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EFFECTIVE STRESS LAW FOR THE PERMEABILITY OF CLAY-RICH SANDSTONES

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ABSTRACT

Two models of clay-rich sandstones are analyzed to explain the variation of permeability with pore pressure and confining pressure. In one model the clay forms a shell-like layer along the pore wall, and in the other it forms particles weakly attached to the pore walls. Both models predict that the permeability is much more sensitive to changes in pore pressure than to changes in confining pressure. The clay particle model gives somewhat better agreement with data from the literature, and with new data on a Stainton sandstone having 8% clay by solid volume.

Keywords: permeability, effective stress, sandstones, clay

INTRODUCTION

Physical properties of sedimentary rocks, such as the permeability k, depend on both the confining pressure, P_c, and the pore pressure, P_p. If hysteresis is neglected, k can always be expressed as some function $k = f(P_c, P_p)$. If the permeability can also be expressed as a function of the *single* parameter $P_c - n_k P_p$, *i.e.*, $k = f(P_c - n_k P_p)$, we then say that it follows an *effective stress law*, where n_k is the effective stress coefficient, and $P_c - n_k P_p$ is the effective stress. For a rock whose mineral phase consists of a single mineral, the effective stress coefficient should not exceed unity (Berryman, 1992). However, Zoback and Byerlee (1975) found that the effective stress coefficient of some clay-rich sandstones can be as high as 3-4. Walls and Nur (1979) found that n_k increased with clay fraction, and reached values as high as 7 for sandstones with volumetric clay fractions of 20%.

To explain this behavior, Zoback and Byerlee proposed a model in which the rock consists of quartz, permeated with cylindrical pores that are lined with an annular layer of clay. As the inner clay layer is more compliant than the outer quartz layer, such a rock should be more sensitive to changes in pore pressure than to changes in confining pressure, and therefore have an effective stress coefficient greater than unity. Although this model has frequently been invoked to explain experimental results, a quantitative discussion of this model has not yet been given. A related model, proposed in this paper, is one in which the clay takes the form of particles that are only tangentially attached to the pore walls. Using recently collected data on the elastic

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deformation of clays (Farber *et al.*, 2001), which show clays to be about twenty times more compliant than quartz, we find that both models yield effective stress coefficients that increase with increasing clay content. The second model gives higher values of n_k, which are in somewhat closer agreement with those found in the literature.

REVIEW OF RESULTS FOR CLAY-FREE ROCKS

Before describing the two models used in our study, it is worth reviewing the results for a rock consisting of a single mineral. If the pores are assumed to be cylinders of radius a, flow through each pore will be governed by Poiseuille's law (Bernabe *et al.*, 1982), which states that the hydraulic conductance of the tube is proportional to a^4. Other factors will influence the overall permeability, such as the interconnectedness of the pores, but these are assumed not to vary with stress. Hence, to find the dependence of permeability on stress, we need only find the variation of the pore radius a with stress.

It follows from the expression $k = f(P_c - n_k P_p)$ that the effective stress coefficient for permeability, n_k, can be defined as the ratio of the sensitivity of permeability to changes in pore pressure, to the sensitivity of the permeability to changes in confining pressure:

$$n_k = \frac{-(\partial k/\partial P_p)_{P_c}}{(\partial k/\partial P_c)_{P_p}}. \tag{1}$$

If k depends on the stresses only through the pore radius a, use of the chain rule gives

$$n_k = \frac{-(\partial k/\partial P_p)_{dP_c=0}}{(\partial k/\partial P_c)_{dP_p=0}} = \frac{-(dk/da)(\partial a/\partial P_p)_{dP_c=0}}{(dk/da)(\partial a/\partial P_c)_{dP_p=0}} = \frac{-(\partial a/\partial P_p)_{dP_c=0}}{(\partial a/\partial P_c)_{dP_p=0}}. \tag{2}$$

Hence, according to this model, the effective stress coefficient for k is essentially the same as that for a (or for the pore volume).

