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LEAST COST CONTROL STRATEGIES IN URBAN DRAINAGE DESIGN- A DYNAMIC PROGRAMMING APPROACH

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Least Cost Control Strategies in Urban Drainage Design --

A Dynamic Programming Approach

by

Syver Froise

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Abstract

An approach to determine least cost control strategies in urban drainage network design has been developed. The networks were defined as combinations of conveyance and storage elements, where the conveyance elements are circular cross-section conduits, junction and diversion structures and pump stations. The storage basins are ponds and retention basins of different configurations. With the network layouts established through a preliminary planning process, it was postulated that there exists an unique least cost combination of conveyance and storage elements to each identified network alternative.

The problem was formulated as an N-stage sequential process for which dynamic programming is used to determine the least cost or optimal solutions. The dynamic programming approach, which is coupled with hydraulic simulation and cost-estimation models, is structured to handle serial and converging networks, and was shown to converge to the global optimum. The model has been programmed in FORTRAN IV, and has been implemented on a digital computer, where the stage variable solution order sequence is chosen such that computation and memory requirements become a minimum.

The hydraulic model consists of two modules; a design module and a simulation module. The desing module determines the geometric dimensions of the network elements, while the simulation module calculates flow propagations by implicit solutions to either the dynamic or the kinematic formulation of the one-dimensional gradually varied unsteady flow problem, depending on the characteristics of the network element under consideration.

Examination of hypothetical drainage networks demonstrated the feasibility of the developed approach and led to the conclusion that significant urban drainage cost reductions could be achieved by using these methodologies. Specifically, when compared to traditionally designed drainage networks, 30% cost reductions appeared to be feasible by using the developed approach. Of this saving, 40% was attributed to use of storage elements as a control option, and the remaining 60% achieved by optimal combinations of conveyance elements and use of advanced hydraulic simulation techniques.

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CHAPTER I

INTRODUCTION AND PROBLEM STATEMENT

> "There it is!" exclaimed my friend, who knew the mountain range like the back of his hand. Before us rose a mighty tower of granite separated from the main ridge by a cleft of softer rock.
> My friend shaded his eyes from the sun and pointed. "See that crack over there? It looks a little more difficult than the rotten gully, but if we can make it go, it will be a truly elegant climb" ---

1.1 Introduction

Urban drainage systems collect, transport and dispose of waste-waters from households, institutions and industries and surface runoff resulting from local precipitation. The first category has been labeled "waste water disposal", while the latter is referred to as "stormwater management". Waste water disposal usually includes some form of treatment which removes objectionable or potentially harmful constituents and produces an effluent that is acceptable for discharge into the receiving waters. Stormwater management, on the other hand, has traditionally consisted of collecting the runoff in gutters that discharge into a conveyance system of buried pipes and open ditches which transport the water to the nearest convenient outfall.

Drainage systems can be either "separate" or "combined"; systems are said to be separate when designed to carry only sanitary waste waters or storm runoff; discharges of other types of wastes into the collection network are prevented. Separate sanitary sewers convey the collected sewage to a waste water treatment plant while separately collected storm runoff is usually discharged without treatment.

The combined system, which was an inevitable economic measure when waterborne waste disposal was socially accepted, is designed to carry household and industrial waste waters together with surface runoff. Unfortunately, as rapidly growing urban areas were served by these systems, the concomitant increase in the amount of pollutants soon led to situations of intolerable receiving water nuisance. At the same time an urban population with increased leisure time and standard of living placed a higher demand on the same waters for recreational purposes.

The immediate remedy to this problem was to add intercepter sewers to the combined networks. Intercepters are designed to separate the "dry weather flow", which are the sanitary and industrial waste waters from the combined discharges, and transport these flows to a treatment plant or deep ocean outfall. As intercepter sewers usually are designed to carry three to five times the dry weather flow, while intensive or prolonged storms may cause combined flows in excess of 100 times the dry weather average, overflows subsequently take place. This pollution potential, the magnitude of which depends on the frequency and the pollutional loads of the overflows, was earlier regarded as an unavoidable concession to economy, but has in recent times been found intolerable by an environmentally aware public.

This pollution problem can be reduced by construction of separate systems in new districts and by sewer separation in districts with existing combined systems. However, this would be a very expensive undertaking since it has been estimated that implementation of separate systems would cost approximately $5 billions annually (ASCE Urban Hydrology Council, 1969), together with an additional $48 billion required for separation of existing combined sewers (Sullivan, 1970). Unfortunately, building of separate sewer systems and treating the sanitary and industrial waste waters would not completely eliminate urban water pollution. Where heavily urbanized areas are served by separate systems, storm runoff has been found to have pollutional characteristics similar to those of sanitary sewage, and it is conceivable that treatment of storm runoff from certain land uses will eventually be required; a situation suggesting use of combined sewer networks.

Traditional storm drainage design practices, which are basically aimed at collecting, conveying and discharging storm floods at peak runoff rates, have also contributed to local flooding and erosion problems. These problems could apparently be solved through provision of increased surface retention or by flow equalization within the collection network. This suggests use of storage or equalization basins in conjunction with the collection network. Such installations should, if properly designed and operated, result in decreased peak flows and significantly reduce the pollution potential by combined sewer overflows and erosion. In addition they could diminish local flooding hazards and allow for use of economically feasible stormwater treatment plants if such are required. Finally, a substantial overall drainage cost reduction could be anticipated since reduced peak flows would permit smaller conveyance facilities.

Design of drainage networks which include storage elements must be based on computational methods which follow the progress of individual storm hydrographs, that is, the method must be able to represent the gradually varied unsteady flow which occurs in storm drains. This approach, however, was unfeasible until high-speed electronic computers and suitable numerical computation techniques were available. Furthermore, the recent interest in applying operations research and system analysis techniques to planning of large scale systems has resulted in identification of a number of techniques that might be useful in urban drainage design.

This study is motivated by these circumstances and based on the hypothesis that urban drainage networks designed as combinations of storage and conveyance elements can have lower overall costs and are functionally better when compared to traditionally designed systems. The proposed approach necessitates use of sophisticated mathematical programming techniques and a hydraulic design based on transient flow phenomena. Fortunately, this is possible by proper use of systems analysis techniques, appropriate numerical methods and high speed digital computers.

1.2 Problem Statement

Traditionally designed urban drainage networks require vast capital commitments, and might be afflicted with functional deficiencies. However, use of detention storages within the collection network appears promising in reducing cost and in improving performance of both existing and new urban drainage systems. A cost reduction is made possible through reduced peak flows which permit smaller conveyance and treatment facilities; a situation which suggests that a least cost combination of storage and conveyance exists in a given situation.

Past efforts in drainage network modeling which included storage elements within the simulated system have either not optimized the use of storage or do not appear to be sufficiently comprehensive for general application. Earlier optimization approaches have also been based on hydraulic simulation of a selected single rainfall or runoff event, an approach which might lead to erroneous conclusions on system performance.

1.3 Research Objectives

The principal objective of this inquiry was to develop an optimizing method for design of urban drainage systems which includes the use of retention storage. Such drainage systems would consist of conveyance and storage elements; storage being provided by both internal and external units, i.e., unused conduit capacity and storage tanks.

Specifically, the study consisted of identifying and developing simulation models and algorithms to represent the various functional relationships that are involved and to make the design decisions and calculations leading to "optimal" solutions. As used in this study, optimization was taken to be identification of the best solution among all feasible alternatives based on least cost.

The inquiry addressed itself to the following five specific objectives:

Objective 1. Identify a suitable optimization technique for the problem.

Objective 2. Identify the interactive system elements and develop an optimization algorithm which determines minimum cost solutions to urban drainage networks.

Objective 3. Develop a digital computer program which applies the optimizing algorithm and demonstrates the feasibility of the approach.

Objective 4. Examine via sensitivity analysis the relationships between network costs and inlet hydrograph properties.

Objective 5. Based on the findings from the fourth objective, make specific recommendations on the method to be used and the steps that should be taken to model urban runoff hydrographs in drainage design studies.

1.4 Scope and Significance

This study differs fundamentally from earlier attempts to model or to optimize the design of urban drainage systems. Firstly, the hydraulic design is based on routing of individual flood waves and has the capability to simulate continuous hydrographs rather than be based on a "design storm," since the latter approach could give rise to erroneous conclusions of flooding or system overload probabilities.

Secondly, a "global" optimum is guaranteed since the model evaluates all feasible combinations of storage and conveyance elements, rather than only user selected combinations which might lead to a "local" optimum.

Thirdly, while the effects of inlet hydrograph properties on cost and performance of urban drainage networks are largely unknown at present, significant information on these relationships should result from experiences with the developed model.

1.5 Limitations

Several important issues can be raised regarding the method and subject of this inquiry. First, the optimization as proposed is based on capital commitments alone, while operation and maintenance are considerations as well. However, drainage networks are characterized by high initial capital requirements and annual operation and maintenance costs are generally accepted to be in direct proportion to the initial investment, thus justifying this comparison base.

Secondly, while several of the elements that comprise the urban drainage problem complex are probabilistic, this study is based on deterministic methods; any impression that a high degree of certainty is associated with the system is not intended.

Finally, although mathematically optimal solutions can be obtained by the use of models, the simplifying assumptions required to model actual systems might make the solution of any analytical model only an approximation to the solution that in reality is the best one (Loucks, 1969).

CHAPTER II

LITERATURE REVIEW

> "When you start on such a climb as this," he said, opening his pack, "you got to make sure you have all your ironmongery in just the right order." He clipped pitons and karabiners to his waist loop in some mysteriously predetermined order, coiled the rope with loving patience and finally closed his pack again. Hoisting it to his back, with the rope over his arm, he started along the broken ridge to the base of the pinnacle. I followed with a rather uneasy feeling in the pit of my stomach ---

2.1 Chapter Introduction

This literature review is divided into three sections and is concluded with a summary. The first section summarizes commonly used urban runoff hydrograph generation techniques; secondly, the problem description, system equations and solution techniques for functional design and hydraulic simulation of urban drainage network elements are identified; and finally, existing urban drainage models is critically reviewed and their utility in terms of the study objectives are examined. For the purpose of this discussion relevant model elements are shown schematically in Figure 2.1.

The model, which is developed through the course of this inquiry, provides a method of finding optimal solutions to urban drainage networks which include retention storages. Thus, the chapter summary attempts to identify the steps that have been taken and the techniques that have been used during the model identification phase.

2.2 Generation of Urban Runoff Hydrographs

The literature defines urban drainage systems in terms of "dry" and "wet" weather flows. Dry weather flows are waste waters from households, industrial, commercial and institutional installations and groundwater seepage or "infiltration" into the drainage network. Quantitative estimates of dry weather flows can be based either on actual measurements or on estimates of typical design flows from the specific sources that are listed above (ASCE, 1970).

Discussion of potentially valuable methodologies for storm runoff or "wet weather" flow estimation is complicated by the vast number of methods which have been suggested in the literature. Fortunately, an examination of the literature reveals the existence of only a few basic models; many methods are only modifications of these. Hence, this discussion will focus on the underlying concepts of urban runoff hydrograph generation techniques, and in particular, those that in terms of the study objectives are of interest to this inquiry.

Only a consideration of the "surface runoff" component of the runoff hydrograph is relevant in the general case, although interflow and groundwater may return to the surface downslope of the point of infiltration and enter the drainage system at a downstream location. The applicable techniques can then be classified as empirical relationships, regression models, frequency analysis, and hydrologic synthesis. Characteristic features of, and common methods belonging to each technique are summarized below.

Empirical Relationships

These methods rely on empirical methods and experience rather than on experimental data and theoretical procedures based on an understanding of fluid mechanics and physical characteristics of the watershed.

The empirical relationship most widely used is the so-called "rational method". Originally proposed by Mulvaney in 1851 and brought to the U. S. by Kuichling (1889), this method has been evaluated in some depth by Chow (1962), Hiemstra and Reich (1967), Schaake et al. (1967) and McPherson (1969). Gregory and Arnold (1932) modified this method to take into consideration the catchment shape and slope.

Used correctly, the rational method can be employed to estimate the peak discharge for a given drainage area, given that the rainfall lasts sufficiently long to allow the area under consideration to reach equilibrium. The rational method itself does not provide the answer to how long this time period is. Furthermore, as the rational method cannot generate continuous runoff hydrographs, its usefulness in modelling urban runoff phenomena is limited.

Other empirical relationships are "The Los Angeles Hydrograph Method" (Hicks, 1944), "Horton's Method" (Horton, 1935) and "The Johns Hopkins University Inlet Method" (Kaltenback, 1963). Common limitations to these techniques are their inabilities to define time to equilibrium, or peak flow, if equilibrium is not obtained, as well as being restricted to uniform storm intensities.

Regression Models

Regression models attempt to relate a causal factor with an effect through use of statistical correlation. Used in hydrology, precipitation and watershed characteristics are related to peak flow and runoff. Based on more data and more sophisticated methods of analysis, these methods are an extension of the empirical relationships. A study on the use of regression models in hydrology was undertaken by Potter (1961).

Regression models are unique for the area and the conditions for which they were derived. Thus, they cannot be developed for a specific watershed during rural conditions and used as the watershed becomes gradually urbanized. This fact together with lack of adequate data on urban stream flow makes regression analysis an unpromising approach to the solution of urban storm runoff problems.

Frequency Analysis

These techniques are used to interpret a past record of events in terms of future probabilities of occurrence. Given a sufficiently long streamflow record at a particular point in a watershed, frequency analysis becomes a powerful tool to determine the probability of peak flows for drainage design.

A frequency analysis method by Anderson (1968) appears to be the only such technique specifically proposed for urban areas to date, but his work is limited to Fairfax County, Virginia.

Unfortunately, lack of streamflow data has prohibited the use of frequency analysis in urban drainage design. Even if such data were gathered, they would be of limited utility because progressive urbanization would have changed the hydraulic characteristics of the watershed and left the observed data inhomogeneous.

Hydrologic Synthesis

These methods have been designed to overcome the problem of limited hydrologic data. Table 2.1. shows a classification by Linsley (1971) of the techniques falling into this group. The listed characteristics indicate that only water balance models, linear systems methods and physical analysis of flow would be of interest to this inquiry and need further review.

Water balance models are based on maintaining a continuous account of the water in soil moisture storage, and are consistent with infiltration theory in which the infiltration capacity is a function of the soil moisture storage. These techniques require a large amount of computation and could not be used until high speed computers were abilable. Water balance models with appropriate functions to simulate interception, depression and detention storages, infiltration and soil moisture movement, appear to be a promising approach for calculating storm runoff, at least for the rural watershed. Unfortunately, these models are usually based on lumped properties of the watershed, rather than on the physical properties constituting it, thus requiring observed data for calibration. This property limits their usefulness in urban drainage design, especially where modifications to the watershed have not yet been made.

The linear system, or unit hydrograph concept was introduced by Sherman (1932) and postulates that shapes of hydrographs resulting from rains of the same durations but different amounts are similar since the physical characteristics of the basin are constant.

All applications of the unit hydrograph concept involve the same basic assumptions and use of the convolution integral:

$$q(t) = \int i(t)\ h(t-\tau)\ d\tau \qquad (2.1)$$

where

$h(t-\tau)$ = basin response or instantaneous unit hydrograph, cf/in-sec

$i(t)$ = "excess" of rainfall hyetogram

$q(t)$ = runoff hydrograph, cfs

Given that the watershed is unchanging in its physical characteristics, there is no reason why reliable estimates could not be obtained by this technique provided that a scheme for handling nonlinear effects was incorporated. Unfortunately, data on urban runoff are not generally available and a program of gaging would be required where a design flow is needed. Clearly, the design problem cannot be resolved in this fashion since the flows cannot be measured in non-existent sewers.

This lack of urban runoff data has prompted suggestions for constructing "synthetic" unit hydrographs, using the length, slope and imperviousness of the basin (Snyder, 1938; Espey, et al., 1965; Eagleson, 1969). However, the hydraulic character of the basin is still poorly defined and the result remains uncertain at best. Thus, models based on the linear system concept do not appear to serve the need of urban hydrology.

Physical analysis of flow treats the runoff problem as one of fluid mechanics. Use of these methods requires the drainage area to be divided into subcatchments with similar physical characteristics and identifiable boundary conditions. It is then possible to calculate the runoff hydrograph at the downstream end, provided the inflow hydrograph and the rainfall hyetogram are known. These calculations are usually coupled with estimates of infiltration losses.

Overland flow is generally assumed to be two dimensional, or take place in a thin sheet of infinite width. Overland flow routing has been based on the complete equations of continuity and momentum (Morgali and Linsley, 1965), the kinematic wave concept (Lighthill and Witham, 1955; Wooding, 1965a, 1965b, 1966; Henderson and Wooding, 1964), and on storage routing principles (Izzard, 1946).

The use of physically based routing methods in urban storm runoff analysis offers advantages like elimination of arbitrary routing constants and assumptions of linearity.

2.3. Hydraulic Simulation of Urban Drainage System Elements.

Problem Description

Drainage conduits are generally designed to have a free surface subjected to atmospheric pressure and should be treated hydraulically as open channels. Thus, the forces causing the flow are due to gravity only, rather than to some external head. This complicates the numerical computations of drainage network flows since the position of the free water surface is likely to change in time and space and also by the fact that the depth of flow, the discharge, and the slopes of the conduit bottom and the water surface are interdependent.

Hydraulic design of storm and combined sewers has traditionally rested on the simplifying assumption of steady uniform flow throughout the network. However, as the flows actually are unsteady and non-uniform, the "steady uniform" design approach neglects several important factors. The most important ones among these have been summarized by Yevjevich and Barnes (1970a) as:

1. Drainage conduits have storage capacities which attenuate flood peaks.
2. Unsteady flow through various drain sections mutually interact such that floods from local heavy storms of limited coverage are significantly alternated in an extensive grid of drains; especially if the conduit slopes are small.
3. As the flood waves progress along the drain conduits, the flood wave modifications due to dynamic effects should not be neglected.

