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TRANSIENT RESPONSE OF RIGID FOUNDATIONS RESTING ON VISCOELASTIC LAYER

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ABSTRACT
A Boundary Element based methodology is presented to analyze the transient response of two-dimensional rigid foundation structures resting on an isotropic viscoelastic layer over bedrock. The transient solution is obtained by applying the Fast Fourier Algorithm (FFT) to previously synthesized frequency domain responses. The stationary responses were obtained by means of an Indirect Version of the Boundary Element Method, which is based on non-singular distributed load solutions for viscoelastic continua. These stationary viscoelastic auxiliary states already incorporate the boundary conditions of the layered soil, so that only the soil-foundation interface needs to be discretized. The auxiliary states were obtained numerically. An accurate integration strategy is used to determine stationary solutions at very high frequencies. The FFT algorithm associated with the accurate broad banded frequency solutions allow the determination of transient responses for very small time steps. The described methodology is applied to investigate the transient response of 2D rigid foundations resting on a viscoelastic layer over bedrock. It should be stressed that, due to the lack of a viscoelastic fundamental solution, current Boundary Element implementations are not able to describe the transient response of foundations layered on this type of continua. In this article the effect of several parameters on the foundation response will be investigated. The layer depth and foundation inertia effects will be addressed. The presented results may be considered benchmarks for BE researchers dealing with transient foundation responses.

Keywords: Dynamic Soil-Structure Interaction, Transient Analysis, Viscoelasticity, Soil Layer

1. INTRODUCTION

When modeling the dynamic response of foundations, the soil is best represented by an unbounded domain. In infinite domains energy is carried away from the perturbation sources in form of non-reflected waves. This phenomenon is known as geometric or radiation damping (Richard et all, 1970). The mathematical expression for this damping mechanism and is known as the Sommerfeld Radiation Condition (SRC). Special techniques are required to include the SRC into domain type methods like the Finite Element Method (FEM) or the Finite Difference Method (FDM). These techniques include de Dirichlet-to-Neumann mapping and the so called Infinite Elements. On the other hand the Boundary Element Method (BEM) can naturally account for the SRC provided an auxiliary state satisfying this condition is available (Beskos 1987, 1997, Dominguez, 1993).
A formulation of the BEM, in which only the boundaries of the domain under consideration need to be discretized, requires a known auxiliary state with the same characteristic, i.e. the same differential operator, of the domain to be solved. These auxiliary states are known as Green`s

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functions (Kane, 1994). A transient analysis of a visco-elastic problem by the BEM asks for a transient visco-elastic Green`s function, with the same internal damping model of the original problem. Unfortunately there is no general transient visco-elastic Green`s function (Beskos, 1987, Beskos, 1997). Consequently, the issue of describing the dynamic transient response of unbounded visco-elastic domains is an unsolved topic. In the last years the authors of the present article have been involved in solving stationary dynamic soil-structure interaction (DSSI) problems. Auxiliary stationary visco-elastic states have been numerically synthesized for homogeneous isotropic 2D and 3D Half-spaces, for 2D and 3D layered isotropic soils and for anisotropic soils as well (Barros & Mesquita, 1999).

These auxiliary, frequency domain, visco-elastodynamic states have been synthesized numerically. Very accurate integration strategies were developed to perform numerically the inverse of the integral transforms. Allowing for the determination of solutions at very high frequencies.

These Green`s functions have been incorporated in the Direct and Indirect versions of the BEM (DBEM, IBEM) to analyze the stationary response of surface, embedded or buried rigid foundation interacting with the described soil-profiles (Barros & Mesquita, 2001). Stationary dynamic compliance matrices for rigid foundations interacting with visco-elastic soils have been obtained accurately even for relative high excitation frequencies. It is possible to use these frequency domain solutions together with the FFT algorithm to determine the transient response of rigid structures in visco-elastic soils (Mesquita et all, 2002).

In the present article the transient response of a surface foundation interacting with the layer is investigated. The influence of the layer deep is analyzed. The effects of the foundation inertia properties on the transient response will also be addressed.

