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A FAST CONTACT DETECTION ALGORITHM FOR DISCRETE ELEMENT METHOD

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ABSTRACT

In discrete element method (DEM), determining the contact forces between interacting particles and the associated contact normals at each time step is a critically important and time consuming step. Common-plane (CP) algorithm is one of the most effective methods for contact detection when dealing with two-dimensional polygonal particles or three-dimensional blocks. This paper proposes a new approach to locate the common plane between polygonal particles. This approach recognizes that a potential plane possesses limiting characteristics, which dramatically reduces the number of potential planes. Our preliminary experiments reveal that in two-dimensions the new method is at least 3 times faster than current methods.

Keywords: common plane, DEM, contact detection

INTRODUCTION

Discrete element method (DEM) was initially introduced by Cundall (1971) to simulate large scale movements of jointed rock masses, and subsequently extended by Cundall and Strack (1979) for the analysis of assemblies of circular disks and spheres. Applicability of DEM to large deformation and localization types of analysis make it quite attractive. In the literature, DEM has been successfully employed to study the mechanical behavior of granular materials (for example, Cundall and Strack 1983, Cundall 1988, Rothenburg and Bathurst 1989, Bathurst and Rothenburg 1990, Matuttis, Luding et al. 2000, and Shodja and Nezami 2003).

However, the application of DEM to large scale simulations, has been impeded by huge computational cost required for detecting new contacts or updating the existing ones at every time step. Among different contact detection methods which have been introduced and implemented in DEM modeling, the class of "Common-Plane" methods (Cundall 1988) seems to be the most efficient when dealing with polyhedral blocks. Cundall (1988) states that "a common plane (CP) is a plane that, in some sense, bisects the space between the two contacting

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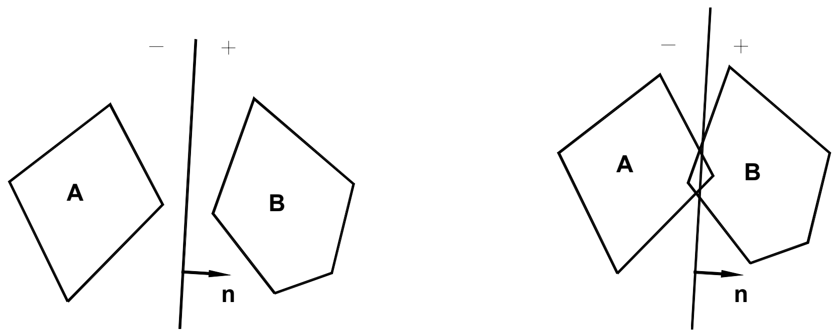
blocks". If two blocks are in contact, both will intersect the CP. If two blocks are not in contact, none of them intersects the CP (Figure 1). As a result, block-block contact detection procedure reduces to a much faster simple plane-block contact detection procedure. Once the CP is established between two particles, the normal to the CP defines the contact normal; and, the closest distances between the CP and the vertices of the blocks are used to define the contact forces. This is advantageous for 2D polygonal or 3D polyhedral particles where the definition of the contact normal is a non trivial problem.



(a) Particles not in contact (b) Particles in contact

Figure 1. Two blocks and their common plane (CP).

CURRENT ALGORITHM FOR FINDING CP

The "distance" d^V of a point V in the space to a plane is defined as

$$d^V = \mathbf{n} \cdot (V_0 - V) \tag{1}$$

where **n** is the unit vector normal to the plane, and V_0 is a point on the plane. d^V can be positive or negative depending on the relative position of vertex V with respect to the plane (Figure 2a).

The "distance" d_A of a block **A** to any plane can be defined as

$$\begin{aligned} d_A &= \min\{d^V\} \text{ if } d^C > 0 \\ d_A &= \max\{d^V\} \text{ if } d^C < 0 \end{aligned} \tag{2}$$

where d^V is the distances of each vertex V of the block **A** to the plane and min{} and max{} are the minimum and maximum respectively, taken over all vertices of the block. d^C is the distance of the centroid of the block (Figure 2b) to the plane. Note that $d^C = 0$ is of no practical interest as the CP will never pass the centroid of a block.