Although storm drainage design based on the assumption of steady uniform flow might be adequate for small projects, such an approach has not, besides the weaknesses summarized above, the capability of supporting design of drainage systems which include storage elements. To facilitate design of such systems, hydraulic routing techniques, or algorithms which follow the progress of individual storm hydrographs are required; the method must be able to represent the gradually varied unsteady flow which occurs in the drain conduits.

Such a computational tool is given by the one-dimensional equations of continuity and momentum, or the so-called "St. Venant" equations, which can be written in dimensionless form as:

$$\frac{A}{VB}\frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V}\frac{\partial y}{\partial t} - \frac{q}{VB} = 0 \tag{2.2}$$

$$\frac{\alpha V}{g}\frac{\partial V}{\partial x} + \frac{\beta}{g}\frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} = S_o - S_f - \frac{\beta V q}{Ag} \tag{2.3}$$

where

A = cross section area of flow, ft^2

B = water surface width, ft

S_o = channel bed slope, ft/ft

S_f = friction slope, ft/ft

V = average velocity, ft/sec

g = gravitational acceleration, ft/sec^2

t = time, sec

x = length along channel, ft

y = water depth, ft

q = distributed lateral inflow, cfs/ft

α = energy distribution coefficient

β = momentum distribution coefficient

A complete derivation of these equations has been given by Chow (1959).

Solution Techniques

Due to inherent mathematical difficulties, exact integration of these equations is practically impossible at this point in time. However, a number of approximate solution procedures have been developed. All of these fall into three categories and are referred to as analytical, graphical and numerical methods.

Analytical solutions are possible only for simplified approximations to equations 2.2 and 2.3. Their utility for the current problem is limited since simplifications which permit analytical solutions might result in departures from the real physical conditions and in invalid solutions. Some discussions and abstracted references on these methods have been given by Yevjevich (1961).

Graphical solution methods have limited application because of the labor requirement involved. However, they might be useful for visualization of digital computer schemes and results obtained by numerical solution procedures.

The St. Venant equations, with appropriate initial and boundary conditions, have been solved numerically by various techniques (Amein, 1968; Gunaratnam and Perkins, 1970; Strelkoff, 1970; and Yevjevich and Barnes, 1970d) The experience gained by using these methods indicates that no one is superior to the other ones under all conditions, but rather that any one scheme might exhibit instability and convergence problems under certain flow conditions (Balzer and Lai, 1968; Gunaratnam and Perkins, 1970).

Numerical solutions can proceed in three general directions:

1. Explicit finite difference schemes, where equations 2.2 and 2.3 are written as linear algebraic equations in finite

difference form which allows explicit evaluation of the unknowns.

2. Implicit finite difference schemes, in which equations 2.2 and 2.3 are written in finite difference form as a set of nonlinear algebraic equations, from which the unknowns are found simultaneously.

3. Method of characteristics, in which equations 2.2 and 2.3 are transformed into four ordinary differential equations, and then written in finite difference form for solution.

Different computational schemes have been proposed for each of these three general solution categories (Richtmeyer, 1962; Amein, 1966; Strelkoff, 1970; and Gunaratnam and Perkins, 1970). Findings pertaining to the convergence, stability, computational speed and simplicity of these methods have been reported by various investigators (Liggett and Woolhiser, 1967; Balzer and Lai, 1968; Strelkoff, 1969; and Yevjevich and Barnes, 1970c).

Explicit Finite Difference Schemes

These schemes are popular because of their directness in expressing the unknowns. Unfortunately, they are afflicted with computational stability problems. Most investigators that use these techniques appear to choose spatial and temporal computation intervals in accordance with the Courant criterion, which postulates that

$$\Delta t \leq \frac{\Delta x}{[V \pm C]} \qquad (2.4)$$

where

C = wave celerity, fps

V = mean velocity of flow over the channel cross section, fps

Δt = computational time increment, sec

Δx = computational spatial increment, ft

However, the Courant criterion is strictly speaking applicable only to explicit formulations which follow the method of characteristics; satisfying the Courant criterion does not necessarily guarantee computational stability (Liggett and Woolhiser, 1967; Gunarathnam and Perkins, 1970).

In drainage network design, the spatial increments are limited to distances between successive nodes. The subsequent limitation on the time increment may therefore result in lengthy computations and render this approach impractical. Furthermore, Yevjevich and Barnes (1970d) showed that for rapidly varying transient flows, explicit methods are inferior to the method of characteristics when accuracy is considered.

Implicit Finite Difference Schemes

These schemes appear to be unconditionally stable (Strelkoff, 1970), and consequently, the computational steps of the independent variables, Δx and Δt, can be chosen independently. However, convergence conditions may limit the maximum increments of these variables.

When "flood wave" propogations in open channels are calculated by numerical methods, the total channel length is divided into N interior reaches, and the continuity and momentum equations, equations 2.2 and 2.3, are written for each reach. This, together with the upstream and the downstream boundary conditions constitutes a system of 2(N + 1) non-linear equations for the solution of the 2(N + 1) unknowns. The unknowns are v_i^{j+1}, y_i^{j+1}, for i=1,2,......,N+1, where v and y are velocities and flow depths, respectively. The upstream boundary is designated by i=1, the interior reaches by i=2,...., N, and the downstream boundary by

$i=N+1$. The superscript $j+1$ indicates the time step where the solution is sought. The generalized Newton iteration method can be used to solve the system (Ralston, 1965).

The reviewed literature indicates that implicit schemes permit numerical solutions over large time steps, but require the solution of large sets of simultaneous algebraic equations at each step. Results from several sources (Sevuk and Yen, 1973) also concluded with a preference for implicit solution schemes for slowly varying long floods when computational time and stability were considered. For rapidly varying and short duration flows, however, the advantage on computer time the implicit scheme has over the explicit scheme might diminish because of timestep limitations imposed by numerical convergence considerations.

Method of Characteristics

The St. Venant equations can be transformed into four ordinary differential equations through an appropriate transformation of coordinates. These equations, which are referred to as the characteristic equations take the form:

$$\frac{dx}{dt} = V + (g\ A/B)^{1/2} \tag{2.5a}$$

$$\frac{dv}{dt} + \frac{dy}{dt}(g\ B/A)^{1/2} = g(S_o - S_f) \tag{2.5b}$$

$$\frac{dx}{dt} = V - (g\ A/B)^{1/2} \tag{2.6a}$$

$$\frac{dv}{dt} - \frac{dy}{dt}(g\ B/A)^{1/2} = g(S_o - S_f) \tag{2.6b}$$

where:

A = cross section area of flow, ft^2

B = water surface width, ft

S_f = friction slope, ft/ft

S_o = conduit bed slope, ft/ft

g = acceleration of gravity, ft/sec^2

t = time, sec

V = average velocity over cross section, fps

x = distance along longitudinal direction of channel, ft

y = depth of flow, ft

The characteristic equations can be written in finite difference form and solved numerically by implicit or explicit shemes on fixed or characteristic grids. The characteristic grid provides high accuracy (Amein and Fang, 1969; Gunaratnam and Perkins, 1970; Liggett and Woolhiser, 1967), but requires two dimensional interpolations which necessitates a considerable amount of bookkeeping and might make the computation inefficient (Gunaratnam and Perkins, 1970). Therefore, a fixed rectangular grid is usually preferred. For all fixed grid schemes, and explicit solutions, the grid size is restricted to conform to the Courant stability criterion given by equation 2.4.

Comparison of Numerical Solution Schemes

The relative merits of the computational methods are usually evaluated on the stability and convergence properties, and the computational speed or time requirement of a particular scheme.

A finite difference scheme is considered computationally stable if small truncation and round-off errors do not amplify in successive computations. Work by Strelkoff (1970) using the von-Neuman technique and additional information by Liggett and Woolhiser (1967) and Gunaratnam and Perkins (1970), indicate that explicit schemes are conditionally stable, whereas, implicit schemes appear to be unconditionally stable, but still subject to the limitations imposed by convergence considerations. In other words, values of Δx and Δt can be chosen independently for the implicit schemes.

For explicit schemes, small steps of Δx and Δt must be used to ensure computational stability, but the fact remains, no universal criterion exists to determine the limits of Δx and Δt that guarantees such stability.

Convergence is usually referred to as the ability of the finite difference scheme to approach the analytical solution of the partial differential equation, thus, it is an indication of accuracy. However, considering the St. Venant equations, the analytical solution is unknown and some indirect means must be taken for convergence considerations. Sevuk and Yen (1973), considered 1) capability to reproduce initial conditions for steady state boundary conditions; and 2) capability of maintaining flood volume balance, and found both implicit and explicit schemes capable of satisfying the first criterion, irrespective of computational step size. Using flood volume balance as a criterion, Sevuk and Yen (1973) also found the implicit scheme for direct and canonical characteristic forms and the second-order characteristic scheme to be the most satisfactory ones. A similar conclusion was arrived at by Yevjevich and Barnes (1970a).

The computer time for explicit schemes depends mainly on the time step, Δt, which is determined by some stability criteria and as a function of the spatial increment, Δx. However, the maximum values of Δx that can be used are often limited by the channel geometry and could result in small time steps and long computation time. The computation time for implicit schemes depends mainly on the number of iterations that are required to generate the solution. This number depends on the value of Δt and to a lesser degree on the value of Δx.

Applied to drainage network design, solutions by the complete equations of continuity and momentum are often too time consuming and therefore undesirable; simplified routing techniques are consequently employed. These methods are commonly based on storage or kinematic wave routing techniques, and might be quite attractive where the dynamic effects are negligible.

Storage routing methods use only the continuity equation and are based on the principle that for any reach of channel the difference between the inflow and the outflow is equal to the stored or depleted water in a given time interval. Yevjevich and Barnes (1970a), however, found storage routing methods to be of limited utility in storm drainage design. The fact remains that their accuracy depends on the accuracy of selecting the basic difference factors Δx and Δt apart from the inaccuracy resulting from using only one equation. A similar conclusion was arrived at by Harris (1970).

Another group of simplified routing methods are those which are based on omitting terms in the momentum equation. These methods generally pre-suppose a balance between gravitational and friction forces, and the flow is then called kinematic. This means that the derivatives in the dynamic equation are negligible when compared to the effects of gravity and the effect of friction, i.e. the friction gradient can be equated to the channel bed slope. Thus, the kinematic scheme can use the following equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (2.7)$$

$$Q = ay^{b} \qquad (2.8)$$

where:

A = water cross section, ft^2

Q = discharge, cfs

a = coefficient

b = coefficient

t = time, sec

x = length along channel, ft

Kinematic wave routing can be done numerically by the method of characteristics or by replacing the partial derivatives by finite differences. Although kinematic wave theory has been worked upon by a number of investigators (Wooding, 1965a, 1965b, 1966; Eagleson, 1970; Harley et al., 1970), quantitative information with regard to their computational economy and accuracy when compared to the equations of continuity and momentum, to this author's knowledge has not been published.

The friction slope, S_f, which is included in equation 2.3, and in the kinematic wave problem, can be evaluated by any uniform flow formula. This inquiry, however, bases these calculations on the Darcy-Weisbach formula. This is in accordance with recommendations by the Committee for Hydrodynamics of the Hydraulics Division of American Society of Civil Engineers (ASCE, 1963).

The Darcy-Weisbach formula postulates that

$$S_f = \frac{f}{g} \frac{V^2}{8R} \tag{2.9}$$

where

R = hydraulic radius, ft

S_f = friction slope, ft/ft

V = average velocity of flow, fps

f = Darcy-Weisbach friction factor, dimensionless

g = acceleration of gravity, ft/sec^2

The friction factor, f, varies with pipe roughness and the Reynolds number, which is defined as:

$$N_R = \frac{R \cdot V}{\nu} \tag{2.10}$$

where

N_R = Reynolds number, dimensionless

R = hydraulic radius, ft

V = average velocity of flow, fps

ν = kinematic viscosity, ft^2/sec

The actual relationship between the Darcy-Weisbach friction factor, Reynolds number and pipe roughness depends on the state of the flow which is governed by the effects of viscosity and gravity relative to the inertial forces of the flow. Depending on the effects of viscosity relative to inertia, the flow may be laminar, turbulent or transitional. However, laminar flow occurs rarely in storm drains, and only turbulent flow needs to be discussed. The turbulent flow region consists of three regimens; an hydraulically smooth, a transitional, and an hydraulically rough regimen. Hydraulically smooth conditions exist when the laminar sublayer completely submerges the effects of the channel wall roughness. von Karman (Daugherty and Franzini, 1965) developed an equation for the friction factor, f, for such cases as

$$1/\sqrt{f} = 2 \log_{10} \mathrm{NR}\sqrt{f}/4 - 0.8 \qquad (2.11)$$

where the Reynolds number, NR, is given by equation 2.10. This equation is valid for any circular conduit as long as the thickness of the laminar sublayer is at least 1.7 times the surface roughness and for all smooth conduits when the Reynolds number exceeds 1,000. Blasius (1913) also showed that for Reynolds numbers between 750 and 25,000, the Darcy-Weisbach friction factor may be expressed approximately as

$$f = 0.223/\mathrm{NR}^{.25} \qquad (2.12)$$

At high Reynolds numbers the thickness of the laminar sublayer decreases, and it has been found that conduits behave as wholly rough pipes when the laminar sublayer is less than 8% of the depth of the surface roughness. For such cases the friction factor is independent of the Reynolds number, and von Karman found that the friction factor could be expressed as

$$1/\sqrt{f} = 2 \log_{10} 2R/\varepsilon + 1.74 \qquad (2.13)$$

In the transition zone, the relationships for hydraulically smooth and wholly rough flows do not apply. Colebrook (1939) found an approximate relationship for this intermediate range:

$$1/\sqrt{f} = -2 \log_{10} (\varepsilon/R \cdot 1/14.83 + 0.63/\mathrm{NR} \cdot \sqrt{f}) \qquad (2.14)$$

where ε is the wall roughness in feet and the other variables are defined above.

Hydraulic Modeling of Junction Structures

Backwater effects, energy losses due to junction geometry and the mathematical complexity of the St. Venant equations makes flood routing through channel junctions very difficult. Thus, a number of inquiries aimed at deriving general expressions for energy losses in junction structures do not appear to have yielded the intended results (Taylor, 1944; Sangster et al. 1958; Yevjevich and Barnes, 1970a,b,c). Even for steady flow, as stated by Chow (1959),

> "The problem is so complicated that only a few simple cases have been studied. The conclusions of such studies indicate that generalization of the problem is not possible or even desirable."

Due to the different hydraulic regimes encountered, several types of junctions need to be considered. In terms of the study objectives, however, junctions which include storage capacities can be eliminated.

The "Sequential type junction" is approximated by the continuity relationship only. The junction is assumed to be represented by a single confluence point without storage capacity; the continuity equation can therefore be written as:

$$Q_o - \sum_{i=1}^{N} Q_i = 0 \qquad (2.15)$$

where

N = total numbers of inflow branches

Q = discharges, cfs

i = subscript indicating inflow branch

o = subscript indicating outflow branch

This approximation has the advantage of computational simplicity, but ignores backwater effects and fails to account for conservation of energy and continuity of water surfaces. Furthermore, as storage changes are neglected, the flow through the junction is implicitly assumed to be instantaneously steady; unsteady effects are not accounted for directly.

A "Drop type junction" assumes an elevation drop between the inflowing and outflowing channels. Consequently, the upstream channel flow can be computed, since the brink section is the one of minimum energy. The effluent channel discharge is again given by equation 2.15. Thus, this approximation is actually a special case of the sequential junction, and both have essentially the same drawbacks and limitations.

The "Point type junction" considers both flow and water surface continuity. The latter relationship, which is a special case of energy consideration, is imposed as:

$$h_o = h_i;\ i = 1, \ldots, N \tag{2.16}$$

in which h is the water surface elevation above a horizontal reference datum, the subscripts o and i refer to outflow and inflow channels, respectively, and N is the total number of inflow channels. The flow continuity is again given by equation 2.15.

Numerical evaluation of this approximation is more involved than those for either of the two previously discussed types. Although equation 2.16 partially and approximately accounts for backwater effects, the junction is still considered as a point without storage. Hence, unsteady effects within the junction cannot be accounted for.

Rules of thumb rather than attempting exact solutions have usually been used to ensure sound hydraulic design of junction structures. Some of these approximations, which also include allowances for energy losses, have been summarized by Fair and Geyer (1954):

1. $H_i = 1/2\ (D_2 - D_1)$ for $D < 24''$ (2.17)

   $H_i = 3/4\ (D_2 - D_1)$ for $D \geq 24''$ (2.18)

   where

   H_i = drop through the junction structure

   D = conduit diameters

2. Keep the "0.8. diameter line" continuous on the principle that it represents the line of maximum velocity.

3. Base flow calculations for junction structures on friction factors or roughness coefficients larger than those used for the channel links.

4. Allowing 0.1 ft drop in through manholes, 0.2 ft drop in the presence of one lateral or bend; and 0.3 ft drop for two laterals.

2.4 Existing Urban Drainage Models

Mathematical models used in design and performance evaluation of urban drainage networks can, according to their objectives, be classified either as simulation or optimization models. The simulation models are synthesized from mathematical or logical statements and are used to predict prototype behavior as responses to applied impulses or forcing functions. Mathematical optimization models, on the other hand, are procedures developed to find the solution defined as being best among all feasible alternatives. Thus, optimization models have, in addition to a mathematical and logical description of the prototype, the added sophistication of an optimizing algorithm.

Elements of Urban Drainage Models

Urban drainage models consist of identifiable functional units or model blocks. Among the sub-models usually included is a runoff model which generates runoff hydrographs at specified points within the drainage area, and an hydraulic model which simulates flow progression through the drainage network elements. More comprehensive models, which consider quality as well as quantity aspects of urban drainage, might also include model blocks to simulate treatment processes and receiving water quality. As this inquiry is addressed to the quantity aspects of urban drainage network design, only techniques related to generation of runoff hydrographs and to hydraulic design and simulation of network components will be discussed.