2. FREQUENCY RESPONSE OF RIGID STRUCTURES

Auxiliary viscoelastodynamic states necessary to formulate the boundary integral equation are the solution of stationary stress boundary value problems (SBPV) governed by Navier equations, which can be expressed in terms of the displacement components u_i as:

$$\mu^* u_{i,jj} + \left(\lambda^* + \mu^*\right) u_{j,ji} + f_i + \rho\omega^2 u_i = 0 \tag{1}$$

In equation (1) ω is the circular frequency, ρ is the continuum density and f_i are the components of the body forces. The constitutive parameters of the visco-elastic medium are the complex Lame constants μ^* and λ^*, which contain the frequency dependent damping function is $\eta(\omega)$,

$$\begin{aligned} \mu^* &= \mu\left[1 + i\eta(\omega)\right] \\ \lambda^* &= \lambda\left[1 + i\eta(\omega)\right] \end{aligned} \tag{2}$$

The boundary conditions for the Stress Boundary Value Problem (SPBV) are given in equations (3) bellow and can also be depicted at figure 1:

$$t_z(x, z=0) = \begin{cases} t_z; |x| \le a \\ 0; |x| > a \end{cases}$$

$$t_x(x, z=0) = 0 \tag{3}$$

$$u_x(x, z=H) = u_z(x, z=H) = 0$$

The auxiliary solutions for the 2D visco-elastic Half-space are obtained numerically. The expressions to be integrated, do present the following general structure:

$$G_i(\omega, x, z) = \int_{-\infty}^{\infty} H_{ij}(\omega, k, x, z)\, t_j(\omega, k, x, z) \exp(i\omega k)\, dk \tag{4}$$

In equation (4) $H_{ij}(\omega,k,x,z)$ are the solution kernels in the wave number domain and $t_j(\omega,k,x,z)$ are the wave number domain (k) representation of the prescribed boundary tractions (Barros & Mesquita, 1999). To obtain the response of a rigid foundation (figure 2) the synthesized auxiliary solutions are, in the sequence, superposed considering the kinematic compatibility and equilibrium equation (Barros & Mesquita, 2001) leading to a stationary dynamic compliance matrix $[N(\omega)]$, relating the external force vector acting upon the foundation $\{F_e\}=\{F_z, F_x, M_y/A\}^T$ and the rigid foundation degrees of freedom $\{U\}=\{W_z, U_x, \phi_y/A\}^T$:

$$\{U\} = \frac{1}{\pi G}[N(\omega)]\{F_e\} \tag{5}$$

or explicitly

$$\begin{Bmatrix} W_z \\ U_x \\ \phi_y A \end{Bmatrix} = \frac{1}{\pi G} \begin{bmatrix} N_{wz} & N_{uz} & N_{\phi z} \\ N_{wx} & N_{ux} & N_{\phi x} \\ N_{wy} & N_{uy} & N_{\phi y} \end{bmatrix} \begin{Bmatrix} F_z \\ F_x \\ M_y/A \end{Bmatrix} \tag{6}$$

In equation (6) A is the foundation half-width. The compliance function $N_{wz}(\omega)$, which represents the vertical foundation response W_z, due to a unit harmonic vertical excitation $F_z=1$ will be analyzed in the examples of this article.

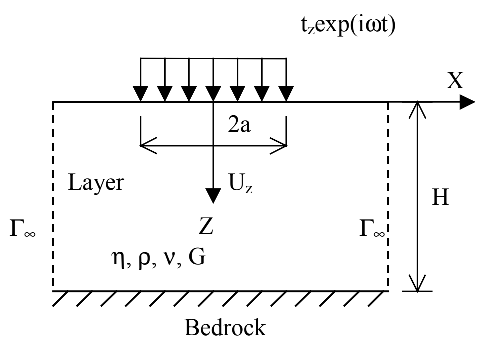

Figure 1. Stess Boundary Value Problem

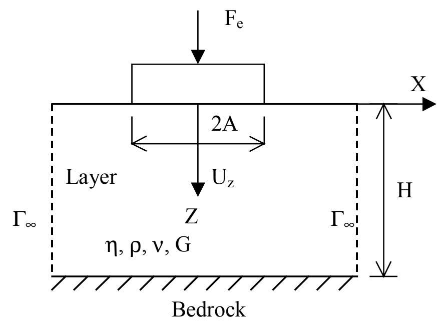

Figure 2. Rigid foundation on viscoelastic layer-Mixed Boundary Value Problem

The typical behavior of the displacement response in frequency domain $N_{wz}(A_o)$ in log scale are showed in figure 3, for the dimensionless frequency parameter$A_o=\omega A/c_s$. In this expression c_s is the shear wave velocity of the elastic layer. Figure 3 shows the stability of the integration strategy for the inverse integral transform, but also shows that the frequency solution decays very slowly in the present case.