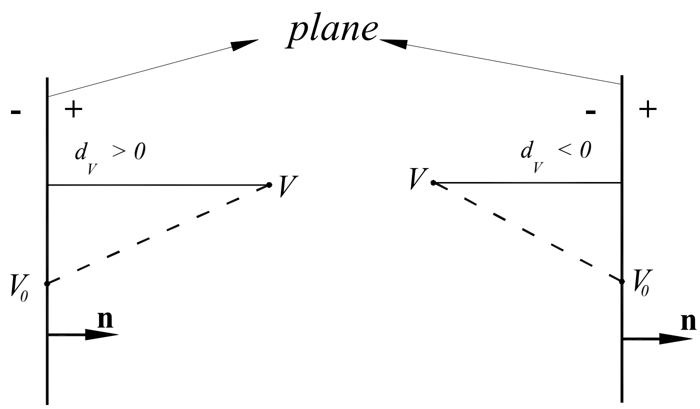
For any two blocks **A** and **B**, a CP is a plane with the following three properties:

1) The centroids of blocks **A** and **B** are located in different sides of it. In this discussion it is assume that the centroid of block **A** is in negative side of the CP and that of the block **B** is in the positive side (Figure 1)

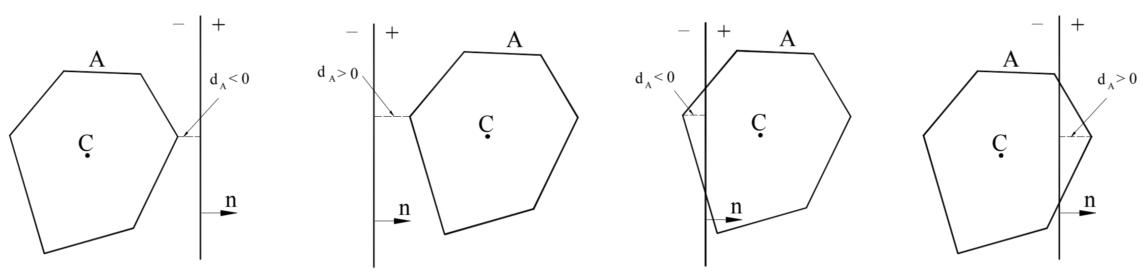
2) $|d_A| = |d_B|$

3) The total gap defined as $d_B - d_A$ is a maximum.

where d_A and d_B are the distances of blocks **A** and **B** to the plane. For given blocks **A** and **B**, the total gap $d_B - d_A$ is only a function of direction **n** of the CP and is independent of the location of the plane in the space. Condition (3) defines the direction of unit vector **n** by maximizing the gap, while Condition (2) specifies the location of the CP by setting $|d_A| = |d_B|$. As a conclusion of Condition (1), d_A and d_B always have different signs. For separated particles, like the ones shown in Figure 1a, the gap is always positive ($d_A < 0$ and $d_B > 0$), while for contacting particles, such as those in Figure 1b, the gap is always negative ($d_A > 0$ and $d_B < 0$).



(a) Distance of a point to a plane;



(b) Distance of a block to a plane

Figure 2. Definition of distances and sign convention for a potential common plane

Cundall (1988) suggests a two stage procedure for finding the CP:

1. The First stage involves detection of one point of the CP (referred to as reference point or contact point). The reference point is chosen as the mid-point between the centroids of the two particles not in contact.
2. The second stage is an iterative process to determine the normal to the CP. The gap

between the particles is maximized by rotating the CP around the reference point. At a given iteration, two arbitrary orthogonal axes passing through the reference point are chosen within the CP and the CP is perturbed around each of these axes. If any perturbation produces a gap larger than that of the current CP, the new CP replaces the current CP. The process continues until the direction of the CP is found with a reasonable accuracy. If the gap, at any stage of iteration, exceeds a pre-defined tolerance *TOL*, the iterative process halts and the contact will be deleted.

One problem associated with this methodology, in addition to large number of iterations required (especially for the first-time formation of CP's), is that during the iterative process the reference point remains constant. In other words, once the reference point is established in the first stage, the iterative process maximizes the gap merely by "rotating" the CP around the reference point. This may lead to a plane that does not necessarily possess the largest gap.

This paper shows that the iterative process of the second stage can be eliminated. In fact, using some characteristics of the CP, it can be shown that there exists a few easy-to-find "candidates" for the CP. Therefore, all that is needed is to check these few candidates to find the one with the largest gap, which is the CP. In two-dimensional problems, the total number of these candidates is only 5.

Throughout this discussion, the words "particle" and "block" are used interchangeably and both of them refer to 2D polygonal. The word Common-Plane (CP) refers to a plane (in 3D problems) or a line (in 2D problems). Also the term "time step" refers to the DEM time step while the terms "step" and "iteration step" refer to the iteration process of contact detection in every DEM time step. All the particles are assumed to be convex, rigid or deformable, while concave particles can be modeled as a combination of several convex particles attached to each other.