Existing Urban Drainage Models

Linsley (1971) estimated that the literature contains description of more than 100 storm runoff models, but very few of these were specifically proposed for urban conditions; even fewer have the capability of hydraulic simulation of man made conveyance systems. This review covers twelve different models.

Among the twelve models that have been examined, all are computer based and have, with the exception of three, been built specifically for urban applications. A summary of the most important model characteristics is given in Table 2.2.

The Environmental Protection Agency, EPA, Storm Water Management Model

(Metcalf & Eddy et al., 1971) is based on the design storm concept, and for "simulating storms which cover a time period of less than one day." The model assumes the watershed to be divided into a series of rectangular subcatchments for which runoff is derived by subtracting infiltration losses computed by Horton's equation from the applied rainfall hyetograph. If the depth of the rainfall excess exceeds a specified detention depth, overland outflow is calculated by Manning's equation. Overland flow enters the gutters and the gutter outflow is again calculated by Manning's equation and a continuity relationship. Dry weather flows and flow quality can be provided as input data or computed from land use characteristics. Hydraulic simulation of the collection network, which could include one major storage installation as an option at the outfall, is based on the kinematic wave concept. Quality routing is coupled with the quantity routing. The model also has the option of providing for treatment at specified overflow points, as well as receiving water quality computations, and prints out quality improvements and treatment costs.

The EPA model is thus a simple infiltration model with kinematic routing of overland, gutter and conduit flows. Tests on only a few storms are reported; these show reasonably good replication of observations. The model cannot perform continuous simulation and assumes empty storage volumes at the onset of the design storm. Thus, the model is unable to define the flooding probability of the system.

The Water Resources Engineers, WRE, Storm Drainage Models, were developed for stormwater runoff studies in San Francisco and Seattle (Roesner et al., 1971) and are an outgrowth of the previously described EPA Stormwater Management Model. The solution techniques used in this model are essentially indentical to those in the EPA model and will not be reviewed here unless they differ fundamentally.

The WRE models omit detailed simulation of gutters and sidesewers, and should, when compared to the EPA model, be better suited for simulation of large urban areas.

The WRE models are claimed by their developers to simulate backwater and surcharges in the collection network. In contrast to the EPA model, which ascribes the property of volume to conduits and subsequently applies the continuity equation to these elements, the WRE models ascribe the property of volume to junctions. The numerical solution technique is based on the improved Euler method (Carnahan, et al. 1969) and consists of applying the continuity equation with storage to each junction in the system and to compute the hydraulic head at each of these elements. These heads, together with friction and momentum forces are used to compute the flows through the conduits; thus, the backwater effects can be approximately propagated through the system. This method has been found to be computationally inefficient (Brandstetter, 1974b). The WRE models prompt conclusions identical to those previously given on the EPA model.

The Stanford Watershed Model, SWM, is the oldest and probably the most well known comprehensive computerized watershed model in existence today. The SWM has been revised and apparently improved several times but only version IV needs to be discussed (Crawford and Linsley, 1966).

The SWM is a continuous water balance model which simulates intercepted water volume as a function of watershed cover and infiltration and as a variable function of soil moisture, where the infiltration capacity is assumed to vary linearly over the watershed. Surface retention is simulated by separating overland flow into various components; overland flow routing is based on the assumption of turbulent flow. Impervious areas are represented by a pre-set factor which represents the "impervious" fraction of the total watershed that is adjacent to or connected to stream channels. The channel flow propagation is calculated by delaying the land surface flows by time-area histogram and routed through a linear reservoir at the outlet.

The SWM's usefulness in urban drainage design is questionable. The functions controlling the various processes in the SWM are fixed in shape and adjusted in level by input parameters which are determined by trials for each watershed. Thus, the SWM requires historical runoff data and is, therefore, of limited utility in urban drainage design. Clearly, one cannot make flow observations in non existant sewers.

The Hydrocomp Simulation Program, HSP, (Hydrocomp, 1969) is an outgrowth of the Stanford Watershed Model, and the simulation techniques that are used in these two models and of interest to this review are quite similar. The techniques applied in HSP are, therefore, described only where they differ significantly from those used in SWM.

The HSP has added a relatively sophisticated routine which performs data management for hydrometerologic data which uses direct access to computer disc storage.

Runoff calculations by the HSP are based on the same principles as those the SWM uses, but the HSP contains fewer input parameters and some algorithms have been reprogrammed to improve the computational efficiency of the model.

Channel routing is stated to be based on the kinematic wave concept, and allows for flood wave routing through reservoirs, diversions structures and natural or man-made trapezoidal or circular channels.

These elements can be simulated in any order. The reservoir simulation assumes level pools and storages being a function of reservoir elevations. Rule curves and discharges as a function of reservoir levels and time of the year must be specified.

The HSP is a continuous water balance model which utilizes non-linear routing procedures. Compared to SWM, HSP also contains a more "refined" description of man-made channels. However, the fact remains, the HSP requires historical data for calibration and is therefore of limited utility in urban drainage design.

The Massachusetts Institute of Technology, MIT, Catchment Model (Harley et al., 1970) is a relatively complex model which is based on relatively involved programming logic and concentrates on the fluid routing processes. The watershed is subdivided into simple planes that are used to model the overland flow by a kinematic wave technique which is solved by the method of characteristics. A linearized solution of the complete equations of continuity and momentum is provided as an option in cases where the kinematic solution would not be appropriate. In its present form the MIT Catchment Model includes a procedure based on theory of flow through porous media for calculating infiltration as a loss from overland flow and evaporation losses from either land or a free water surface. However, these functions are not presently part of a water balance computation but simple computations of losses from a given rainfall excess.

The MIT Catchment Model includes provisions for flood wave routing through partially filled circular pipes. Again, the routing can be done either by the kinematic wave method, solved by the method of characteristics, or by a linearized version of the complete equations of continuity and momentum.

The model was originally tested only on two wholly impervious areas of less than two acres in size, and excellent reproduction of observed flows was obtained. Two later applications of the model which have been published by Schaake et al. (1973) also show good reproduction of observed flows.

The University of Cincinnati Urban Runoff Model, UCURM (Preul and Papdakis, 1970) simulates runoff resulting from a preselected "design storm." Separate submodels simulate infiltration, surface retention, overland flow, gutter flow and pipe flow.

Infiltration simulation is based on Horton's exponential relationship (Horton, 1940) modified as suggested by Tholin and Keifer (1960) for cases where the initial precipitation is less than the initial infiltration capacity. The model neglects losses due to interception and evapotranspiration and uses a method based on work by Linsley et al. (1949) to model depression storage. Overland flow routing is based on a storage method by Crawford and Linsley (1966). The spatially varied and unsteady gutter flows are approximated by uniform flow equations for routing and the inflows are "routed" through the collection network simply by shifting the hydrographs by a time step corresponding to the average flow time for the hydrograph and the pipe under consideration.

In conclusion, the apparent oversimplification in modeling the prototype behavior seems to make this model of limited value in most design situations.

Merritt's Optimization Program (Merritt, 1970) differs from the previously reviewed models in that it is an optimizing algorithm which determines the least cost combination of pipe sizes and trench depths for a specified drainage network. The optimization is based on discrete dynamic programming, where the hydraulic model is based on the assumption of uniform flow.

The Norwegian Institute for Water Research, NIVA, Model (Lindholm, 1973) consists of separate submodels for simulation of sewer networks, treatment plants and sludge treatment as well as a model for cost calculations. Only the sewer network model will be reviewed here.

Overland flow supply rate is assumed to be a percentage of the rainfall, and is calculated by using a runoff coefficient and runoff hydrograph generation is based on a linear routing procedure. Runoff quality is computed as a function of time after start of rainfall with the total quantity of pollutants given as input data. Flood wave routing through the pipe network is done by a storage routing procedure where the hydraulic calculations for the sewer network are made under the assumption of steady uniform flow conditions at each time step. No comparison with actual data has been given, and it is impossible to judge the accuracy of this model.

The Battelle Model (Brandstetter et al. 1973) was designed to facilitate optimum design of additions to and control of the drainage network of the City of Cleveland. The model provides a method of finding the least cost combination of user specified alternatives of new facilities that are needed to reduce waste discharges to specified amounts. The variables are pipe sizes, treatment plants, and storage volumes. The design optimization is based on a modified gradient technique which is used in conjunction with a dynamic programming optimization of optimum control for a fixed design.

The model simulates retention storages by exponentially decaying functions of time after an initial loss is satisfied. Infiltration losses are approximated by Holtan's infiltration equation as modified by Huggins and Monke (1966) and the precipitation excess is convoluted with a unit hydrograph to determine the storm runoff of the catchment area. The unit hydrographs for ungaged catchments are derived by a method developed by the Soil Conservation Service (Kent, 1971) and are calculated using physical catchment characteristics which include drainage area, length, slope, soil and vegetation.

The network flow routing is accomplished by a kinematic wave technique which is solved by the method of characteristics. The present version simulates flow in circular pipes only; backwater effects are neglected.

The computational efficiency and accuracy of the Battelle Model are not known and it is neither proved nor apparent that the design optimization converges to the global optimum. The computation requirements could also be excessive.

The Illinois Storm Sewer System Model, ISSM, is essentially a Ph.D. dissertation by Sevuk (Sevuk et al., 1973) and concentrates on routing user specified runoff hydrographs through a branched collection network. The routing problem is based on the complete St. Venant equations which are solved by the method of characteristics. Only nonpressurized turbulent flow is considered and the friction slopes are evaluated by the Darcy-Weisbach formula.

This simulation effort appears to be more of academic than of practical interest, and documented experience with the model is not at hand at present. However, the model should provide a method for checking the computational accuracy of models based on less refined simulation methods.

The Dorsch Consult Hydrograph-Volume Method (Klym, et al. 1972) is a proprietary model of Dorsch Consult of Munich, West Germany, and appears to be one of the most complete models for the computation of runoff from urban catchments and the routing of flows in sewer networks. The model is limited to the simulation of single runoff events, but considers a system consisting of several catchments and a sewer and/or open channel network consisting of loops and converging and diverging branches.

Dry-weather flow which represents the base flow is computed as the sum of a constant groundwater seepage into the system and a constant sanitary flow computed from the number of residents and per capita contribution. Storm runoff is computed separately for pervious and impervious areas with and without depression storage; extreme detail is used to describe individual lots. Horton's or Holtan's equation can be used to compute infiltration or pervious areas. Overland and gutter flow routing is accomplished by a kinematic wave formulation.

Flow routing in sewers and open channels is accomplished by an implicit finite difference solution of the dynamic wave equations. Several cross sections are modeled. The formulation considers drop structures and retention basins and is coupled with equations for overflow and diversion structures. An iterative solution technique is used which considers backwater and downstream hydraulic control but neglects flow reversal. Special formulations are included to handle surcharging and pressure flow. The implicit solution of the dynamic wave equations appears to provide an accurate means for flow routing, (Brandstetter, 1974a). The overall numerical solution, however, appears to be very time consuming and justified only if backwater effects and surcharging are significant or where effects of downstream hydraulic conditions on diversion and retention basin performance have to be considered. However, the channel routing procedure seems to have reasonable computer time requirements (Brandstetter, 1974b).

The Utah Simulation Model (Narayana et al.1969) is based on analog simulation and considers a number of individual subwatersheds. Runoff hydrographs are generated by linear routing of overland flow supply which is computed by subtracting interception, infiltration and depression storage from the precipitation hyetogram. Subwatershed hydrographs are then combined and routed to the outlet, again using a linear routing method.

Although modeling results have been reported only for single storm events, the model could probably be used for continuous simulation. However, model parameters are determined by fitting the model results to observed data, thus, modeling of ungaged watersheds would involve assumption of transferable loss functions as well. The general validity of this assumption is doubtful.

2.5 Chapter Summary

This inquiry has focused on the problem of finding least cost solutions to urban drainage systems that are integrated systems of convergence and storage elements. An examination of existing urban drainage models failed to identify one that satisfies this objective.

Optimum design based on mathematical simulation would be inefficient, if at all feasible. This task could theoretically be achieved through comparison of all feasible solutions which for a large size and complex system would result in virtually an infinite number of possibilities. Fortunately, sound engineering judgement, intuition and experience can generally make a vast reduction in the number of solutions needing detailed investigation, but such an approach would, except in the simplest case, still be a nearly impossible undertaking even with the most advanced computational hardware. This property makes simulation an unpromising approach to optimization and suggests use of mathematical optimizing techniques for this particular problem. Such an approach should significantly reduce the number of cases that need to be examined.

The literature does not satisfactorily address the issue of whether calculations of flood wave propogations in urban drainage networks should be based on dynamic or kinematic routing techniques. While much work has been done on both categories, no quantitative comparison with regard to computational economy and accuracy has been undertaken. This situation necessitates this inquiry to include both approaches in the developed model, and to develop a set of selection criteria which allows the flow routing method with maximum utility to be selected in a particular case.

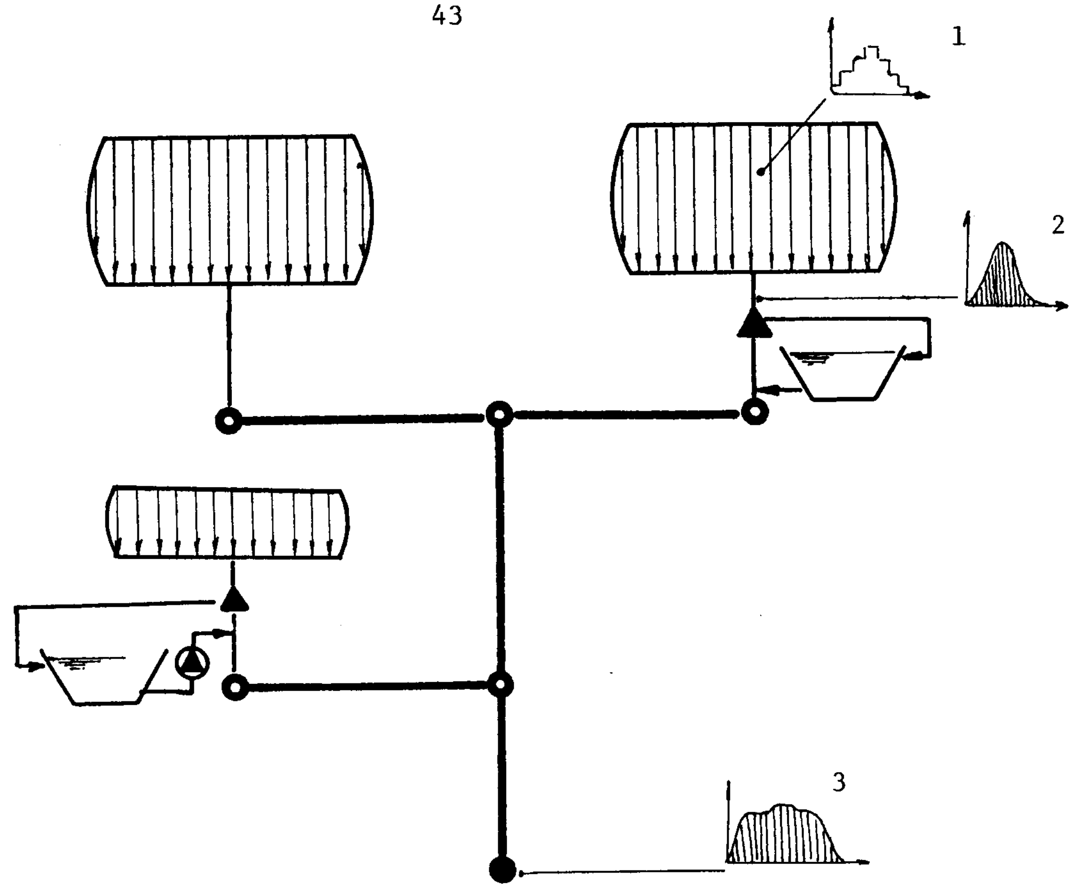
Table 2.1 Classification and Characteristics of Hydrologic Synthesis Methods

Method			
Basic relations	Technique	Characteristics	References
Rainfall-runoff Relationships (estimate of runoff volume or rainfall excess	Regression Relations	Estimate direct runoff from precipitation data, antecedent rainfall and time of the year by correlation techniques.	Kohler and Linsley (1951), Kohler and Richards (1962), Betson, et al. (1969)
	Infiltration Indicies	Estimate runoff from precipitation by use of a selected infiltration index which has previously been derived from precipitation and runoff data	Linsley, et al. (1949)
	Water Balance Models	Estimate runoff from precipitation by subtracting losses as functions of the prevailing, continuously simulated soil moisture storage	Linsley and Ackerman (1942) Crawford and Linsley (1966)
Hydrograph Synthesis (determines the shape of flow hydrograph from estimated runoff volumes)	Linear Systems Concept	Estimates runoff by convoluting precipitation with the basin response to a unit impulse of excess rain	Sherman (1932)
	Physical Analysis	Estimates watershed runoff as a problem of fluid mechanics	Thomas (1935), Isacson, et al. (1953, 1954, 1956), Morgali and Linsley (1965)