If the elasticity equations are solved for a cylindrical pore in an elastic body, we can find the sensitivity of pore radius a to the inner and outer pressures. From this solution, we then find that the effective stress coefficient depends on the porosity ϕ and the Poisson ratio ν of the medium, but never exceeds unity (Bernabe *et al.*, 1982; Zimmerman, 1991). For a typical value of ν, such as 0.25,

$$n_k = (2+\phi)/3. \tag{3}$$

If the pores were assumed to be elliptical rather than cylindrical, the effective stress coefficient would increase, approaching unity in the limit of thin crack-like pores, but never exceeding it (Bernabe *et al.*, 1982; Zimmerman, 1991). Hence, it seems that values of $n_k > 1$ should only be expected to occur in a sandstone if it contains clay. Indeed, the data collected from various sources by Kwon *et al.* (2001) shows that n_k increases almost linearly with clay content, reaching values as high as 7 when the clay content is 20%.

MODELS FOR CLAY-RICH SANDSTONES

To explain the results mentioned above in a quantitative way, two different pore-clay models have been examined. The first model is the clay shell model proposed by Zoback and Byerlee (1975) in which the rock consists mainly of a single mineral, say quartz, permeated with cylindrical pores that are lined with shell-like layer of clay (Fig. 1a). In the second model, the rock again consists mainly of quartz permeated with cylindrical pores, but with the clay situated as particles that are touching, but only weakly coupled to, the rock matrix (Fig. 1b). These two somewhat idealized models may be expected to represent extreme cases with regards to the extent of coupling between the clay and rock.

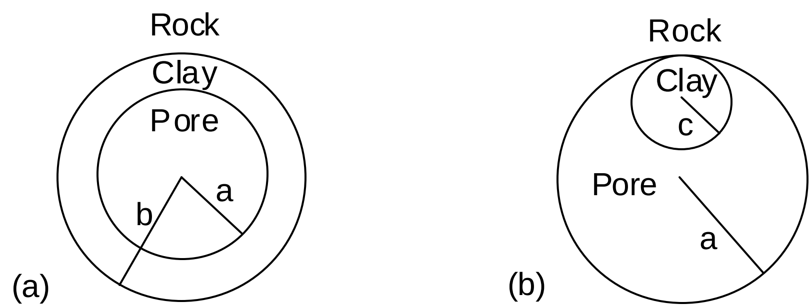

FIG. 1. Pore clay models: (a) clay shell model; (b) clay particle model.

Clay Shell Model

In this model, the clay is equally distributed over all of the pore walls, forming a thin layer (Fig. 1a). As in the clay-free case, the permeability will depend only on the radius of the pore tube, a, and the effective stress coefficient will be given by the ratio shown in equation (2). The dependence of a on the two applied stresses will of course be different in this case. This dependence can be found by solving the appropriate elasticity problem; see Al-Wardy (2003) for details.

Clay Particle Model

In this model the clay exists in the form of particles that are tangentially connected to the pore walls (Fig. 1b). In this configuration, the clays will have essentially no influence on the effect that the confining pressure has on the pore geometry. An increase in confining stress will cause the pore channel to deform in the same manner as if the clay were not present, and furthermore will have essentially no influence on the geometry of the clay particles. The pore pressure will cause the pore wall at $r = a$ to expand radially, exactly as in the clay-free case. But the pore pressure, which acts over essentially the entire outer boundary of the clay particle, will also cause a uniform hydrostatic compression of the clay particle. Hence, this model does not require the solution to any new elasticity problems.

However, the permeability will depend on the geometry of the region of the pore that is not occupied by the clay particles. Hence, we need a solution for the viscous flow problem within the open pore space. In order to yield a tractable, two-dimensional flow problem, we assume that the clay exists as a solid cylinder of radius c, touching the pore wall. (Alternative models could include, for example, spherical clay particles attached to the pore walls at random locations.) The region available for fluid flow is then the region between two eccentric cylinders, of radii a and c, in the limiting case in which the inner cylinder is touching the outer one. This solution can be found in White (1974), from which the sensitivity of the permeability to the pore pressure can be calculated from

$$\frac{dk}{dP_p} = \frac{\partial k}{\partial a}\frac{da}{dP_p} + \frac{\partial k}{\partial c}\frac{dc}{dP_p}, \tag{4}$$

and similarly for the confining pressure. The partial derivatives of *a* and *c* with respect to pressure are already known, as explained in the previous paragraph. The derivatives of *k* with respect to *a* and *c* are found by differentiating the solution given by White (1974); again, details can be found in Al-Wardy (2003).