PROPOSED COMMON PLANE DETECTION ALGORITHM

An enhanced algorithm to locate the CP is summarized below:

At every time step,

- If there is no CP available from the previous time step, the algorithm described in "particles not in contact" section is performed to find the CP and the associated gap. If the gap is larger than a user-defined value *TOL*, then the two particles are recognized as "not in contact" and no CP is established between them. If the gap is smaller than the *TOL* and larger than zero, then it is marked as a "potential contact", for which the particles are separate and no contact force is assigned to them.
- If the CP is available from the previous time step (whether it corresponds to a "potential contact", where the particles are separated and the gap takes a positive value, or to a "real contact", where the gap is negative) the separation method (described in "particles in contact" section) is applied. Whenever the gap is smaller than zero, a "real contact" is generated. Whenever the gap is larger than *TOL*, the contact is deleted.

Particles not in contact

Assume that a CP is already found for two convex blocks **A** and **B** with their vertices A and B respectively having the shortest distances to the CP (Figure 3). According to property (2) of the common plane, the CP should pass through the mid-point of the line AB. Furthermore, the CP cannot intersect the line CC_1 which is parallel to edge AA_1, otherwise the vertex A_1 (and not A) would be the closet point to the CP. For the same reason, the CP cannot intersect lines CC_2,

CC_3, and CC_4 which are parallel to edges AA_2, BB_1, and BB_2 respectively. So, the CP should be in the area bounded by the lines CC_1, CC_2, CC_3 and CC_4 (shaded area in Figure 3b).

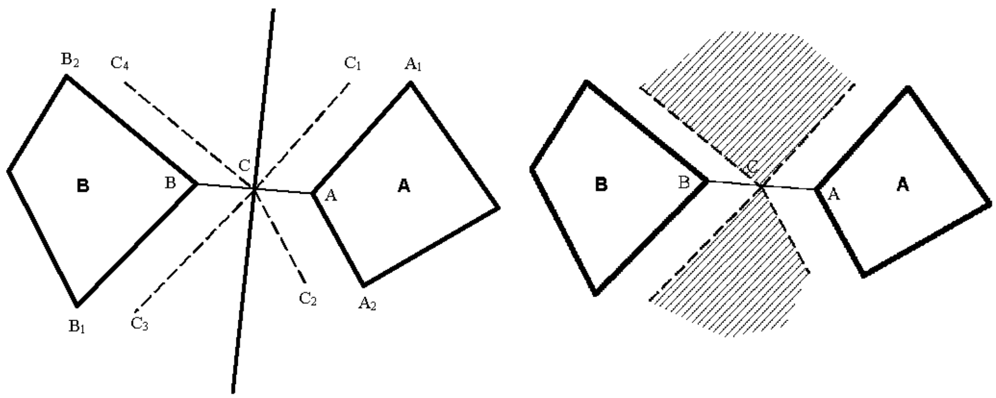
On the other hand, according to property (3), any rotation of CP around the reference point C produces a gap which is less than $d_B - d_A$. Thus, the perpendicular bisector of the line AB, if completely located in the shaded zone of Figure 3b, is the CP because its distance from the two points A and B is the largest. If the perpendicular bisector is not completely located in the shaded area, then the CP is the line that is the "closest" to the CP while still completely located in that shaded area. That is, one of the boundary lines.

So the candidates for being a CP are:

a) The perpendicular bisector of the line AB.

b) The lines passing the mid-point of line AB and parallel to edges AA_1 and AA_2 of particle **A.**

c) The lines passing the mid-point of line AB and parallel to the edges BB1 and BB2 of particle **B**.



(a) **(b)**

Figure 3. (a) Two particles and their associated CP; (b) Allowable zone for CP's

In each of these cases, the common plane is found to be either parallel to one of the edges of the polygons or normal to the line connecting the closest points. The following algorithm is suggested to find the CP between particles not in contact:

Step 1) Initial guess: set the CP as the perpendicular bisector of the line connecting the centroids of the two particles.

Step 2) For the new CP, find the vertices A and B associated with the shortest distances.

Step 3) For the vertices A and B found in step 2 check all five candidate planes to find CP.

Step 4) If the CP obtained in step 3 is the same as the CP used in step 2 to find the vertices A and B, then halt. Otherwise go to step 2.