Table 2.2 Summary of Urban Drainage Model Properties

Model (Reference) / Characteristics	GENERAL: Model specially derived for urban applications	GENERAL: Can be used in forecast mode 1)	GENERAL: Simulates continuous hydrographs	INPUT DATA: Rainfall	INPUT DATA: Needs historical record for calibration	INPUT DATA: Relies on judgement factors 2)	RUNOFF MODEL: Includes interception losses	RUNOFF MODEL: Accounts for depression storage	RUNOFF MODEL: Simulates evapotranspiration	RUNOFF MODEL: Infiltration	RUNOFF MODEL: Simulates water balance	RUNOFF MODEL: Overland flow routing technique	RUNOFF MODEL: Gutter flow routing technique	RUNOFF MODEL: Side sewer routing technique	HYDRAULIC MODEL: Pipe routing method	HYDRAULIC MODEL: Backwater effects simulated	HYDRAULIC MODEL: Simulation of storage elements	QUALITY MODEL: Simulates quality parameters	QUALITY MODEL: Simulates quality changes in drainage network	QUALITY MODEL: Simulates treatment processes	QUALITY MODEL: Simulates receiving water quality
Environmental Protection Agency Storm Water Management Model (Metcalf & Eddy, et al., 1971)	Yes	Yes	No	Single design storm	No	Yes	Input data	*	No	Simple decay function	No	Kinema-tic wave	*	*	*	No	Yes	Yes	Yes	Yes 8)	Yes
Water Resources Engineers Storm Drainage Model (Roesner et al. 1971)	Yes	Yes	No	Single design storm	No	Yes	Input data	*	No	Simple decay function	No	Kinema-tic wave	*		Kinema-tic wave	Approxi-mated	Yes	Yes	No	No	No
Stanford Watershed Model IV (Crawford and Linsley, 1966)	No	No	Yes	Cont. rainfall record	Yes	No	Yes	Yes	Yes	Soil Moisture Concept	Yes	Storage routing			Storage routing	No	No	No	No	No	No
Hydrocomp Simulation Program (Hydrocomp International, 1969)	No	No	Yes	Cont. rainfall record	Yes	Yes 3) No	Yes	Yes	Yes	Soil Moisture Concept	Yes	Storage routing		Kinema-tic wave	*	No	Yes	No	No	No	No
M.I.T. Catchment Model (Harley, et al., 1970)	No	Yes	No	Single design storm	Yes	Yes	No		Yes	Yes?	Not directly	Kin.wave or lin.soil to St.Ven	*	*	*	No	Yes	No	No	No	No
University of Cincinnati Urban Runoff Model (Preul and Papadakis, 1970)	Yes	Yes	No	Single design storm	No	Yes	No	Yes	No	Simple decay function	No	Storage routing	*	*	Empiri-cal	No	No	No	No	No	No
LaVere Merritt's Simulation Program (Merritt, 1970)	Yes	Yes	No	Max.5Min. flow rates as data 7)	No	Yes			No	Empiri-cal 4)	No	Not Included	*	Empiri-cal 5)	*	No	No	No	No	No	No
Norwegian Institute for Water Research Model (Lindholm, 1973)	Yes	Yes	No	Single design storm	No	Yes			No	Empiri-cal	No	Storage routing	*	*	*	No	No	Yes	No	Yes	No
Battelle Pacific Northwest Research Laboratories (Brandstetter, et al., 1973)	Yes	Yes	No	Single design storm 6)	Yes	Yes	Yes	Yes	No	Empiri-cal 4)	No	Not Included	*	Kinema-tic wave	*	No	Yes	Yes	No	Yes	No
University of Illinois Storm Sewer System Simulation Model (Sevuk, et al., 1973)	Yes	Yes	No	Single design storm 4)	No	Yes	No	No	No	Empiri-cal 4)	No	Not Included	*	St. Ven. equations	*	Yes	Yes	No	No	No	No
Dorsch Consult Hydrograph - Volume Method (Klym, et al., 1972)	Yes	Yes	No	Single design storm	No	Yes	No	Yes/No	No	Simple decay function	No	Kinematic wave	*	St. Ven. equations	*	Yes	Yes	No	No	No	No
Utah Simulation Model (Narayana, et al., 1969)	Yes	No	Yes	Single design storm	Yes	Yes	Yes	Yes	No	Empirical	No	Linear	Not included	*	*	*	No	No	No	No	No

1) Runoff prediction based on physical characteristics of watershed and does not need to be fitted to observed data.
2) Based on experience from similar situations.
3) In case of no calibration data.
4) Runoff calculated externally and read as data.
5) Assumes steady uniform flow; solved by Mannings equation.
6) Runoff volumes calculated by unit hydrograph techniques.
7) Runoff values calculated externally; peak flow read as data.
8) Simulation of overflow treatment only.



LEGEND:

Catchment Element:

Subcatchments

Network Elements:

Storage basin

Pump station

Diversion structure

Junction structure

Outfall structure

Network Channel

1 Rainfall hyetogram

2 Runoff hydrograph

3 Outflow hydrograph

Figure 2.1 Simplified Illustration of the Drainage System Elements

CHAPTER III

CONCEPTUAL DEVELOPMENTS

> I stood at the bottom of the crack and saw my friend disappearing out of sight around a bulge, and I heard the ringing of his hammer as he drove a piton into a crack in the solid rock. "It's your turn now", he called. I tried to wedge my boot in the crack so it wouldn't be too hard to get it out again, and felt with my hand for something to hang on to. I knew by now that it would take all my skill and strength to follow his lead - - -

3.1 Chapter Introduction

This chapter outlines the conceptual developments of an optimization approach which determines least cost control strategies in urban drainage network design. The use of this method within the overall urban drainage planning and implementation process is defined and the assumptions underlying the approach are presented.

The optimal control problem is solved by coupling an hydraulic simulation model and a control cost determination model with an optimization procedure. The techniques that are implemented and the rationale behind the selection of system constraints, hydraulic simulation techniques and cost calculations are discussed. A solution sequence model which controls the implementation of the developed procedure over the drainage system is presented, and finally, a convergence proof for the optimization scheme is offered.

3.2 Planning of Urban Drainage Network Alternatives

Planning in its broadest sense is fundamentally a social process which ascertains needs, problems, objectives and goals, and formulates alternatives for implementation. Within the planning process, engineering technology is used to assess alternatives quantitatively. Urban drainage planning, in particular, is complicated by the fact that it is woven into a web of interrelated factors which makes the overall problem an inherently complex one. The primary factor contributing to that complexity is the dynamic interaction of the elements shown in Figure 3.1.

Further complication of the urban drainage planning process is due to the nature of the problem variables involved. In simple terms, urban drainage needs are determined by urban growth, hydrological variables, governmental regulations and public desires. All of these variables include random components and are probabilistic rather than deterministic. Thus, it is clear that planning and design of urban drainage systems occur in a state of uncertainty, and in particular, the objective becomes an issue of decision itself since it depends on random components.

The overall task of planning and implementing urban drainage systems can be broken down to a sequence of specific activities as shown in Figure 3.2. A number of feedback loops other than those shown might result during the course of a particular study.

Urban drainage planning is commenced by governmental regulations and initiative by citizens and governmental agencies. These efforts should identify needs and community desires and establish drainage management on a watershed basis. The activities in the planning domain start with formulation of goals. This step translates the objectives formulated by public activities into dogmatic terms. Clearly, periodic review and appraisal of adopted goals is required.

The next activity in the planning process is identification of alternatives that achieve the adopted goals and should be studied further. The importance of this activity cannot be over-emphasized since the plan finally selected as best is among those that are identified in this step, while an in-depth investigation of too many alternatives is undesirable for economical reasons. Thus, screening techniques that eliminate the most unpromising alternatives might be applied at this step.

Next, the defined alternatives are studied in detail and compared, and the optimization method developed in this inquiry; identification of least cost solutions to given network alternatives, is addressed to this stage of the analysis. While costs remain an important element of comparison, it should be realized that factors like effectiveness, potential environmental impact, human values and ease of implementation enter into the analysis.

If the alternative finally proposed for implementation is found acceptable by citizens and governing agencies, the decision to implement the alternative is reserved for elected officials. Upon decision to implement an alternative, design efforts are undertaken and plans are translated into physical facilities.

The management process, however, should not end with the construction of facilities. The performance of a facility should be monitored and compared to standards, and, as performance standards often are changing,additional action might be needed to satisfy new goals.

3.3 Theoretical Development and Conceptualization of the Problem

The drainage system, as envisioned was shown in Figure 2.1, and is assumed to consist of nodes which are connected with links. A schematic illustration of the problem variables is shown in Figure 3.3, where the nodes are labeled in increasing numbers upstream from the discharge point. Any node could include a storage volume and lift station besides a junction structure which connects the incoming and outgoing channels. The channels could be natural or man-made.

Although the necessary information on topography and location of nodes are known from the planning phase, the problem still appears to have an infinite number of feasible solutions. As a large channel will have the same hydraulic capacity as a smaller one laid at a steeper slope, a trade off point between cost of excavation and junction structure requirements on one hand and conduits on the other apparently exists. Different channel alternatives will also attenuate the inflow hydrograph differently, thus, imposing different hydraulic requirements on the downstream elements. Furthermore, decisions to impound as well as to release previously impounded waters will affect channel and storage requirements. Finally, deep and concomitant expensive trenches require consideration of lift stations and impose new trade off considerations.

The number of possible solutions is limited by a number of constraints. These include minimum and maximum velocity requirements to prevent deposits of solids carried in suspension by the runoff and abrasion or erosion damages, respectively; trenches being between a maximum and minimum depth; and a number of restrictions to insure a sound functional design of the system components.

The problem, as imposed on the decision maker, consists of determining the combination of elements among all feasible ones that will carry the runoff volumes to the predetermined point at minimum cost, or phrased another way, the problem consists of making a string of decisions represented by a decision on control vector $\overline{d}_n$; (n = N, ...,1) such that the associated control costs, $\overline{C}_n$, (n = N, ...,1) yield a minimum total cost.

Assuming that each node could contain a storage volume and considering the outflows from the nodes as decisions to be made and restricting the channels to commercial conduit sizes, each decision will lead to a finite number of solution sets; each set might consist of a storage volume, a number of pipes laid at slopes that will carry the flow without violating any constraints, pump stations, and the associated costs. Thus, at each decision point, or stage, hydrographs of flow, storage volumes, channel sizes, pump station capacities, invert elevations and solution costs become variables that describe the state of the system at the stage and will be labeled state variables. The variable that determines the order in which the event occurs in the system, the stage variable, specifies the node solution order of the system. As the decisions at any stage exert an influence on the state variable at later stages, the consequences of a serial string of decisions must be considered.

In matrix notation, the problem can then be conceptualized as the discrete process illustrated in Figure 3.4. The value of the state variable vector, $\overline{X}$, and the decision or control vector, $\overline{d}$, at stage n are denoted by $\overline{X}_n$ and $\overline{d}_n$, respectively. The cost resulting from the imposed degree of control at stage n is denoted by the performance criterion, $\overline{r}_n$. The performance criterion provides an evaluation of a given control sequence and depends on the control sequence string and on the state vector elements.

The optimization problem associated with the N-stage sequential process illustrated in Figure 3.4 can then be stated as follows:

Given: (i) A system equation, that relates the state variable at stage n + 1 to the state variables at stage n described by the non-linear relationship:

$$\overline{X}(n) = g\{\overline{X}(n+1), \overline{d}(n+1), n+1\} \quad (3.1)$$

where

$\overline{X}$ = k - dimensional state vector

$\overline{d}$ = m - dimensional decision or control vector

n = stage index

g = k - dimensional vector function

(ii) A variational performance criterion

$$R = \sum_{n=1}^{N} r_n = \sum_{n=1}^{N} \{\sum_{m=1}^{M} \ell_m[(\overline{X}(n+1), \overline{d}(n+1), n+1]\} \quad (3.2)$$

where

R = total cost of controls which is to be minimized

r_n = control cost for a single stage

ℓ_m = scalar cost function for the m-th control option

N = total number of stages

M = total number of control options

(iii) A set of system constraints that place restrictions on the values that the state variables and the control variables can assume:

$$\overline{X}(n) \varepsilon \; \chi\,(n)$$

$$\overline{d}(n) \varepsilon \; D\{\overline{X}(n),\, n\} \quad (3.3)$$

where

$\chi(n)$ = set of admissable states at all stages

$D\{\overline{X}(n),\, n\}$ = set of admissable control policies that can vary with $\overline{X}$ and n.

(iv) An initial state which gives the conditions at stage n=N

$$\overline{X}(N) = \overline{c} \quad (3.4)$$

Find: The control sequence $\overline{d}^*(N)$, $\overline{d}^*(N-1)$, ..., $\overline{d}^*(1)$ that minimizes the total control cost, R, as given by equation 3.2 and subject to the system constraints.

In cases where the performance criterion, $\ell_m\{\overline{X}(n), \overline{d}(n), n\}$, is concave or linear, and differentiable, classical optimization techniques such as the Euler-Lagrange and gradient techniques, or linear and nonlinear programming can be used to determine the optimal solution set in a convex policy space. Unfortunately, if the objective function is convex and non-differentiable as in this case, these techniques are difficult or impossible to apply. In these cases, the computational techniques of dynamic programming as developed essentially by Bellman (Bellman 1957; Bellman and Dreyfus, 1962) can be used. This approach is based on Bellman's postulated principle of optimality:

> An optimal policy has the property that whatever the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

Phrased another way, the principle of optimality means that the end stages in a serial structure must be optimized with respect to the input from the previous stages, and the dynamic programming solution to the previously given problem is obtained by an iterative functional equation that determines the optimal control for any admissible value of the state variable at any stage. Before this equation can be written, the minimum cost function, $I(\overline{X}, n)$, must be defined. This function determines the minimum cost that can be obtained by the admissible control in going from any admissible state X, at the initial state N and to the end stage 1. The defining equation is:

$$I(\overline{X},n) = \underset{\substack{\overline{d}(j)\varepsilon D \\ j=n,..,1}}{\text{Min}} \{ \sum_{j} \ell[\overline{X}(j), \overline{d}(j), j] \} \tag{3.5}$$

Applying Bellman's principle of optimality, the iterative equation can be written as:

$$I(\overline{X},n) = \underset{\overline{d}\varepsilon D}{\text{Min}} \{\ell[\overline{X}, \overline{d}, n] + I[g(\overline{X}, \overline{d}, n), n + 1]\} \tag{3.6}$$

This equation describes an iterative relation for determining the minimum cost, $I(\overline{X},n)$, for all $\overline{X}\varepsilon X$ from knowledge of $I(\overline{X}, n+1)$ for all $\overline{X}\varepsilon X$.

The optimal control, $\bar{d}^*(\bar{X},n)$, is defined as the control that satisfies the iterative expression given by equation 3.6, and is related to the principle of optimality in that the minimum cost at state k and stage n is found by minimizing the sum of the cost of the present stage, n, and the minimum cost in going to the end of the sequence from the resulting state at the next stage n-1. The iterative equation is solved by successively evaluating $I(\bar{X},N)$, $I(\bar{X},N-1)\ldots$, $I(\bar{X},1)$; the state value at stage N being the initial boundary condition. When the final stage is reached in the iterative process, the optimal policy in terms of least overall total cost can be determined by retracing the optimal path from the optimal state in the last stage. The result of this procedure is that the optimization over a sequence of controls is reduced to an optimization sequence over a single control.

In the conventional method of solving the iterative equation, both the state and the control variable are quantized into discrete levels. At given quantized values of the state variable, X, each quantized control, d, is applied and the corresponding next quantized state, $g(\bar{X},\bar{d}, n-1)$ is evaluated. The cost to the next state is then computed based on the present value of $I(\bar{X},n)$ and the transition cost from the present stage. The minimum cost, $I(\bar{X}, n-1)$ is obtained by direct comparison of the quantity bracketed in equation 3.6 for all quantized control states.

3.4 Solution Procedure

<u>Serial Structures</u>

In a dynamic programming framework, the optimization problem is structured as shown in Figure 3.5. At each network node (the stage variable), invert elevations (the state variable) are quantized into K discrete levels. The decisions or control options (the decision variables are maximum discharges, channel dimensions and slopes, lift stations and storage volumes. The channels are restricted to the discrete commercial pipe sizes normally used in sewerage systems, and maximum discharges are restricted to a finite set of values. The sum of the individual control option costs gives r_n, the total transition cost between states over the stage interval. Dynamic programming is used to determine the path from the initial stage, at n = N, to the final stage, at N = 1, which results in the least total cost.

The transition matrix for stage n, $[M_n, k]$, that is shown in Figure 3.5 can be written as:

$[M_{n,k}] =$

$$\begin{bmatrix} N_{n,1} & CC_{n,1} & CH_{n,1} & CJ_{n,1} & CP_{n,1} & CS_{n,1} & D_{n,1} & V_{n,1} & Q_{n,1} & Z1_{n,1} & Z2_{n,1} & Q_{n,1,t} \\ N_{n,2} & CC_{n,2} & CH_{n,2} & CH_{n,2} & CP_{n,2} & CS_{n,2} & D_{n,2} & V_{n,2} & Q_{n,2} & Z1_{n,2} & Z2_{n,2} & Q_{n,2,t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N_{n,k} & CC_{n,k} & CH_{n,k} & CJ_{n,k} & CP_{n,k} & CS_{n,k} & D_{n,k} & V_{n,k} & Q_{n,k} & Z1_{n,k} & Z2_{n,k} & Q_{n,k,t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N_{n,K} & CC_{n,K} & CH_{n,K} & CJ_{n,K} & CP_{n,K} & CS_{n,K} & D_{n,K} & V_{n,K} & Q_{n,K} & Z1_{n,K} & Z2_{n,K} & Q_{n,K,t} \end{bmatrix} \tag{3.7}$$

where

$CC_{n,k}$ = cummulative cost to state k at stage n

$CH_{n,k}$ = conduit cost at state k over stage n

$CJ_{n,k}$ = junction cost at state k over stage n

$CP_{n,k}$ = pump cost at state k over stage n

$CS_{n,k}$ = retention basin cost at stage n + 1 on the optimal trajectory to state k at stage n

$D_{n,k}$ = inflow conduit diameter to state k at state n

$N_{n,k}$ = state at stage n + 1 on the optimal trajectory to state k at stage n.

$V_{n,k}$ = retention basin cost at stage n + 1 on the optimal trajectory to state k at stage n

$Q_{n,k}$ = design flow at state k over stage n

$Z1_{n,k}$ = invert elevation of inflow conduit to state k at stage n

$Z2_{n,k}$ = invert elevation of outflow conduit on the optimal trajectory to state k at stage n

k = subscript indicating state increment

n = subscript indicating stage

$Q_{n,k,t}$ = inflow hydrograph to state k at stage n

t = subscript indicating time

The conceptualization of the optimization problem as illustrated in Figure 3.5 is the application of the state transformation function, $g(\overline{X}_n,\overline{d}_n,n)$, which is given by equation 3.1. The solution manipulation within this framework is shown in Figure 3.6. This procedure is the implementation of the performance criterion, which is given by equation 3.2, and the iterative equation for cost minimization, equation 3.6.