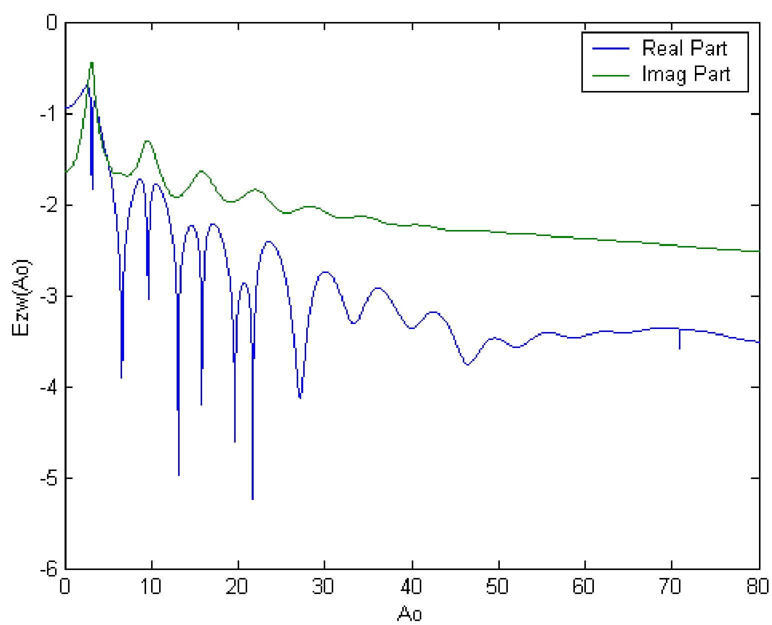

Figure 3. Frequency domain response for a rigid surface foundation.

3. TRANSIENT RESPONSES

The transient solution is obtained by applying the Fast Fourier Transform (FFT) algorithm with respect to the pair (ω,t) to the solutions given in equations (6). One important issue of the discrete FT is the relation between the frequency step $\Delta\omega=\omega_{k+1}-\omega_k$ and the maximum time reachable in the transient process, $T_{max}=2\pi/\Delta\omega$. Analogously, the relation between the time step Δt and the maximum sampling frequency ω_{max} is: $\Delta t=2\pi/\ \omega_{max}$. So it is desirable to have frequency solutions at very small steps $\Delta\omega$ and also at very large maximum sampling frequencies ω_{max}. However, it should be stressed that the determination of frequency domain solutions is computationally very expensive. So, in order to yield small steps and large final frequencies the domain solutions $u_i(\omega)$ must undergo a mathematical treatment. The same reasoning applies to the dimensionless frequency A_o. Figure 4 shows, schematically, the mathematical manipulation that is applied to the frequency solutions. The functions $u_i(A_o)$ (i=x,z) are calculated numerically, with a frequency step ΔA_o in the range $0<A_o<A_{o\ calc}$. To obtain a smaller frequency step in this range, the function is interpolated with cubic splines. Beyond the frequency $A_{o\ calc}$ the numerical integration scheme fails or it is extremely time consuming. Above the cut-off frequency $A_{o\ zero}$ the functions are assumed to vanish, $u_i(A_o> A_{o\ zero})=0$. In the range between $A_{o\ calc}<A_o<A_{ozero}$ the functions are also interpolated by cubic splines, leading to a smooth transition at the extremes $A_{o\ calc}$ and $A_{o\ zero}$. For low damping values the stationary response presents high oscillations at $A_{o\ calc}$. In this case splines do not lead to a smooth transition and an exponential filter can be applied.

Beyond the cut-off frequency $A_{o\ zero}$ the solution is filled with zeros $u_i(A_{o\ zero}<A_o<A_{of})=0$. If all significant information of the frequency signals has been determined in the range $0<A_o<A_{o\ zero}$, then filling the response with zeros should only lead to smaller time steps, without changing the transient response itself (Mesquita et all, 2002).