Although this algorithm is iterative, the number of iterations is generally less than 3 and in most cases is equal to one. It should be noted that no contact force or contact point exists as long as the particles are separated (assuming no tension forces act between particles). The CP is established to determine how close the particles are to a real contact, and to keep track of those particles which are expected to form a real contact in the next few time steps.

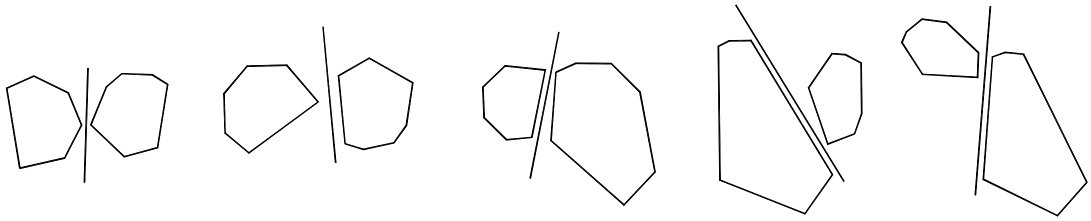
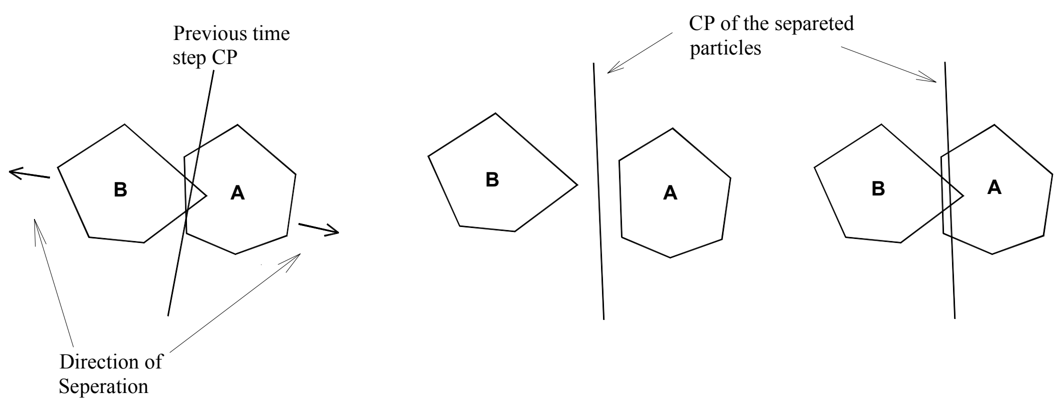


Figure 4. Examples of CP determined using the proposed alorithm

Particles in contact

For particles in contact an additional step is performed to temporarily separate the particles. This is accomplished by translating them, at each time step, in a direction perpendicular to the CP from the previous time step (Figure 5). When the blocks are separate, the algorithm described for particles not in contact is employed to determine the new CP. In Figure 5a, the position of the two particles whose CP is to be found at the present time step are shown. Also the CP known from the previous time step is depicted. Figure 5b shows the same particles as Figure 5a but now separated by translating them in a direction perpendicular to the CP from previous time step. The new CP obtained based on this separated configuration is also calculated and shown in the same figure. This new CP is assumed to be that of the particles A and B in their original configuration of Figure 5a (as shown in Figure 5c). The gap $d_B - d_A$ is then calculated using the original configuration and the new CP as shown in Figure 5c.