The solution at each state starts by adding the inlet and the inflow hydrographs and scanning the total hydrograph for the largest discharge rate, $Qmax_{n,k}$. A set of maximum controlled outflows, $Qci_{n,k}$, which are the design flows for the downstream conduit links, is then established through the application of a set of flow controls, i, or

$$Qci_{n,k} = \frac{Qmax_{n,k}}{i};\ i = i_1, i_2, \ldots, i_I \tag{3.8}$$

where

$Qci_{n,k}$ = maximum controlled discharge from state k at stage n for discharge control i.

$Qmax_{n,k}$ = maximum inflow discharge to state k at stage n

A qualitative relationship between the degree of equalization and the associated storage volumes is shown in Figure 3.7. This relationship shows that the flow control vector, $\bar{i}$; $\bar{i} = [i_1, i_2, \ldots, i_I]$, that needs to be considered in the optimization problem, only has to span a limited interval of the possible solution domain. As the numerical values of the flow controls increase, the maximum controlled discharges, $Qci_{n,k}$, approach the average flow for the considered time period. Thus, the storage requirements are greatly increased while the hydraulic capacity requirements of the associated conduits are not appreciably reduced. The flow control set is shown as vector $\bar{i}$ in Figures 3.5 and 3.6.

The pseudocontrols, which are the channel sizes, are shown as vector $\bar{\ell}$ in Figures 3.5 and 3.6. The channels that are considered are limited to commercial sizes of circular conduits. All channel sizes are considered for each flow control that is applied. Thus, the resulting controls at each state are the entries in a matrix with dimensions IxL, where I is the total number of flow controls and L the total number of conduit sizes.

Hydraulic design of the conduits is based on the assumption of steady uniform flow, where the design flow is the maximum ordinate on the inflow hydrograph. Under these assumptions the conduit slope is equal to the friction slope, and these slopes can be written explicitly by any uniform flow equation:

$$s = \Phi(f, g, D, Q) \tag{3.9}$$

where

D = conduit diameter, ft

Q = discharge, cfs

f = friction factor, dimensionless

g = acceleration of gravity, ft/sec^2

s = conduit slope, ft/ft

Used in the optimization model, this relationship and the cost-effectiveness consequence as illustrated by Figure 3.8, show that each conduit size only needs to be considered at one specific slope for each state and for each quantized increment of the flow control vector.

For steady uniform flow, the headloss, h_L, is proportional to the square of the average velocity, or

$$h_L \propto V^2 \tag{3.10}$$

Further, the discharge increases with increased depth of flow, y, when conduits with constant cross sections that are installed at uniform slopes carry steady flows, or

$$Q \propto y \tag{3.11}$$

The slope that should be considered is the one that results in the conduit flowing full at the maximum discharge, or if this decision violates the minimum velocity constraint, the slope that results in the minimum allowable velocity of flow should be used. In situations where the solutions violate the downstream depth constraints, solutions which include use of drop structures or pump stations are selected.

The specific techniques that are used in the hydraulic design of the system components are discussed in Section 3.6.

The total number of solutions at each state is shown as a volume with dimensions I, L, K in Figure 3.5, where I is the total number of flow controls, L the available conduit sizes and K is the number of quantized states. However, sound functional and hydraulic design of the drainage system components bounds the number of feasible solutions. These considerations are included in the model as a set of constraints, and include velocity limitations, minimum conduit sizes, alignment and diameter progression constraints of conduits, and junction spacing requirements. A quantitative discussion of these constraints is given in Section 3.5.

The number of feasible solutions at stage n, S_n, is

$$S_n = \sum_{\substack{k=1 \\ \lambda \varepsilon L \\ \iota \varepsilon I}}^{K} [i_{n,k,\iota} \times \ell_{n,k,\lambda}] \tag{3.12}$$

where

I = maximum number of flow controls

K = total number of state increments

L = total number of available conduit dimensions

i = flow control

k = subscript indicating state increment

ℓ = conduit dimension

n = subscript indicating stage

ι = subscript indicating flow control

λ = subscript indicating conduit dimension

Each solution, which is illustrated by the cube $S_{n,k,i,\ell}$ in Figure 3.5, contains a solution number, a pointer to the trajectory state at stage n, design flow, conduit diameter, invert elevations, storage basin and pumping requirements, cummulative cost, and control costs over the stage.

The procedure implemented to determine the optimal state transition between successive stages is shown in Figure 3.6 and consists of the following steps:

1. Calculate all feasible solutions; i.e. the transitions between stages n and n-1.

2. Load the feasible solutions into a solution array.

3. Sort the solutions with respect to invert elevations at the downstream junction; i.e. at stage n-1.

4. Quantize the state vector at the downstream junction, or stage n-1 by partitioning the difference between highest and lowest terminal elevations among the feasible solutions into K equal increments.

5. Search the solutions corresponding to each state at the downstream junction, or stage n-1, and select as optimal control to the state the solution with least cummulative cost.

When the state transition has been determined, the inlet hydrograph for the state at stage n is routed to stage n-1. The routing technique is described in Section 3.6. Upon completion of the hydrograph routing to all states at stage n-1, the transition matrix for that stage, $[M_{n-1,} k]$, as given by equation 3.7, can be written.

The dynamic programming solution to the optimization problem is identical to the solution procedure presented in Section 3.3. The minimum cost function is defined for all feasible values of the state variable as

$$I'\{\overline{X}(n-1),\ m\} = \min \sum_m \ell_m \{\overline{X}(n),\ \overline{d}(n),\ n\} \tag{3.13}$$

Again, using Bellman's principle of optimality, the recursive equation can be expressed as

$$I'\ \{\overline{X}(k,n-1,\ m\} = \min\{I[\overline{X}(k,n),\ n] + \sum_m \ell_m[\overline{X}(n),\ \overline{d}(n),\ n]\} \tag{3.14}$$

This equation states that the minimum cost solution for some state at stage n-1 is found by choosing the control that minimizes the sum of the cost at the present stage and the cost in going from the present stage to stage n-1.

Non-Serial Structures

Nodes with more than one tributary link pose a special problem. Thus, the solution procedure outlined above, which applies to serial systems only, needs to be modified. Fortunately, this task is simplified by one consequence of dynamic programming and the specific approach taken in this inquiry.

As equation 3.14 leads to the optimal path for all feasible quantized states at stage n-1, optimal control is specified not only along the optimal trajectory to every stage, but for every admissable state at every stage. The non-serial stage problem is then reduced to finding the least cost combination of all tributary conduit links to each state at the stage, and the solution can proceed as outlined for serial structures.

3.5 Constraints

The system constraints, which are described by equation 3.3, form an integral part of the solution procedure, and are imposed on the system to insure a satisfactory functional design of the system components.

1. Minimum conduit size D_{min}, is specified to avoid clogging and subsequent maintenance problems, or

$$D \geq D_{min} \tag{3.15}$$

Minimum diameters usually employed in sanitary and storm sewer designs are 8 (Fair and Geyer, 1954) and 12 inches (Metcalf and Eddy, 1972), respectively.

2. Pipe progression constraint which specifies that the diameter of any downstream conduit link must be equal to or greater than the upstream link is a common requirement, particularly in the smaller pipe sizes, or

$$D_{downstream} \geq D_{upstream} \tag{3.16}$$

This criterion is based on the rationale that any object small enough to pass through the upstream link must in turn be passed through the downstream lines.

3. Straight alignment is commonly required for sewer sizes less than 24 inches. Curved alignment is considered for economical reasons while regarded objectionable for reasons of difficulty of maintenance and increased joint separation problems. Use of curved sewers has increased in recent years (Federal Housing Administration, 1959); they are usually considered acceptable when laid with radii greater than 100 ft.

4. Manhold spacing requirements are specified for reasons of ease of inspection and maintenance. Maximum spacings are often in the range of 300 to 500 feet for conduit sizes not large enough to permit maintenance personnel to enter.

5. Conduit depths are usually limited by minimum and maximum values which define the trajectory corridor for all feasible solutions. Typical minimum depths are 6 to 8 feet for sanitary sewers and 2 feet for storm sewers (Babbitt and Bauman, 1958). The values to be used in a given case are generally determined by either the depth required for service connections or by a minimum soil cover required for frost protection or to avoid conduit

damages by surface loads.

Maximum depth is not a rigid requirement and is usually set due to the uncertainty associated with problems caused by ground water, unstable soil, excessive pipe loads, and maintenance and repair at deep excavations. Rather arbitrary maximum depth limits are used; open trench construction is normally limited to approximately 30 feet.

6. Velocity restrictions are intended to prevent solids separation and deposition on one hand and pipe abrasion and impact on the other. In both cases, the solids and grit which are present in all urban drainage waters in varying amounts are the major sources of the problem. Capacity reduction, septicity, and clogging may result from solids deposition. Excessive pipe wear as well as pipe separation may occur at high velocities.

For the purpose of achieving self-cleansing action, minimum velocities of 2 fps and 3 fps for sanitary and storm sewers, respectively, are considered adequate (ASCE, 1966). The minimum velocity is assumed to be independent of sewer size.

The minimum velocity approach is not unlike the critical velocity concept used in channel design. However, there is one major difference. The critical velocity for the initiation of motion of sediment particles depends on the boundary characteristics and properties of the sediment particles as well as the flow in the canal (ASCE, 1966).

The results of an experimental study of flow of sand-water mixtures in pipes indicates that the critical velocity to prevent sand particles from depositing in the pipe increases with the square root of the pipe diameter (Robinson and Graf, 1972). By approaching the self-cleansing problem of sewers on the basis of critical shear stress, Yao (1974) concluded that by using a fixed minimum velocity for all sewer sizes the self-cleansing action would be less effective for larger sewers. Either the smaller sewers are over designed or the larger sewers are underdesigned.

Maximum limiting velocities are usually taken to be 10 and 15 fps for sanitary and storm sewers, respectively. These constraints reflect the need to accomodate infrequent but large flows for storm sewers.

3.6 Hydraulic Model

General Description

The hydraulic model consists of two separate modules; a design module and a simulation module. The design module, which performs the hydraulic design of the network elements, is coupled with the dynamic programming scheme. Hydraulic design is based on the assumption of steady uniform flow for the maximum controlled inflows to the system elements.

The hydraulic simulation model calculates wave progressions through the drainage network components, and unlike the design module, which is part of the dynamic optimization model, the simulation block is implemented over the stage variable after the optimal state transitions have been determined. Since the simulation model is applied on the upstream reaches before the one immediately downstream is designed, transient pipe storages and time lags are included in the optimal design.

This "two pass application" of the hydraulic model modules over the stage variable has significant implications. The design module has negligible computational requirements and is used on all feasible solutions in the vector space spanned by vectors K, I, L shown in Figure 3.5. The simulation module, however, which has significant computational requirements, is used only on the transitions between successive stages, i.e. a maximum of K times for each stage. Thus, an order of magnitude reduction in computational requirements has been achieved without loss of accuracy.

Hydraulic Design

The Solution Procedure (Section 3.4) showed the channel design problem that is imposed on the optimization model to consist of finding a specific slope for a circular conduit that carries a steady uniform flow. Specifically, for the flow and conduit under consideration, the design problem is to determine

1. the slope at which the conduit carries the flow while flowing full while satisfying all velocity and depth constraints, or
2. the slope at which the conduit carries the flow while flowing less than full and satisfying the depth and velocity constraints.

Conduit design is based on the continuity relationship for an incompressible fluid,

$$Qci_{n,k} = A\ V_a \tag{3.17}$$

where

A = cross-sectional area of flow, ft^2

$Qci_{n,k}$ = the steady uniform design flow evaluated by equation 3.8.

V_a = average velocity of flow, fps

For a circular conduit flowing full, the average velocity of flow is

$$V_a = Qci_{n,k}/(\pi D^2/4) \tag{3.18}$$

where

D = conduit diameter, ft

Violation of the maximum velocity constraint, V_{max}, by the average velocity determined by equation 3,18,i.e.,

$$V_a > V_{max} \tag{3.19}$$

eliminates the conduit from the feasible solution set for the state and the design algorithm is advanced to the next larger conduit size.

If both velocity constraints are satisfied, or

$$V_{min} \leq V_a \leq V_{max} \tag{3.20}$$

the friction slope can be calculated directly by the Darcy-Weisbach formula:

$$S_f = f \frac{1}{D} \frac{V_a^2}{2g} \tag{3.21}$$

where

S_f = friction slope, ft/ft

f = Darcy-Weisbach friction factor, dimensionless

V_a = average velocity, fps

g = acceleration of gravity, ft/sec^2

D = conduit diameter, ft

The steady uniform design flow assumption allows the friction slope, S_f, to be equated to the conduit invert slope, S_o,

$$S_o = S_f \tag{3.22}$$

Finally, if the solution does not violate the conduit cover constraints, it is added to the feasible solution set for the state, and the design model continues with the next larger conduit dimension.

Where the minimum velocity constraint is violated when the conduit is flowing full,

$$V_a < V_{min} \tag{3.23}$$

the conduit slope must be increased to the slope which maintains the allowable minimum velocity of flow. The required flow area can be determined from the continuity relationship,

$$A = Qci_{n,k}/V_{min} \tag{3.24}$$

where

A = required flow area, ft

$Qci_{n,k}$ = the design flow given by equation 3.8, cfs

V_{min} = minimum average velocity, fps

Using the relationship between flow area, central angle and conduit size from Figure 3.9, the flow area given by equation 3.24 can be written as

$$A = \frac{D^2}{8}(\theta - \sin\theta) \tag{3.25}$$

where A = cross sectional area of flow, ft^2

D = conduit diameter, ft

θ = central angle, radians

Thus, equations 3.24 and 3.25 can be combined to an implicit relationship for θ:

$$Qci_{n,k}/V_{min} - \frac{D^2}{8}(\theta - \sin\theta) = 0 \tag{3.26}$$

Writing equations 3.26 as a function of θ,

$$F(\theta) = Qci_{n,k}/V_{min} - \frac{D^2}{8}(\theta - \sin\theta) \tag{3.27}$$

and adding and subtracting the central angle, θ, gives

$$\Theta = \Theta - F(\Theta) \qquad (3.28)$$

which is a recursive relationship for the central angle that can be used to generate the sequence $\{\Theta_i\}$ for i=1,2,..... Specifically, by the Newton-Raphson iteration technique, Θ can be found by successive evaluation of

$$\Theta_{i+1} = \Theta_i - F(\Theta_i) \frac{\partial}{\partial\Theta} (F(\Theta_i)) \qquad (3.29)$$

which is the technique that is utilized in the model.

Knowing the central angle, the friction slope can be evaluated by the Darcy-Weisbach formula where the hydraulic radius is written in terms of the conduit diameter and the central angle of the flow area,

$$S_f = f\frac{V^2}{2g} \frac{1}{D} \frac{\Theta}{(\Theta - \sin\Theta)} \qquad (3.30)$$

Again, by the assumption of steady uniform flow, the friction slope is equal to the invert slope,

$$S_f = S_o \qquad (3.31)$$

The Darcy-Weisbach friction factor, f, varies with the Reynolds number and the relative roughness of the conduit wall. Neglecting the transition zone between the hydraulically smooth and rough conditions, the threshold Reynolds number, $\mathbb{R}^*$, which separates these regions is determined by eliminating f from the Blasius and von Karman equations, equations 2.12 and 2.13, respectively, to yield the transition Reynolds number,

$$\mathbb{R}^* = .633 \cdot (\log_{10}\frac{2R}{\varepsilon} + .87)^8 \qquad (3.32)$$

where

R = hydraulic radius, ft

ε = conduit wall roughness, ft

The model computes f by the Blasius or the von Karman equation depending on whether $\mathbb{R}^*$ is greater or less than the actual Reynolds number, $\mathbb{R}$. The specific relationships are:

$$\mathbb{R} = VR/\nu \tag{3.33}$$

$$f = .223/\mathbb{R}^{.25} \qquad \text{if } \mathbb{R} < \mathbb{R}^* \tag{3.34}$$

$$f = 1/(2 \log_{10} \frac{2\ R}{\varepsilon} + 1.74) \qquad \text{if } \mathbb{R} \geq \mathbb{R}^* \tag{3.35}$$

The slope determination in cases where equation 3.23 is valid consist of two iterative loops. The first iterative loop is based on equation 3.29 and determines Θ. The second one includes equations 3.30 and 3.31 and equations 3.33 through 3.35 and determines the friction slope.

Hydraulic Simulation

The equations that govern the gradually varied and unsteady flow occurring in storm drains, the equations of continuity and momentum, were given as equations 2.2 and 2.3, respectively. Without lateral inflow and assuming the coefficients of velocity and momentum distribution to be unity, these equations can be written as:

$$Y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0 \tag{3.36}$$

$$v \frac{\partial v}{\partial x} + \frac{\partial y}{\partial x} + \frac{\partial v}{\partial t} - g\ (S_o - S_f) = 0 \tag{3.37}$$

where

S_o = slope of channel invert, ft/ft

S_f = friction slope, ft/ft

Y = hydraulic depth (flow area/flow surface width), ft

g = gravitational acceleration, ft/sec^2

t = time, sec

v = average flow velocity, ft/sec

x = length along channel, ft

y = flow depth, ft

The dynamic wave description can be approximated by the kinematic wave equations which were given by equations 2.7 and 2.8. The validity of this approximation depends on the properties of the wave and of the network element. Again, without lateral inflow the kinematic equation set can be written as:

$$Y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0 \qquad (3.38)$$

$$Q - ay^{b} = 0 \qquad (3.39)$$

where

Q = discharge, cfs

Y = hydraulic depth, ft

a = parameter determined by channel slope, size and roughness

b = parameter determined by channel slope, size and roughness

t = time, sec

v = average flow velocity, ft/sec

x = length along channel, ft

y = wave depth, ft

The kinematic approach is preferred for reasons of computational efficiency, but is deficient relative to the dynamic solution where the inertia and pressure terms are significant in comparison to the slope and the friction terms in the momentum equations. The hydraulic simulation model, as developed, is capable of solving the routing problem by both dynamic and kinematic simulations. The solution domains appropriate to each method are discussed in Appendix A.