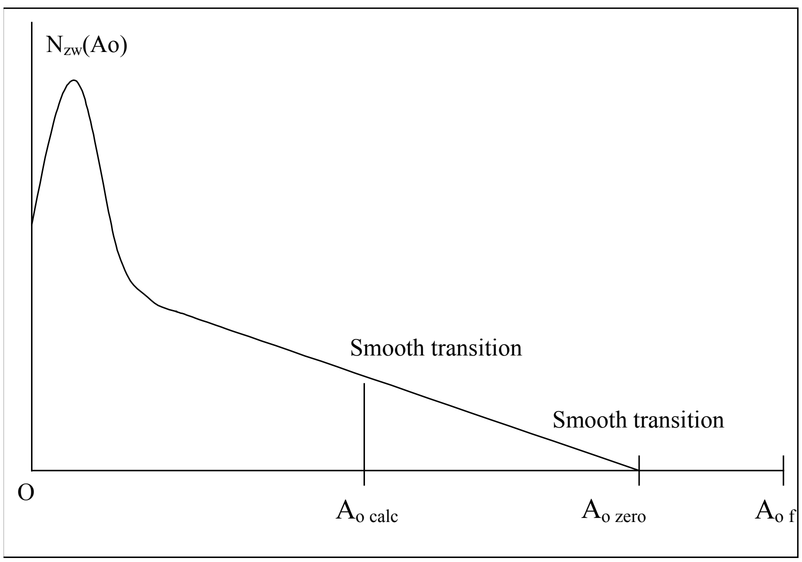

Figure 4. Scheme for treating the frequency solutions

4. NUMERICAL RESULTS

In this section, numerically synthesized transient responses for the layer over the bedrock will be reported. The influence of the layer depth, damping parameters and the inertia effects will be investigated.

Influence of the Layer Depth. Consider a rigid strip foundation, A=1, resting on an isotropic viscoelastic, with depth H, possessing hysteretic damping with coefficient $\eta=0.2$, density

ρ=1kg/m^3 and elastic constitutive parameters G=1 N/m^2, Poisson coefficient ν=1/3 , subjected to a Dirac's Delta impulse in time f(t)=δ (0). With these constitutive parameters the layer presents a dilatational wave velocity p_1=2 [A/s], shear velocity c_1=1 [A/s] and a Rayleigh wave velocity $c_R \approx 0.95$ [A/s]. In the range ($0.0 < A_o < A_{ocalc}$) the stationary responses were generated with a dimensionless frequency step ΔA_o=0.05. The other frequency parameters are $A_{ocalc}=A_{ozero}$=200, A_{of}= 409.6. A total of 8192 terms in frequency were used. The correspondent transient response has a period T_{max}= 125.6560 s and a time step Δt= 0.0077 s.

To analyze the transient behavior at larger time steps, the wave reflections at the rigid base must be considered. Figure 5 shows the transient response a foundation resting on a layer over bedrock with depth H=2A. At the time instante t=2s, the first reflected dilatational wave reaches back the foundation. The same figure 5 also shows the transient response of a half-space in which no waves are reflected back to the surface. At the initial time steps, the half-space response and the layer response are coincident. But as time evolves the wave reflection at the bedrock base change markedly the foundation response, as can be seen in figure 8.

The response for different layer depths is presented in figure 6. The layer depth causes two distinct effects on the transient response. For deeper layers the path that must be traveled by the waves prior to the arrival back at the surface increases. The consequences are that the reflected waves impinge the surface at later times with smaller amplitudes, due larger energy dissipation (geometric and internal). All these phenomena are well characterized in the response shown in figure 6.