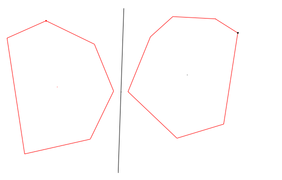
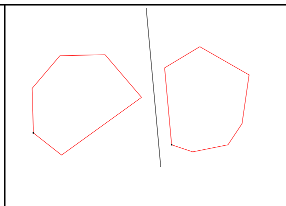
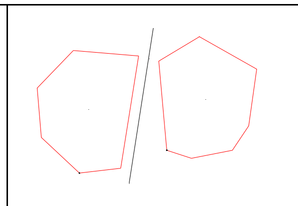
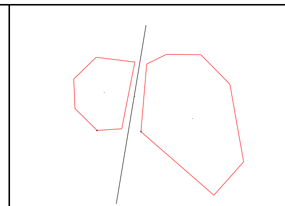
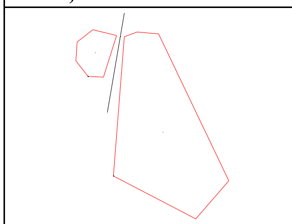
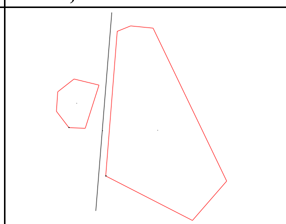
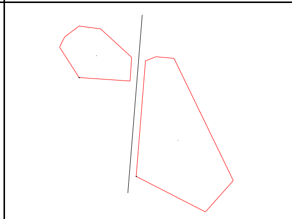
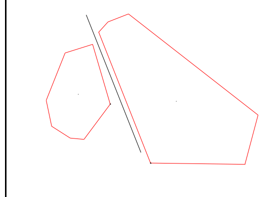
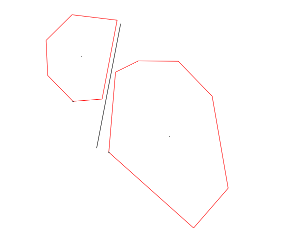
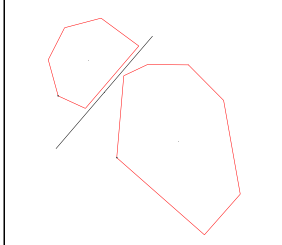
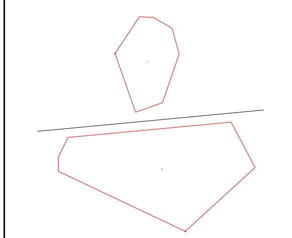
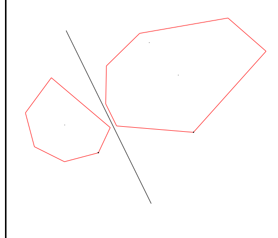
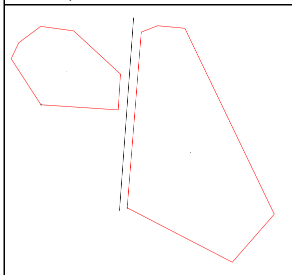
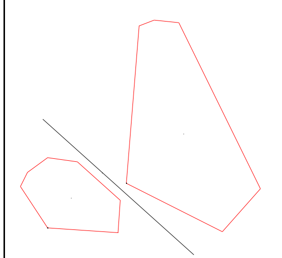
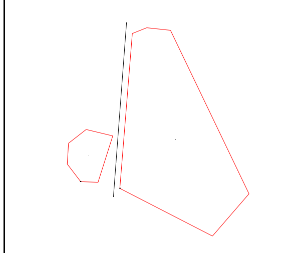
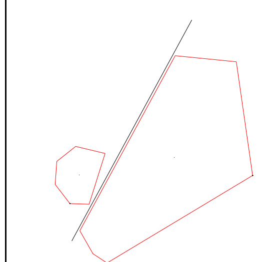


(a) **(b)** **(c)**

Figure 5. The method of translation of particles; (a) original configuration of particles and the CP from previous time step; (b) the particles after translation normal to the previous CP, and their corresponding CP; (c) original configuration and the associated CP.

Table 1 shows 16 different configurations of particles for which the performance of the proposed algorithm is compared with that described by (Cundall 1988). For all these cases the resulting CP's from both algorithms are identical. For the purpose of comparison, the number of iteration steps, *I,* required in Cundall's algorithm, and the speed up ratio *R* of the CPU running time in Cundall's method to that in the proposed method are shown. Note that for all configurations shown in Table 1, for the proposed method, the number of candidate planes is always five. Consequently, the number of iterations required to find the CP in the proposed method is either one (if the bisector of the line AB is the answer) or five (if the bisector is not the answer). Based on the results presented in Table 1, and some other results not shown here, it is concluded that the proposed algorithm for two-dimensional problems, on average, is 3 times faster than Cundall's algorithm.

Table 1. Comparison of the performance of Cundall's (1988) and the proposed algorithm.

			
I=14, R=2.0	*I=14, R=2.6*	*I=17, R=3.3*	*I=14, R=3.1*
			
I=23, R=4.6	*I=15, R=1.23*	*I=19, R=3.6*	*I=18, R=3.6*
			
I=19, R=3.7	*I=15, R=3.0*	*I=14, R=2.1*	*I=15, R=3.0*
			
I=19, R=3.2	*I=16, R=2.3*	*I=14, R=2.5*	*I=22, R=2.1*

CONCLUSIONS

An efficient common plane algorithm is developed to find the common plane between two-dimensional polygonal particles. The algorithm takes advantage of characteristics of the

CP to limit the search space for the plane. The algorithm is approximately 3 times faster than the iterative procedure suggested by Cundall (1988). The algorithm is currently being extended to 3-D and a paper describing this development is under preparation.

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