Dynamic simulation

Numerical solutions to equations 3.36 and 3.37 are obtained over a discrete rectangular net of points in the x-t plane. Figure 3.10a shows the point mesh that is used in the development of the numerical integration scheme. The equations of continuity and momentum have two independent variables, t and x, and two dependent variables, y and v. Designating the dependent variables by u, the finite difference approximation by the four point non-central scheme that was selected for the problem solution can be written as:

$$u \approx \frac{1}{2} (u_i^{j+1} + u_{i+1}^{j+1}) \tag{3.40}$$

$$\frac{\partial u}{\partial x} \approx \frac{1}{\Delta x} (u_{i+1}^{j+1} - u_i^{j+1}) \tag{3.41}$$

$$\frac{\partial u}{\partial t} \approx \frac{1}{2\Delta t} (u_i^{j+1} + u_{i+1}^{j+1} - u_i^j - u_{i+1}^j) \tag{3.42}$$

where the subscripts refer to the spatial positions and the superscripts to the temporal positions on the point mesh grid. This particular scheme was selected because it gave best reproduction of observed data during the course of this inquiry, and reportedly avoids stability and convergence problems (Sevuk and Yen, 1973) as opposed to other schemes (Balzer and Lai, 1968; Gunaratnam and Perkins, 1970).

Equations 3.36 and 3.37 written in terms of the finite difference approximations given by equations 3.40 through 3.42 give the following finite difference equations:

$$\frac{1}{\Delta x} (Y_i^{j+1} + Y_{i+1}^{j+1}) (v_{i+1}^{j+1} - v_i^{j+1}) + \frac{1}{\Delta x} (v_i^{j+1} + v_{i+1}^{j+1}) (y_{i+1}^{j+1} - y_i^{j+1})$$

$$+ \frac{1}{\Delta t} (y_{i+1}^{j+1} + y_i^{j+1} - y_{i+1}^j - y_i^j) = 0 \tag{3.43}$$

$$\frac{1}{\Delta t} (v_i^{j+1} + v_{i+1}^{j+1} - v_i^j - v_{i+1}^j) + \frac{1}{2\Delta x} (v_{i+1}^{j+1} \cdot v_{i+1}^{j+1} - v_i^{j+1} \cdot$$

$$v_i^{j+1}) + \frac{g}{\Delta x} (y_{i+1}^{j+1} - y_i^{j+1}) + \frac{g}{2} (S_{f_i}{}^{j+1} + S_{f_{i+1}}{}^{j+1}) - g\, S_o = 0 \tag{3.44}$$

Equations 3.43 and 3.44 are two simultaneous nonlinear algebraic equations in four dependent variables, y_i^{j+1}, v_i^{j+1}, y_{i+1}^{j+1} and v_{i+1}^{j+1}. Although the friction slopes $S_{f_i}^{j+1}$ and $S_{f_{i+1}}^{j+1}$ also are unknown, their values are explicit functions of the four dependent variables.

The solution to the equation set is attempted by an implicit method. These methods are unconditionally stable, and the time and distance steps can therefore be selected independently. Unfortunately, convergence requirements might impose limitations on the maximum temporal and spatial integration steps that can be used in a given situation. Thus, the developed solution scheme is based on dividing the channel length into N interior reaches, where N is an integer variable that can be chosen independently or be imposed as a function of the total channel length in each case. The resulting finite difference grid is shown on Figure 3.10b.

Dividing the total channel length into N internal reaches gives N-1 points centered on row j+1 in the x,t-plane, and by equations 3.43 and 3.44, a total of 2(N-1) simultaneous equations become available. Altogether there are 2(N+1) unknowns.

The two additional relationships that are used are the upstream and downstream boundary conditions. The upstream boundary condition is given by the inflow hydrograph and the approximating assumption of uniforn flow conditions, or

$$Q_1^{j+1} - \Phi(y_1^{j+1}, v_1^{j+1}) = 0 \qquad (3.45)$$

where

Q_1^{j+1} = discharge to the channel at time j+1, cfs

y_1^{j+1} = normal depth for discharge Q_1 at time j+1, ft

v_1^{j+1} = uniform velocity for discharge Q_1 at time j+1, ft/sec

The functional relationship between depth and velocity for a given discharge is based on the Darcy-Weisbach formula.

The downstream boundary conditions are also derived under assumption of uniform flow, and again based on the Darcy-Weisbach uniform flow formula, or

$$v_N^{j+1} = \Phi(S_o, f, D, y_N^{j+1}) \tag{3.46}$$

where

D = conduit diameter, ft

S_o = invert slope of conduit, ft/ft

f = Darcy-Weisbach friction factor, dimensionless

v_N^{j+1} = average velocity at downstream boundary, ft/sec

y_N^{j+1} = flow depth at downstream boundary, ft

A set of initial conditions must be known before the solution can start. This requires that all depths and velocities along the channel are known at a given time. This inquiry assumes a constant baseflow throughout all reaches at the initial time. Thus, the problem can be treated as one of steady flow, where velocities and depths along the channel are determined by backwater and drawdown surface profiles, depending on the element geometry and control conditions.

For combined sewers, initial depths and velocities can be estimated from the dry-weather flow conditions. For storm sewers, however, generally the only known initial condition is the one of zero depth and velocity throughout the network. This dry-bed initial condition, unfortunately, constitutes a singularity and numerical integration methods fail to advance the solution to the immediate next time level. Thus, a small steady baseflow was assumed along all conveyance elements. Fortunately, no backwater effects can result from the baseflow that is imposed on the system. The initial conditions are therefore taken to be the normal depths and velocities for the specified steady baseflow occurring throughout all channel reaches.

Equation 3.45 written for the upstream boundary, equations 3.43 and 3.44 written at each interior reach and equation 3.46 at the downstream boundary, constitute a system of 2(N+1) non-linear equations for solution of the 2(N+1) unknowns; viz, v_i^{j+1}, and y_i^{j+1} for $i = 1, 2, 3, \ldots\ldots, N+1$. An N-dimensional analog to the Newton-Raphson iteration method is used to solve the system (Ralston, 1965).

Within the iterative solution, boundary conditions and continuity and momentum equations for all interior reaches are written as a system equation vector, $\overline{F}$, which is defined as:

$$\overline{F} = \begin{bmatrix} F_1 \\ \cdot \\ \cdot \\ F_n \\ \\ F_{n+1} \\ \cdot \\ \cdot \\ F_{2(N+1)} \end{bmatrix} = \begin{bmatrix} F_1(v_1^{j+1}, y_1^{j+1}, Q_1^{j+1}) \\ \\ \\ F_C(v_i^{j+1}, y_i^{j+1}, v_{i+1}^{j+1}, y_{i+1}^{j+1}, v_i^j, y_i^j, v_{i+1}^j, y_{i+1}^j, \Delta x, \Delta t) \\ \\ F_M(v_i^{j+1}, y_i^{j+1}, v_{i+1}^{j+1}, y_{i+1}^{j+1}, v_i^j, y_i^j, v_{i+1}^j, y_{i+1}^j, S_{f_{i+1}}^{j+1}, S_{f_i}^{j+1}, g, S_o, \Delta x, \Delta t) \\ \\ \\ F_2(v_{N+1}^{j+1}, y_{N+1}^{j+1}, S_o, f_{N+1}^{j+1}, D) \end{bmatrix}$$

$$i = 1, 2, ., N+1 \qquad (3.47)$$

where

D = conduit diameter, ft

$F_C(\ )_i$ = continuity equation at interior reach i

$F_M(\ )_i$ = momentum equation at interior reach i

$F_1(\ )$ = upstream boundary condition

$F_2(\ )$ = downstream boundary condition

N = total number of interior reaches

Q = discharge, cfs

S_o = slope of conduit invert, ft/ft

f = Darcy-Weisbach friction factor, dimensionless

g = acceleration of gravity, ft/sec^2

i = subscript for interior reach

j = superscript for time

v = flow velocity, fps

y = flow depth, ft

Δx = distance step, ft

Δt = time step, sec

The unknowns, v_i^{j+1}, v_{i+1}^{j+1}, y_i^{j+1}, y_{i+1}^{j+1} are written as vector $\bar{u}$,

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ u_{2(N+1)} \end{bmatrix} = \begin{bmatrix} v_1^{j+1} \\ y_1^{j+1} \\ v_2^{j+1} \\ \cdot \\ \cdot \\ y_{N+1}^{j+1} \end{bmatrix} \tag{3.48}$$

The Jacobian matrix for the simultaneous system equation set can then be written as:

$$[J]_{2(N+1)x2(N+1)} = \begin{bmatrix} \frac{\partial F1}{\partial u_1} & \frac{\partial F1}{\partial u_2} & \cdots & \frac{\partial F1}{\partial u_{2(N+1)}} \\ \frac{\partial F2}{\partial u_1} & \frac{\partial F2}{\partial u_2} & \cdots & \frac{\partial F2}{\partial u_{2(N+1)}} \\ \cdot & & & \\ \cdot & & & \\ \frac{\partial F2(N+1)}{\partial u_1} & \frac{\partial F2(N+1)}{\partial u_2} & \cdots & \frac{\partial F2(N+1)}{\partial u_{2(N+1)}} \end{bmatrix} \quad (3.49)$$

The iterative system used to solve the simultaneous non-linear equation set is:

$$\bar{u}_{p+1} = \bar{u}_p - \{[J]^{-1} \cdot \bar{F}\}_p \quad (3.50)$$

where p is the iteration counter.

Kinematic simulation

Numerical solutions to the kinematic equation set, equations 3.38 and 3.39 are also obtained over the discrete rectangular net of points in the x,t-plane shown on Figure 3.10. The solution uses the continuity equation, equation 3.38, written in terms of its finite difference approximations:

$$\frac{1}{\Delta x}(Y_i^{j+1} + Y_{i+1}^{j+1})(v_{i+1}^{j+1} - v_i^{j+1}) + \frac{1}{\Delta x}(v_i^{j+1} + v_{i+1}^{j+1})(y_{i+1}^{j+1} - y_i^{j+1})$$

$$+ \frac{1}{\Delta t}(y_i^{j+1} + y_{i+1}^{j+1} - y_i^j - y_{i+1}^j) = 0 \quad (3.51)$$

with the unknown velocities written in terms of the Darcy-Weisbach equation

$$v_i^{j+1} = \left[S_o \cdot \frac{2 \cdot g \cdot D \cdot R_{i+1}^{j+1}}{f_{i+1}^{j+1}}\right]^{.5} \quad (3.52)$$

$$v_{i+1}^{j+1} = \left[S_o \cdot \frac{2 \cdot g \cdot D \cdot R_{i+1}^{j+1}}{f_{i+1}^{j+1}}\right]^{.5} \qquad (3.53)$$

where

D = conduit diameter, ft

R = hydraulic radius, ft

Y = hydraulic depth, ft

S_o = slope of conduit invert, ft/ft

f = Darcy-Weisbach friction factor, dimensionless

g = acceleration of gravity, ft/sec^2

i = subscript for interior reach

j = superscript for time

t = time step, sec

v = average flow velocity, fps

x = distance step, ft

y = flow depth, ft

The kinematic problem contains N+1 unknowns, viz. y_i^{j+1} for i=1, 2, ..., N+1. The continuity equation, equation 3.5., written at each internal grid point and the downstream boundary provides for N equations and the remaining one by the upstream boundary equation,

$$Q_1 - \Phi(y_1^{j+1}, v_1^{j+1}) = 0 \qquad (3.54)$$

The N+1 non-linear simultaneous equations are again written as a system equation vector $\overline{F}$,

$$\overline{F} = \begin{bmatrix} F_1 \\ F_i \\ \cdot \\ \cdot \\ F_{N+1} \end{bmatrix} = \begin{bmatrix} F_1(v_1^{j+1}, y_1^{j+1}, Q_1^{j+1}) \\ F_C(v_i^{j+1}, y_i^{j+1}, N_{i+1}^{j+1}, y_{i+1}^{j+1}, v_i^j, y_i^j, v_{i+1}^j, y_{i+1}^j, \Delta x, \Delta t) \\ \\ F_C(v_N^{j+1}, y_N^{j+1}, v_{N+1}^{j+1}. y_{N+1}^{j+1}. v_N^j. y_N^j. v_{N+1}^j. y_{N+1}^j. \Delta x, \Delta t) \end{bmatrix} \qquad (3.55)$$

i=1,2,...N+1

where the definition of the terms are identical to those given for equation 3.47.

The unknowns, y_i^{j+1}, i = 1,2, ..., N+1 are written as vector $\bar{u}$,

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_{N+1} \end{bmatrix} = \begin{bmatrix} y_1^{j+1} \\ y_2^{j+1} \\ \cdot \\ \cdot \\ y_{N+1}^{j+1} \end{bmatrix} \qquad (3.56)$$

The Jacobian matrix for the simultaneous equation set is then

$$[J]_{(N+1)x(N+1)} = \begin{bmatrix} \frac{\partial F1}{\partial u_1} & \frac{\partial F1}{\partial u_2} & \cdots & \frac{\partial F1}{\partial u_{N+1}} \\ \frac{\partial F2}{\partial u_1} & \frac{\partial F2}{\partial u_2} & & \frac{\partial F2}{\partial u_{N+1}} \\ \cdot & & & \\ \cdot & & & \\ \frac{\partial F(N+1)}{\partial u_1} & \frac{\partial F(N+1)}{\partial u_2} & \cdots & \frac{\partial F(N+1)}{\partial u_{N+1}} \end{bmatrix} \qquad (3.57)$$

The iterative solution for the equation set is then

$$\bar{u}_{p+1} = \bar{u}_p - \{[J]^{-1} \cdot \bar{F}\}_p \qquad (3.58)$$

where p is the iteration counter.

3.7 Transition Cost Determination

The control strategy which is optimized over the stage variable is represented by the decision or control vector $\bar{d}$. Associated with each individual control vector component d, at stage n is a cost r_n, which is the sum of the m applied controls, or $r_n = \sum_{m=1}^{M} r(m,n)$.

Drainage control options could include open channels and closed gravity and pressurized conduits, junction structures or manholes, storage basins, pump stations, diversion and outfall structures, energy dissipators and siphons. This inquiry considers only circular gravity flow conduits, junction structures, storage basins, and lift stations. The exclusion of the remaining ones is justified because of their limited use as well as being of negligible interest when the objectives of this inquiry are considered.

Control efficiency evaluation is based on comparison of initial capital requirements only, thus, neglecting operation and maintenance requirements as well as control option lifetime. Another approach would have been to consider the total annual expenses of the applied controls. These consist of the annuity of the initial capital investment, and the annual operation and maintenance, which can be written as:

$$r_a(n) = \sum_{m=1}^{M} r(m,n) \, \iota/(1-(1-\iota)^{-T_m}) + C_o(m) + C_t(m) \tag{3.59}$$

where

$C_o(m)$ = annual operation cost for control option m; fraction of initial capital requirement

$C_t(m)$ = annual maintenance cost for control option m; fraction of initial capital requirement

M = total number of control options

T_m = lifetime of control option m, years

m = control option

n = stage number

r(m,n) = cost of control m at stage n, \$

$r_a(n)$ = annual cost of total controls at stage n, \$

i = annual interest rate, %

This approach was rejected for several reasons. They include the difficulty of determining a realistic service life for the various system components; the necessity of basing the decimal fractions for operation and maintenance to a large degree on judgement, besides the fact that the initial capital requirements of these systems are commonly used for efficiency comparisons. The control option cost determination basis is summarized in the remaining portion of this section.

Conduits

The components that comprise total costs of installed concrete conduits are shown in Figure 3.11, and developed by subprogram SEWRCO which is not interfaced with the optimization model. The costs are expressed in \$/lin.ft and derived as functions of conduit diameters and invert depths. Structural requirements are functions of specified static and dynamic loads and calculated in accordance with ASCE Manuals and Reports No. 37 (ASCE, 1970). Each entry in the final cost table is the least cost combination of pipe and bedding costs to satisfy the structural requirements where the total costs are some of construction costs (Dodge Guide, 1974; Engineering News-Record, 1975) and materials prices (Assoc. Sand and Gravel, 1975).

Conduit Junction Structures

All junction structures are designed in accordance with standard structural specifications, developed by the American Public Works

Association (APWA, 1964). The general configuration of these elements is shown in Figure 3.12. For conduit diameters not larger than 42 inches precast circular concrete sections are used, and the junctions have concrete channels made to conform to the conduit grade and carried vertically to the conduit crown. For conduit dimensions 48 inches and larger, the junction structure consists of a cast in place monolithic base with access through a vertical shaft.

Unit costs for junction structures are calculated by subprogram JSTRUC and expressed in $/unit as functions of conduit diameters and invert depths. The total costs are the sum of construction costs (Dodge Guide, 1974; Engineering News-Record, 1975) and materials prices (Assoc. Sand and Gravel, 1975).

The JSTRUC program is not interfaced with the optimization model but used to generate relevant cost data which are used as input to the optimization model.

Storage Elements

Two conceptually different storage elements are considered in this inquiry, viz, pre-entry ponding and off-line storage. Pre-entry ponding is taken to mean planned ponding on roof-tops, plazas or parking lots, open spaces or recreational fields and other upstream areas. Significant reductions in the peak rate of runoff appears possible through use of these elements (Rice, 1971) and the developed model can be used to explore the cost effectiveness of these measures by modifying the inlet hydrographs for various degrees of ponding and repeating the network design for each set of new conditions.

Off-line storages are specially constructed retention basins located within the conveyance system. Functional design of these units could be carried out in a number of ways depending on the local conditions. Clearly, gravity inflows and outflows are advantageous; in situations with sufficient head available, this would be the desirable solution. In special cases, where large areas could be acquired inexpensively, shallow gravity flow basins could be designed with a minimum of available head. This concept could possibly be combined with locating the detention ponds in parks to function as decorative lakes, as suggested among others by Linsley (1971). In some cases, however, the requirements for storage will exist at locations where both area and head are at a premium. Thus, a deep tank will be required to conserve area and pumping will be required to restore lost head.