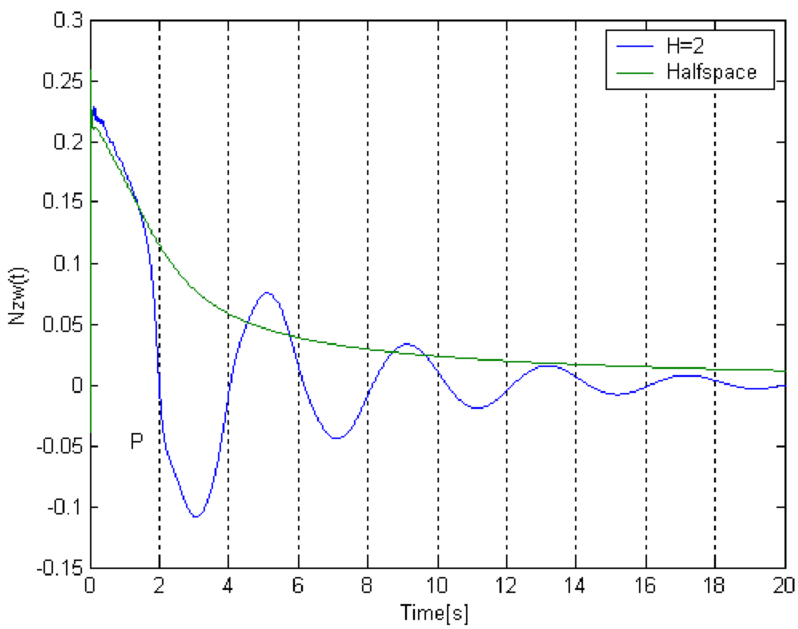

Figure 5. Comparison between the response of the halfspace problem and a layer over bedrock.

Inertia Effects. The effects of foundation inertia on the transient response are now investigated. The differential equations governing the transient response of the foundation possessing inertia can be readily incorporated at the stationary response given by equation (4) (Barros & Mesquita, 2001). The inertia is included with the use of an auxiliary dimensionless mass factor $B=M_f/M_{soil}$, which relates the mass of the foundation M_f, with an equivalent mass of soil M_{soil} existing in a semi-circle under the foundation, and defined by $M_{soil}=\pi\rho A^2$. Figure 7 gives the transient response of the foundation with inertia submitted to vertical Dirac´s delta, resting over a layer with deep H=1, and constant hysteretic damping factor η=0.2, the other layer proprieties are equal to those used in the previous analysis. Figure 7 shows that for larger foundation masses the final amplitude will be reduced and the peaks responses will be achieved times latter. Larger inertia factors tend to retard the rigid body motion of the foundation with respect to time. For higher B values, B=10 or B=25, the inertia effect leads to longer foundation oscillation periods compared to the smaller inertia factors.

Influence of the Damping. In Figure 8 the response for a constant hystheretic damping model for distinct values of η, ranging from η=0.05 to η=0.2 is reported. Other parameters are those of the previous example. It can be seen that an increase in the damping coefficient implies in a decrease of the amplitude response peak.

Analysis of Causality. It is well known that the FFT algorithm furnishes periodic responses. So the response for the largest time T_{max} is connected to the initial time T=0. This means that the response at T_{max} gives information about the causality of the obtained transient solution. Figure 9 shows the effect of the damping ratio η for the constant hysteretic model. It is well known that this is a non-causal viscoelastic model. In this model the static or quasi-static solution (A_o–>0) presents a non-vanishing complex part related to the material damping. In figure 9, as the damping decreases and the solution tends to the causal elastic case and the transient solution becomes increasingly causal. The results reported indicated that the present methodology may be used to assess the relation between damping models and causality.

