations due to vibrations of the pump during injection. To make the comparison with numerical pressure possible, only the average experimental pressure is taken into account. In this case the numerical pressure matches with the experimental one.

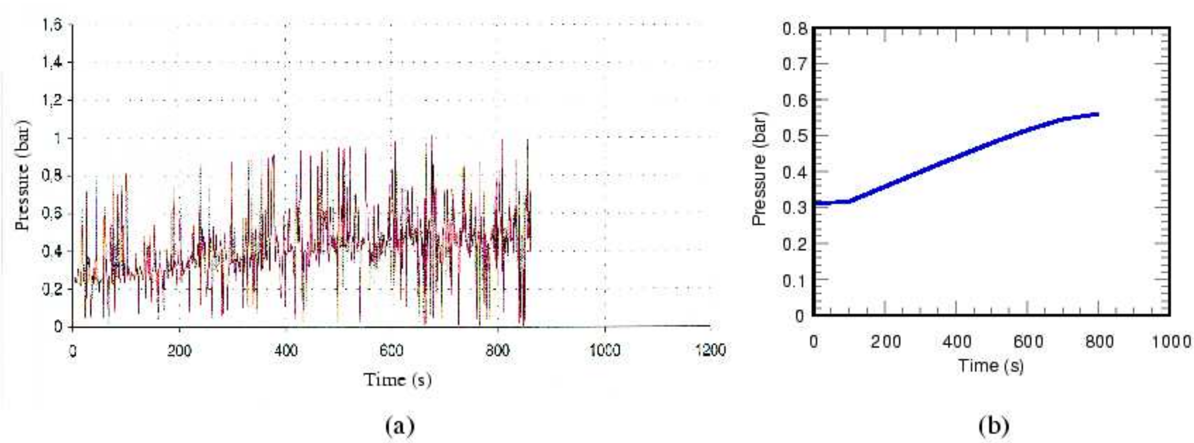

Figure 2. Evolution of the injection pressure with time. (a) Experimental. (b) Numerical pressure.

CONCLUSION

The proposed model based on Bear's transport equation enables to simulate the grout propagation in a saturated sand column. It provides a good approximation of the pore pressure and the grout concentration in the column during injection. Nevertheless the fine discretization used to avoid numerical oscillations or diffusion introduced by solving the transport equation leads to very important calculation times for larger problems. Consequently other discretization methods need to be considered.

An acoustic technique to detect the grout front position was successfully tested. The latter permits the grout front to be localized when it can not be visually followed (e.g. injection tests in tanks).

Afterwards a phenomenological filtration law reflecting the permeability reduction during grouting will be considered in order to improve the present model.

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REFERENCES

Bear J. (1979). Hydraulics of groundwater. McGraw-Hill, New York.

Bear J. and Bachmat Y. (1991). Introduction to Modeling of transport phenomena in porous media. Kluwer Academic Publishers, Dordrecht.

Bouchelaghem F., Vulliet L. (2001). Mathematical and Numerical Filtration-Advection-Dispersion Model of Miscible Grout Propagation in Saturated Porous Media. Int. J. for Num. and Analytical Methods in Geomechanics 25, NO. 12, 1195-1227.

Dano C., Hareb H., Hicher P.Y. (2003). Characterization of Loire river sand in the small strain domain using new bender extender elements. Proceedings 16th engineering mechanics conference. Seattle, US.

Diaw E.B., Lehmann F., Ackerer Ph. (2001). One dimensional simulation of solute transfer in saturated-unsaturated porous media using the discontinuous finite elements method. Journal of Contaminant Hydrology 51,197-213.

Hutakorn P.S and Pinder G.F. (1983). Computational methods in subsurface flow. Academic Press, London, UK.

Jason L., Saiyouri N., Hicher P.Y. (2002). Propagation study of grouts in sands : numerical modelling. Proceedings 4th International Conference on Ground Improvement Techniques. Kuala Lumpur, Malaysia.

Langtangen H.P. (1999). Computational partial differential equations. Numerical methods and diffpack programming. Springer-Verlag, Berlin.