RESULTS AND DISCUSSION

The effective stress coefficients for permeability, as predicted by the two models, are plotted in Fig. 2a as a function of clay fraction. The porosity is taken to be 20%, and, the Poisson's ratio of both the rock and the clay are taken to be 0.25. The different curves represent different values of the stiffness ratio. At zero clay content, all curves begin at the value 0.733, given by equation (3). In the limiting case on which the stiffness ratio is 1, *i.e.*, the clay and rock have the same elastic properties, the system is equivalent to a uniform rock without clay, and the effective stress coefficient is consequently insensitive to clay fraction. For higher stiffness ratios, the effective stress coefficient increases with clay content, at a rate that increases with increasing stiffness ratio. The rate of increase is greater for the clay particle model than for the clay shell model.

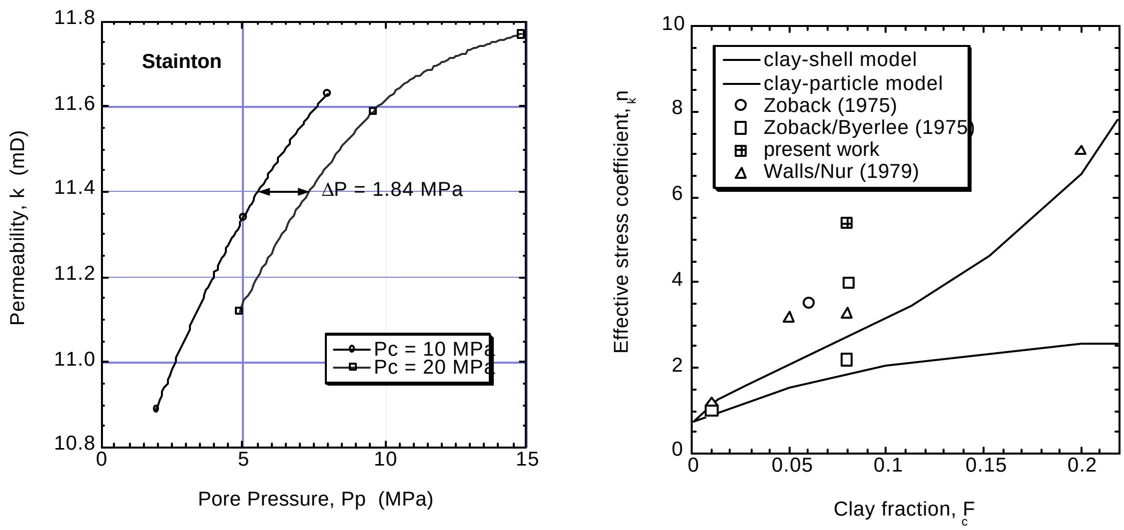

FIG. 2. (a) Permeability of Stainton sandstone as a function of pore pressure, at two different confining pressures. Curves essentially coincide if shifted by DP_p = 1.84 MPa. (b) Effective stress coefficient n_k, as a function of clay fraction F_c. The points refer to data from the present work and from the literature, and the lines refer to the two models.

To test our models, the permeability was measured on a core of Stainton sandstone, having 16% porosity and 8% volumetric clay content. Details of the rock and the experimental procedure can be found in Al-Wardy (2003). Permeability was measured as a function of pore pressure, with the external hydrostatic confining pressure held constant at 10 MPa and 20 MPa in the two tests (Fig. 2a). According to the effective stress concept, we can find n_k by shifting one of these curves along the P_p axis by an amount ΔP_p, until the two curves coincide. The effective stress coefficient is then given by

$$n_k = -\left(\frac{DP_c}{DP_p}\right)_{\text{constant}\,k} \tag{4}$$

The best agreement between the two curves occurs if $DP_p = 1.84$ MPa, which implies that the effective stress coefficient is 5.4 (Fig. 2b). Also plotted on Fig. 2b are values measured for several different clay-bearing sandstones, as collected by Kwon *et al.* (2001) from various sources (Zoback, 1975; Zoback and Byerlee, 1975; Walls and Nur, 1979). Both models give the same trend as is observed in the data, with the clay particle model giving the better fit.

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