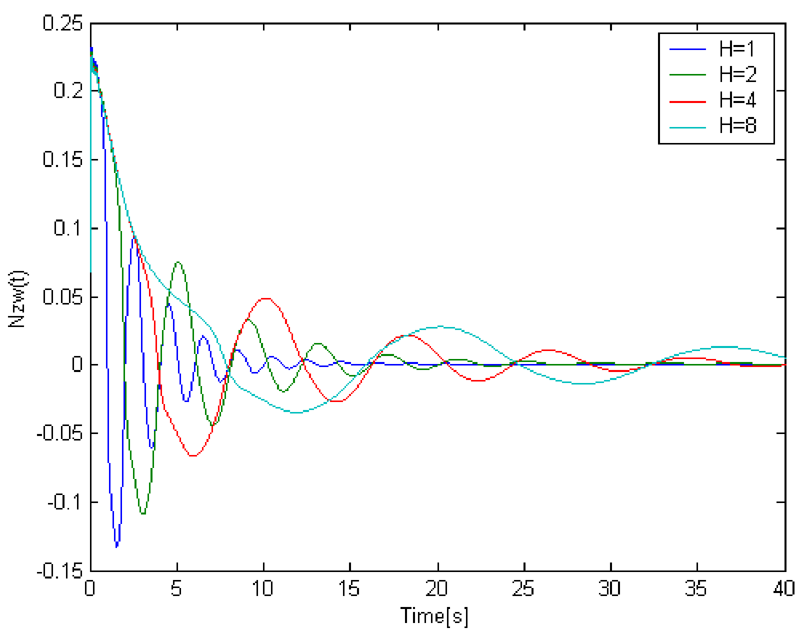

Figure 6. Effect of the layer depth on the transient response

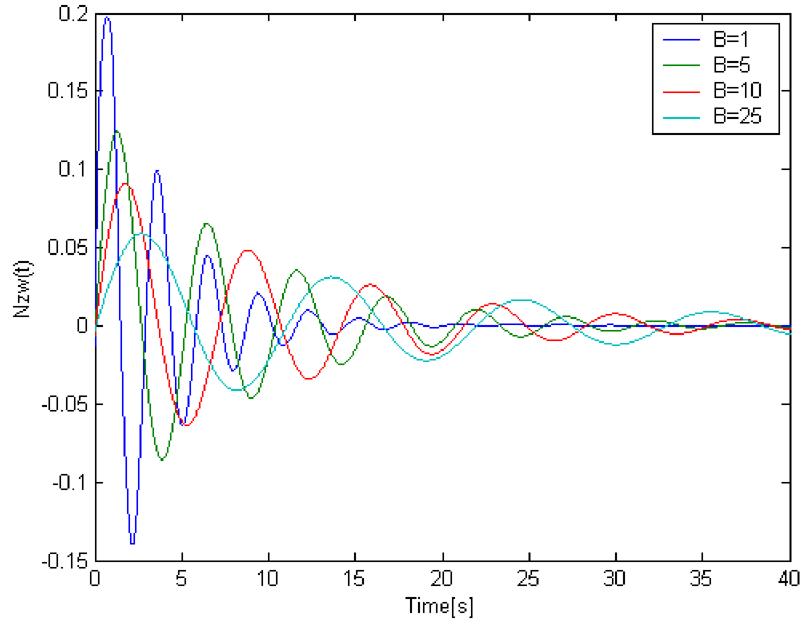

Figure 7.Transient Response to impulsive loading, foundation with mass.