Storage element configurations also depend on their functional objectives. This inquiry assumes these elements to have the following functions:

1. Equalize a variable inflow, and
2. Allow for controllable outflows.

For the purpose of the optimization problem, the storage elements can be limited to three general types. These are shown on Figure 3.13 and are 1) shallow basin with gravity inflow and outflow, 2) shallow basin with gravity inflow and pumped outflow, and 3) deep basin with gravity inflow and pumped outflow; where the shallow basins are assumed to be of earthen construction and the deep basin of reinforced concrete construction. Based on work by EPA (1974), Consoer, Townsend and Assoc.(1974), and Waller, (1972), cost functions for the various basin alternatives are:

Type I basin

$$FB_I = ENRCCI/2000\ (1.95 \cdot V - 1.09 \cdot 10^{-5} \cdot V^2 + 3.95 \cdot 10^{-11} \cdot V^3 - 7.28 \cdot 10^{-17} \cdot V^4 + 6.61 \cdot 10^{-23} \cdot V^5 - 2.36 \cdot 10^{-29} \cdot V^6) \quad (3.60)$$

$$FB_{II} = ENRCCI/2000\ (6.34 \cdot V - 4.69 \cdot 10^{-5} \cdot V^2 + 2.28 \cdot 10^{-10} \cdot V^3 - 5.55 \cdot 10^{-16} \cdot V^4 + 6.36 \cdot 10^{-22} \cdot V^5 - 2.74 \cdot 10^{-28} \cdot V^6) \quad (3.61)$$

$$FB_{III} = ENRCCI/2000\ (50{,}000 + 15.83 \cdot V - 9.09 \cdot 10^{-5} \cdot V^2 + 3.16 \cdot 10^{-10} \cdot V^3 - 6.11 \cdot 10^{-16} \cdot V^4 + 6.04 \cdot 10^{-22} \cdot V^5 - 2.36 \cdot 10^{-28} \cdot V^6) \quad (3.62)$$

where

ENRCCI = Engineering News-Record Construction Cost Index

V = retention basin volume, cf

Pump Stations

Merritt (1970) indicates that the high capital requirements associated with pump stations generally restricts their use to cases where no other solution is possible, i.e. when trench depths become excessive. There usually exists a trench depth beyond which construction becomes impractical. Before that depth is reached, however, trenches and conduits become very expensive and suggest tradeoffs between pumping and conduit depth. Merritt's work (1970) also concludes that the initial cost of pumping is very large while the incremental cost of additional capacity is relatively small.

The construction cost relationship for pump stations that is shown on Figure 3.14 is based on Merritt's work which is updated through the Engineering News-Record Construction Cost Index. The cost function can be written as:

$$FP = ENRCCI/1000 \cdot \times 105{,}500 \times Q^{.516} \tag{3.63}$$

where

ENRCCI = Engineering News-Record Construction Cost Index

Q = pump capacity, cfs

3.8 Stage Solution Order Control

The model is implemented on a digital computer, and over the stage variable (drainage system nodes) such that computer memory requirements and execution time become a minimum. Before a general programming algorithm was devised, it was necessary to examine the general characteristics of urban drainage networks such that a stage solution sequence that achieved these objectives could be developed.

Urban drainage networks can be conceptualized as nodes (junction structures) that are connected by links (conveyance channels). The system nodes contain a junction structure to connect one or more inflow channels to an outflow channel, and depending upon the applied controls, the node could also include a retention basin and one or more lift stations. Because the drainage system configuration depends on the drainage area topography, system objectives and constraints etc., the network cannot be described in general terms. This situation suggests that the proposed model should be advanced over the stage variable by a computerized routine which considers the specific characteristics of the system being designed. The model should also have the capability of loading the input data associated with the stage variable in a random order. These features have been implemented in the model through a stage solution order control routine as described below.

The stage variable control technique can be described by the illustrative example shown on Figure 3.15. A general network has four types of nodes; two types of exterior nodes and two types of interior nodes. The exterior inflow nodes, which are not limited by any specific number, all have one inflow hydrograph, one outflow link, but do not have an inflow link. A typical network contains only one exterior outflow node, and this node is characterized by not having any outflow links.

Interior nodes are classified according to the number of inflow pipe links. Serial interior nodes are those with only one inflow pipe link, while converging nodes are those with two or more inflow links. Both types of interior nodes can have an inlet hydrograph.

The data sets that are associated with each node can be read in a random order by the model. These sets contain the node number, the downstream and upstream node numbers, an inlet hydrograph, if any, and the necessary topographical information. After the node data are loaded, the stage variable control routine will locate the exterior inflow nodes. When such a node is located, all serial nodes downstream of that node are identified. Defining an "exterior string" as an exterior inflow node, the downstream serial nodes and a converging interior node, the control model identifies a series of such strings and calls the appropriate routines for solution of the serial sequences. The example problem shown in Figure 3.15 has seven such exterior strings, which would be executed in the following order: (7,6,5), (8,5), (10,9), (11,9), (14,13,12,1), (16,15), and (19,18,17,15). The stage matrix, defined by equation 3.7, is stored for the end node in each string and identified by the number of that node. In the example problem the downstream node sequence is

{5,5,9,9,1,15,15}. The total number of occurrences of a given node in the end node sequence is then compared to the number of inflow links to that node. If the number of inflow links to the node equals the number of occurrences of the node in the end node sequence, the solution is found by calling the converging node routine described in paragraph 3.4. This operation is indicated by the blocks containing double numbers on figure 3.15b; where the first number identifies the end node and the second number the immediate upstream node. Each time a converging node solution is obtained, the downstream serial nodes are added and the serial solution sequence repeated. This solution sequence is repeated until the exterior outflow node is reached.

3.9 Chapter Summary

The problem of selecting optimal control strategies in urban drainage design has been conceptualized as an N-stage discrete decision process for which dynamic programming techniques were applied in the optimization procedure. The dynamic programming implementation over the stage variable is controlled by a computerized routine that attempts to minimize computer memory requirements and execution time. Coupled with this optimization approach are submodels and algorithms for network element design, hydraulic simulation and control cost determination.

Before discussing the model application in the next chapter, there are several crucial questions pertinent to the algorithms and the programs developed in this chapter. Most significant among these are the ones of model validity and whether or not the optimizing scheme will converge to a global optimum. The decisions leading to optimal design rest on the assumption of steady design flow. This approach significantly reduces the calculation requirements without loss in solution accuracy. The conclusion with regard to accuracy can be drawn since this assumption is consistent with the upstream boundary conditions during the hydraulic simulation phase and with the design constraint that requires the channels to be of constant cross sections between subsequent stages. As hydraulic design is based on the simulated outflows from the upstream elements, transient pipe storages are included in the optimal design.

The question of convergence is not trivial, and a global optimum is not guaranteed regardless of how fine the quantization of the state and control vectors are. Fortunately, the dynamic programming routine as structured and applied in the model allows a convergence proof to be derived rather easily. Figure 3.16 shows the bounded solution domain which is spanned by control vectors $\overline{d}_i$, maximum inflow to the stage, and $\overline{d}_\ell$, commercial conduit sizes applied over the stage. As structured, the dynamic programming routine evaluates successively each grid intersection on the solution domain by applying the performance criterion, $R = \sum_{n=1}^{N} r(n)$, as given by equation 3.2, and the optimum solution is the one given by the minimum value of R over all trajectory and control sequences, $\{\overline{X}_n\}_{n=1}^{N}, \{\overline{d}_n\}_{n=1}^{N}$, while satisfying all the system constraints. Thus, it can be concluded that a true optimum is found if all solutions are located on the grid intersections in figure 3.16, or in other words, a global optimum is found if no feasible solutions can be located outside the grid intersections. Such a solution is illustrated by Point B on the figure.

Considering control variable d_ℓ, conduit sizes, which are restricted by the model to commercial conduit sizes only, it is clear that the optimal solution will always fall on a horizontal grid line. This follows without further proof since this control is a discontinuous function which has all possible values located on the grid lines and no possible value outside. The horizontal gridlines represent controlled outflows to the stage intervals. This control is a continuous function which theoretically can take on any value between zero and the maximum inflow. However, as the performance criterion selects the most cost-effective combination of storage and conveyance capacity, and the flow control is a continuous function, the calculated solution can be as close to the global optimum as desired only by making the decision vector quantization finer and finer.

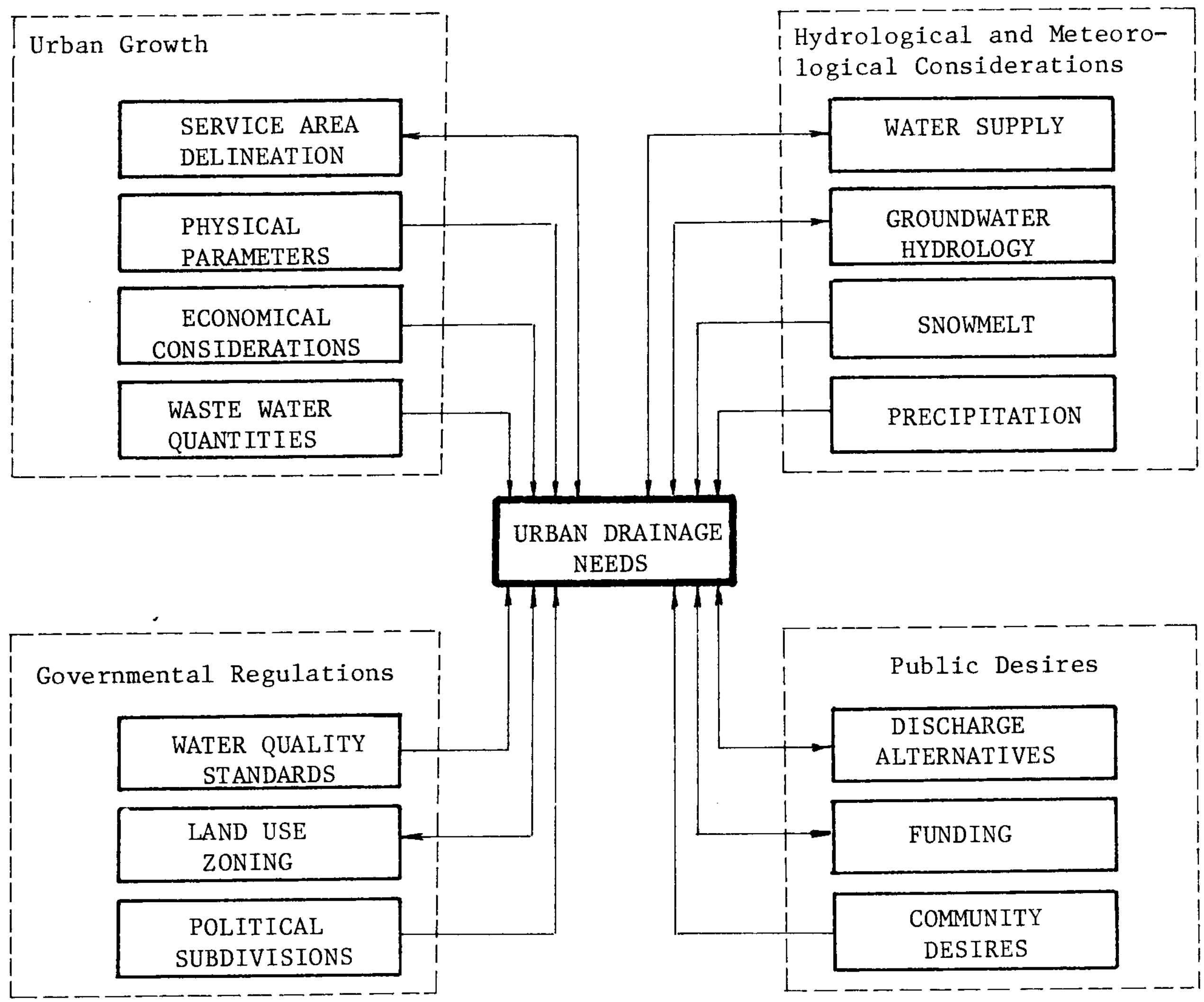


Figure 3.1 Basic Structure of the Urban Drainage Problem Complex

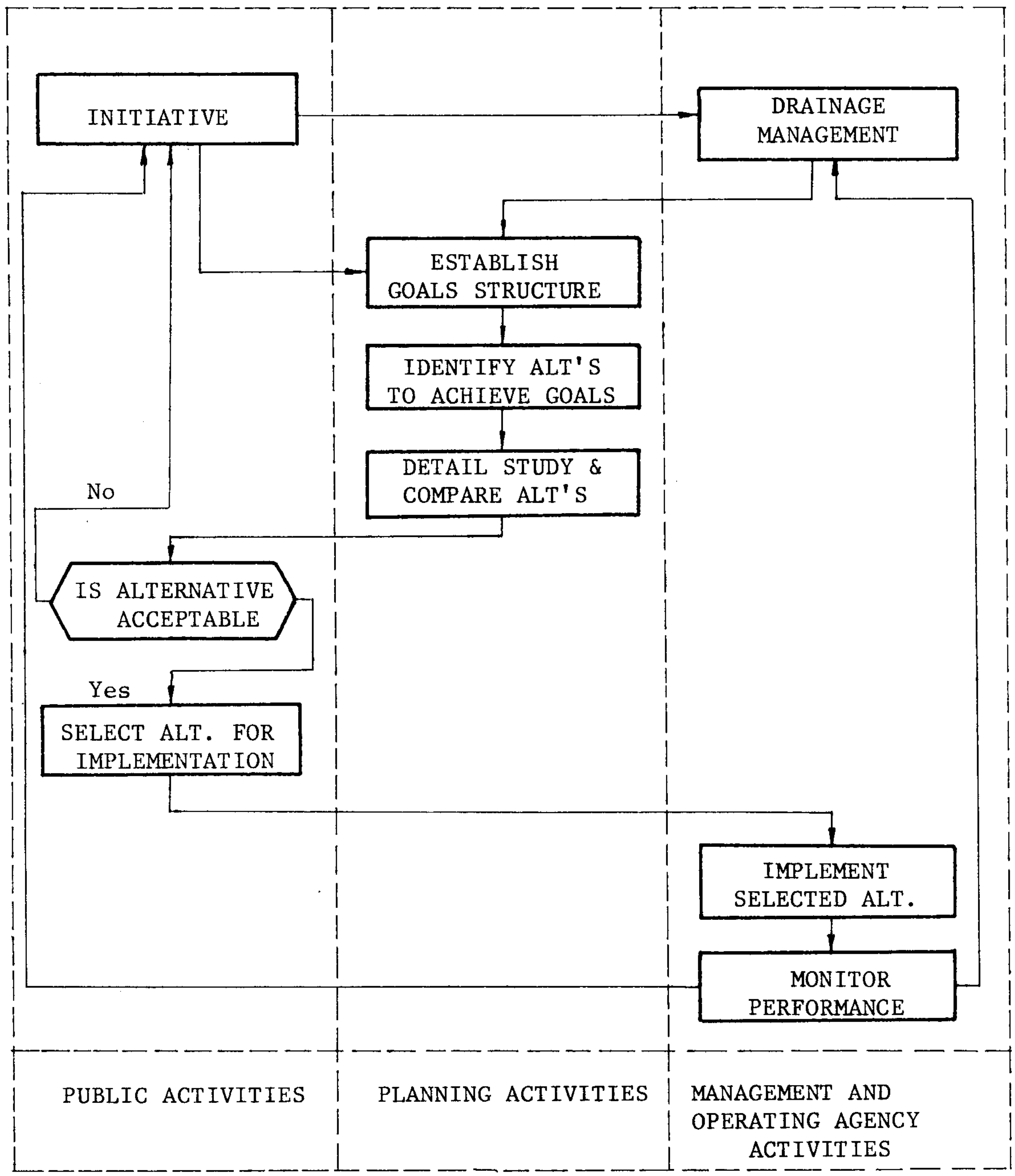
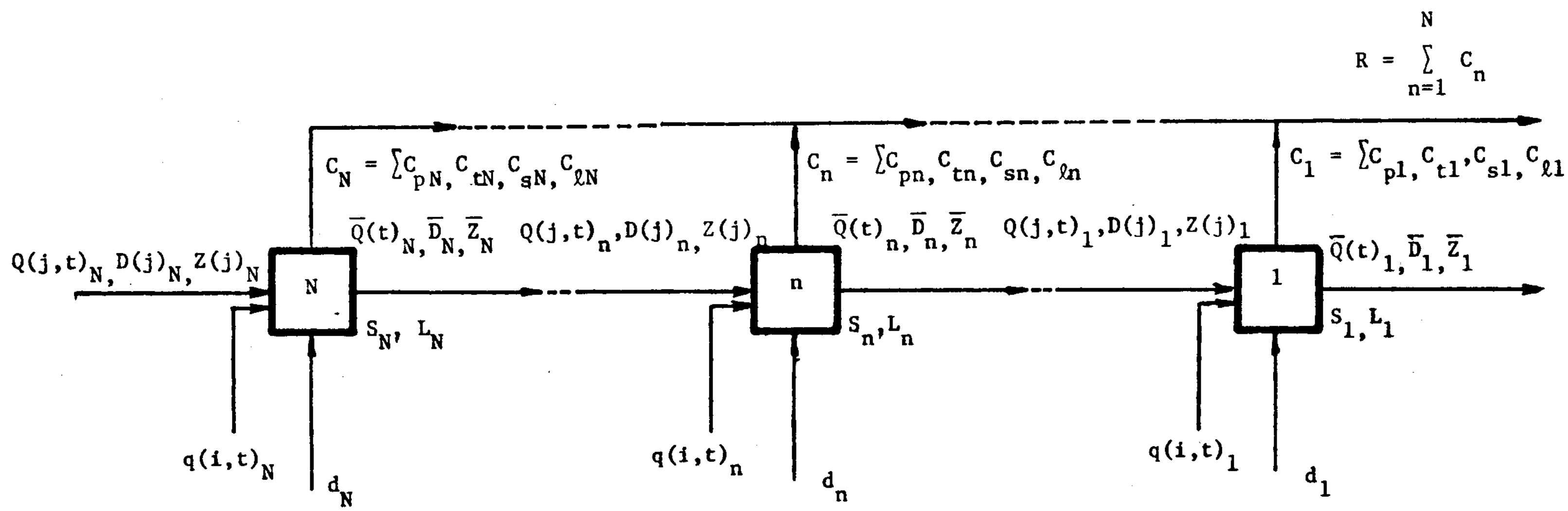