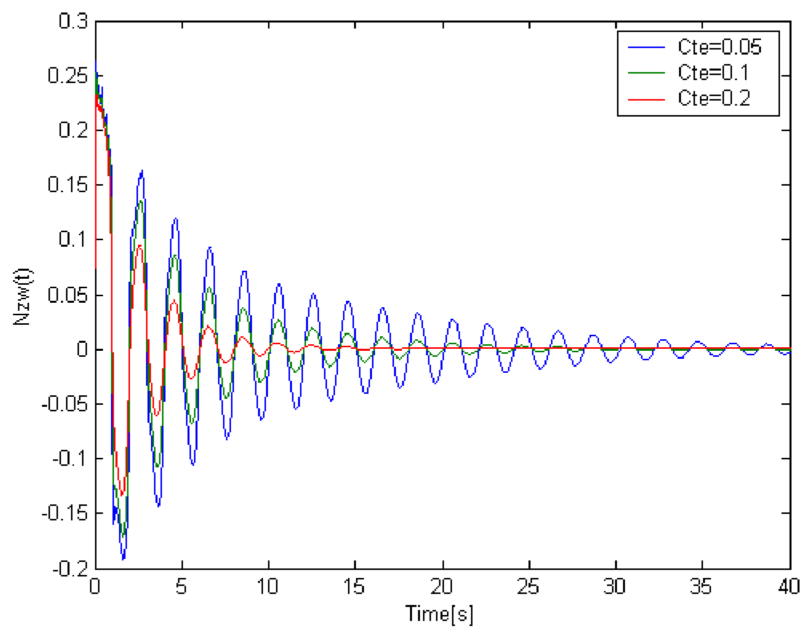

Figure 8. Influence off the damping coefficient

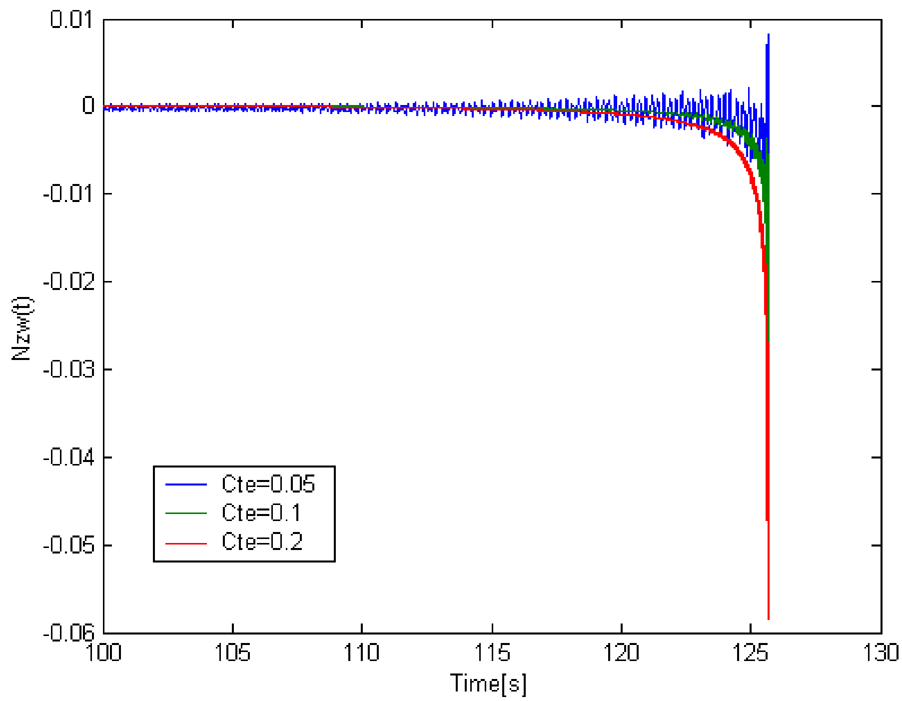

Figure 9. Analysis of causality- constant hysteretic model

5. CONCLUDING REMARKS

A methodology to synthesize the transient response of 2D rigid surface foundations resting on an isotropic viscoelastic layer is outlined in the present paper. The method is based on response functions calculated in the frequency domain and transformed to the time domain by the FFT algorithm. The procedure is applied to investigate the effects of distinct viscoelastic soil models and parameters. The effect of the damping coefficient on the causality of the time response is also reported. The influences of the layer depth are investigated. The foundation inertia influence is addressed.

6. ACKNOWLEDGEMENT

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7. REFERENCES

Barros, P.L.A., Mesquita, E., 1999, Elastodynamic Green's Functions for Orthotropic Plane Strain Continua with inclined Axis of Symmetry, International Journal for Solids and Structures, vol. 36, pp. 4767-4788.

Barros, P. L. A., Mesquita, E., 2001, On the Dynamic Interaction and Cross-Interaction of 2D Rigid Structures with Orthotropic Elastic Media Possessing General Principal Axes Orientation, Meccanica (2001), vol.36, n.4, pp.367-378.

Beskos, D, 1987, Boundary Element Methods in Dynamics Analysis, Applied Mechanic Review, vol. 40, n. 1.

Beskos, D, 1997, Boundary Element Methods in Dynamics Analysis: Part II (1986-1996), Applied Mechanic Review, vol. 50, n. 3.

Domingues, J., 1993, Boundary Elements in Dynamics. Computational Mechanics Publications, Southampton, UK.

Kane, J. H., 1994, Boundary Element Analysis in Engineering Continuum Mechanics, Prentice Hall.

Mesquita, E., Adolph, M., Barros, P.L.A., Romanini, E., 2002, Transient Green and Influence Functions for Plane Strain Visco-elastic Half-spaces. IABEM 2002, International Association for Boundary Element Methods, UT Austin, TX, USA, May 28-31, 2002, CD-ROM.

Richard Jr, F. E., Hall, J. R., Woods, R. D., 1970, Vibration of Soils and Foundations, Prentice Hall, Englewood Cliffs, New Jersey.