Figure 3.2 Suggested Phases of Urban Drainage Planning and Implementation



NOTATION:

C_n = cost at node n

$C_{\ell n}$ = cost of lift station at node n

C_{pn} = cost of conduit between node n and node n-1

C_{sn} = cost of storage at node n

C_{tn} = cost of trench between node n and node n-1

$D(j)_n$ = diameter of influent conduit(s) to node n

$\bar{D}_n$ = diameter of effluent conduit from node n

d_n = decision at node n

i = number of runoff hydrographs entering node n

j = influent conduit number to node n

L_n = lift station at node n

n = node number

$Q(j,t)_n$ = influent hydrograph to node n from conduit j

$\bar{Q}(t)_n$ = effluent hydrograph from node n

$q(i,t)_n$ = runoff hydrograph i to node n

R = total system cost

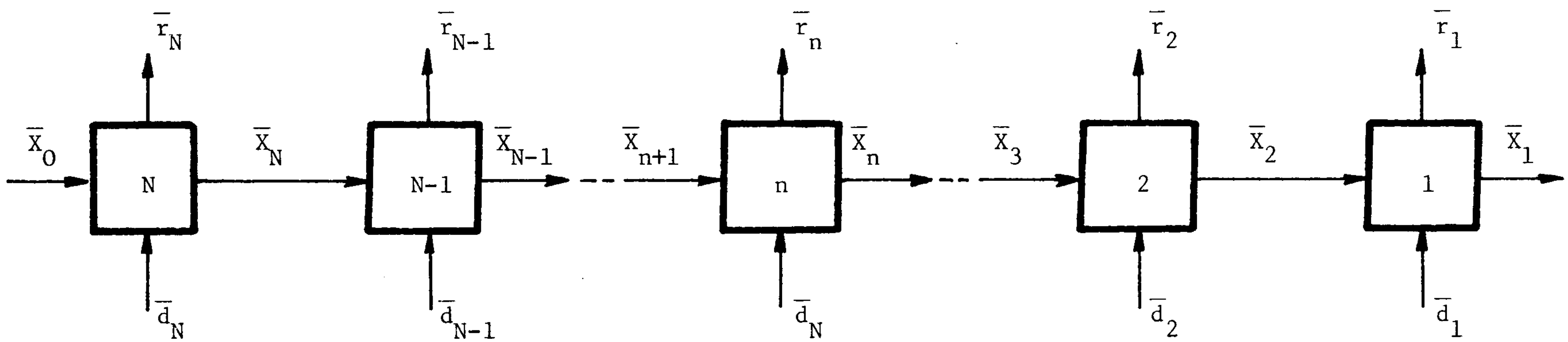
S_n = storage volume at node n

t = time

$Z(j)_n$ = invert elevation of influent conduit j to node n

$\bar{Z}_n$ = invert elevation of effluent conduit from node n

Figure 3.3 Schematic Problem Illustration



N = total number of stages

n = stage number

$\bar{d}_n$ = decision variable at stage n

$\bar{r}_n$ = control cost at stage n

$\bar{X}_n$ = state variable

Figure 3.4 General Description of the Optimization Problem

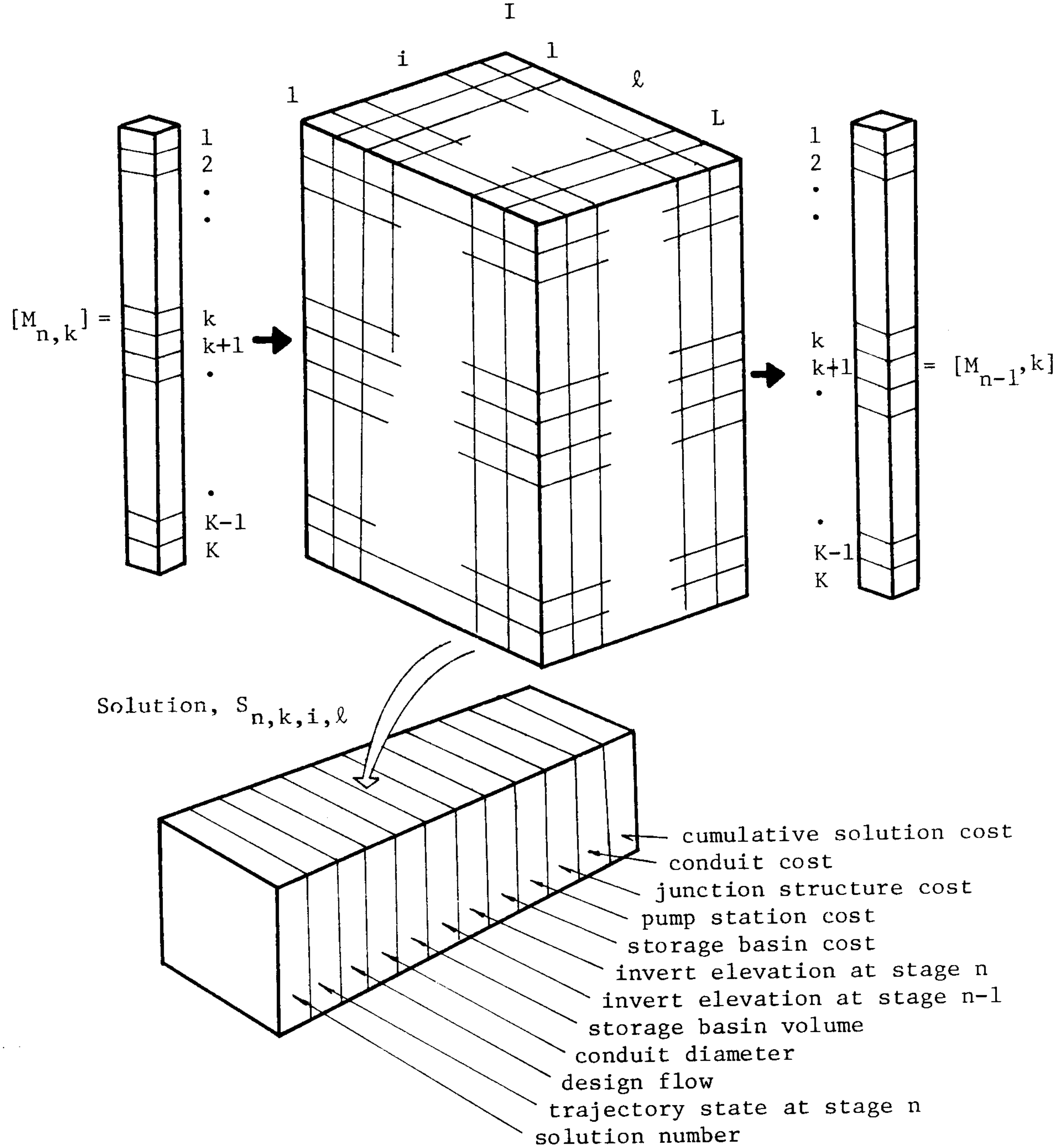
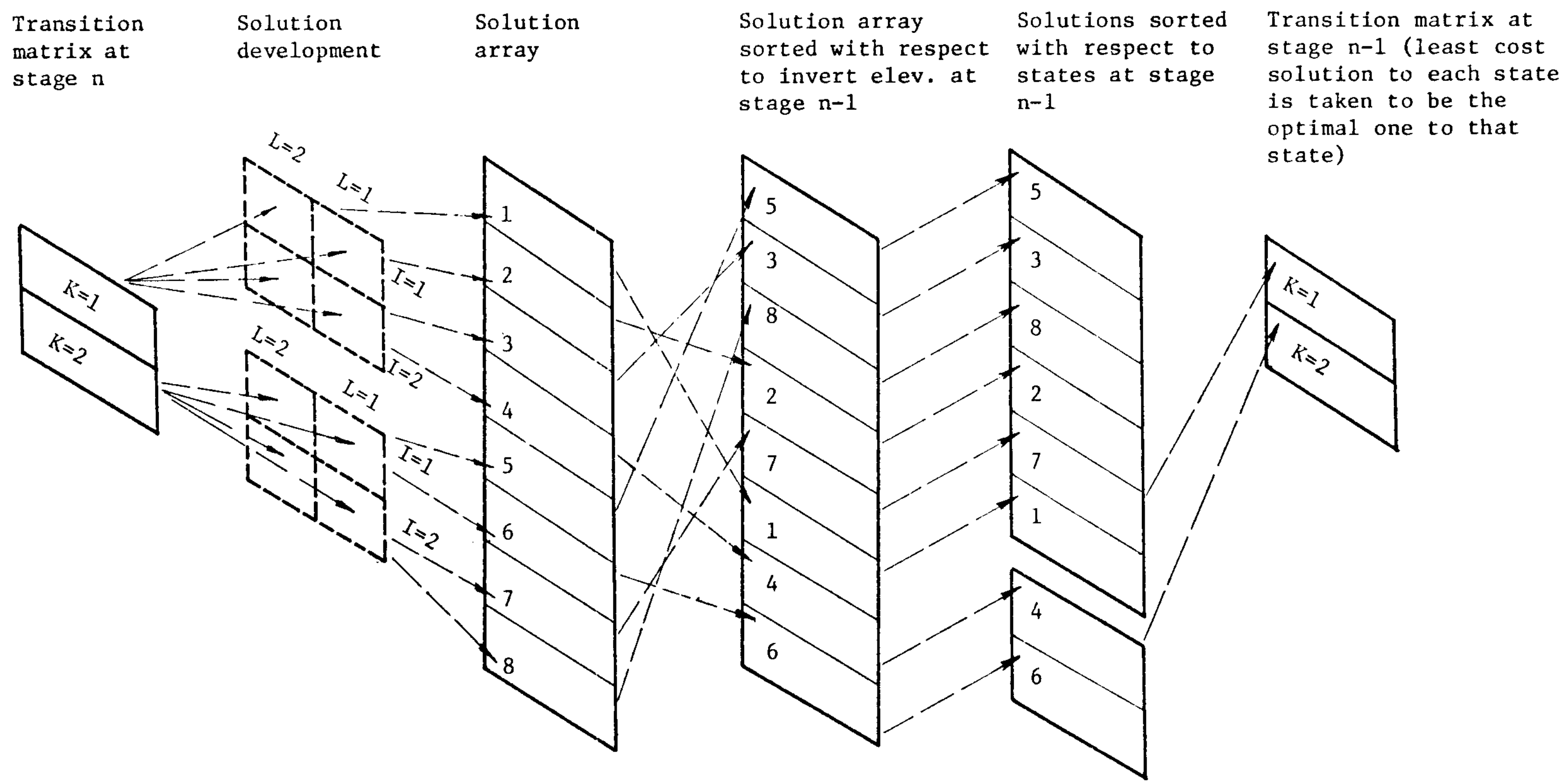


Figure 3.5 Conceptualization of the Optimization Problem in Dymamic Programming Terms



K = states
I = control i (maximum discharges)
L = control ℓ (conduit sizes)

Figure 3.6 Solution Manipulation

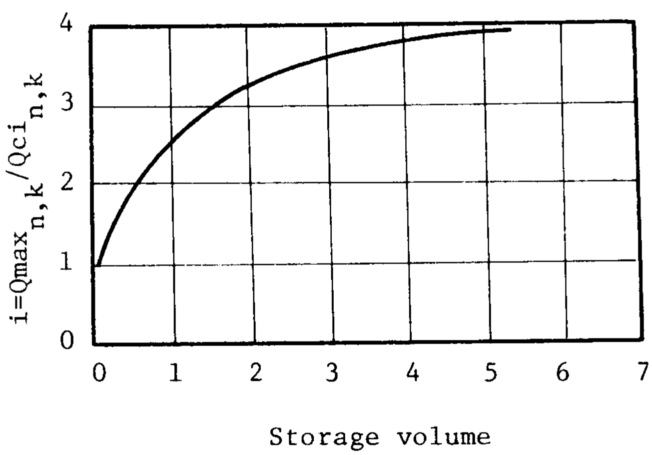
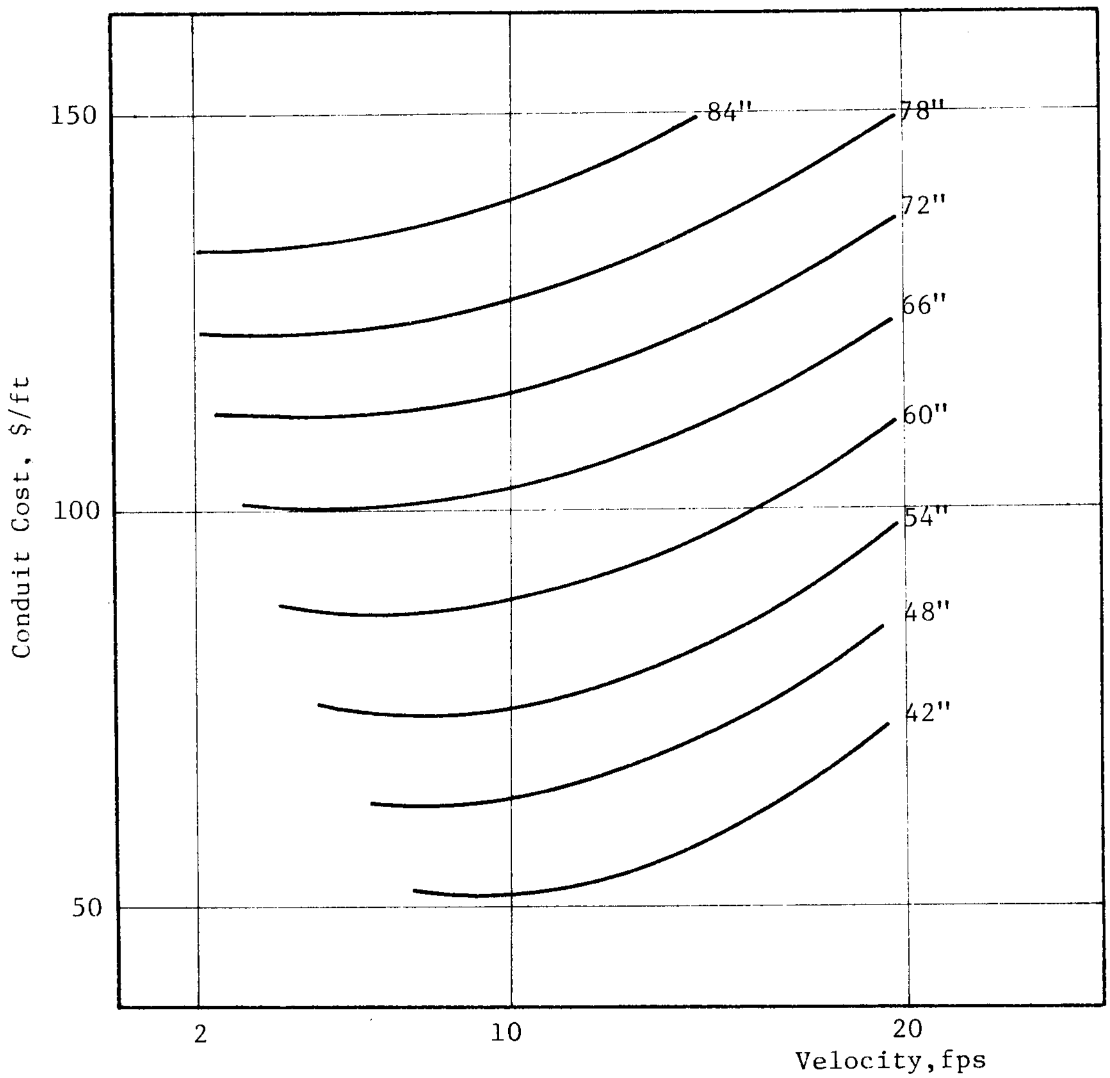


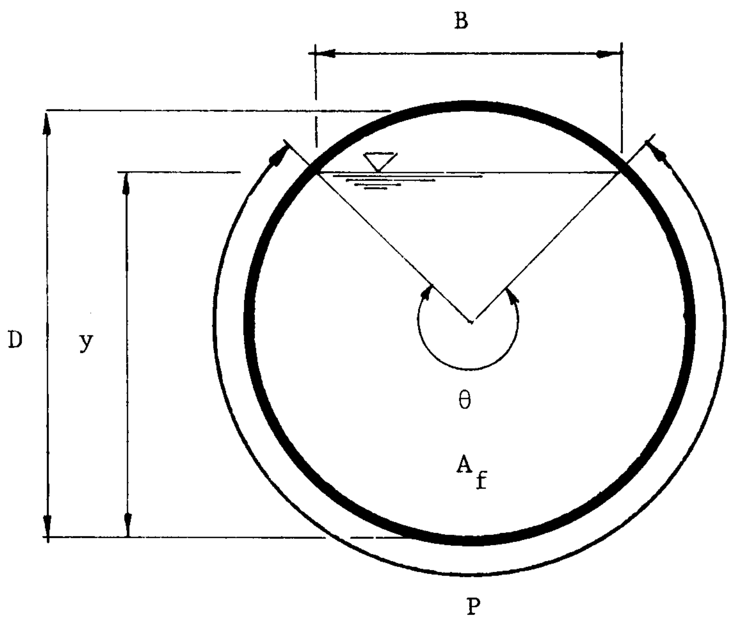
Figure 3.7 General Relationship Between Degree of Equalization and Storage Basin Volume



Basis for Cost Estimate

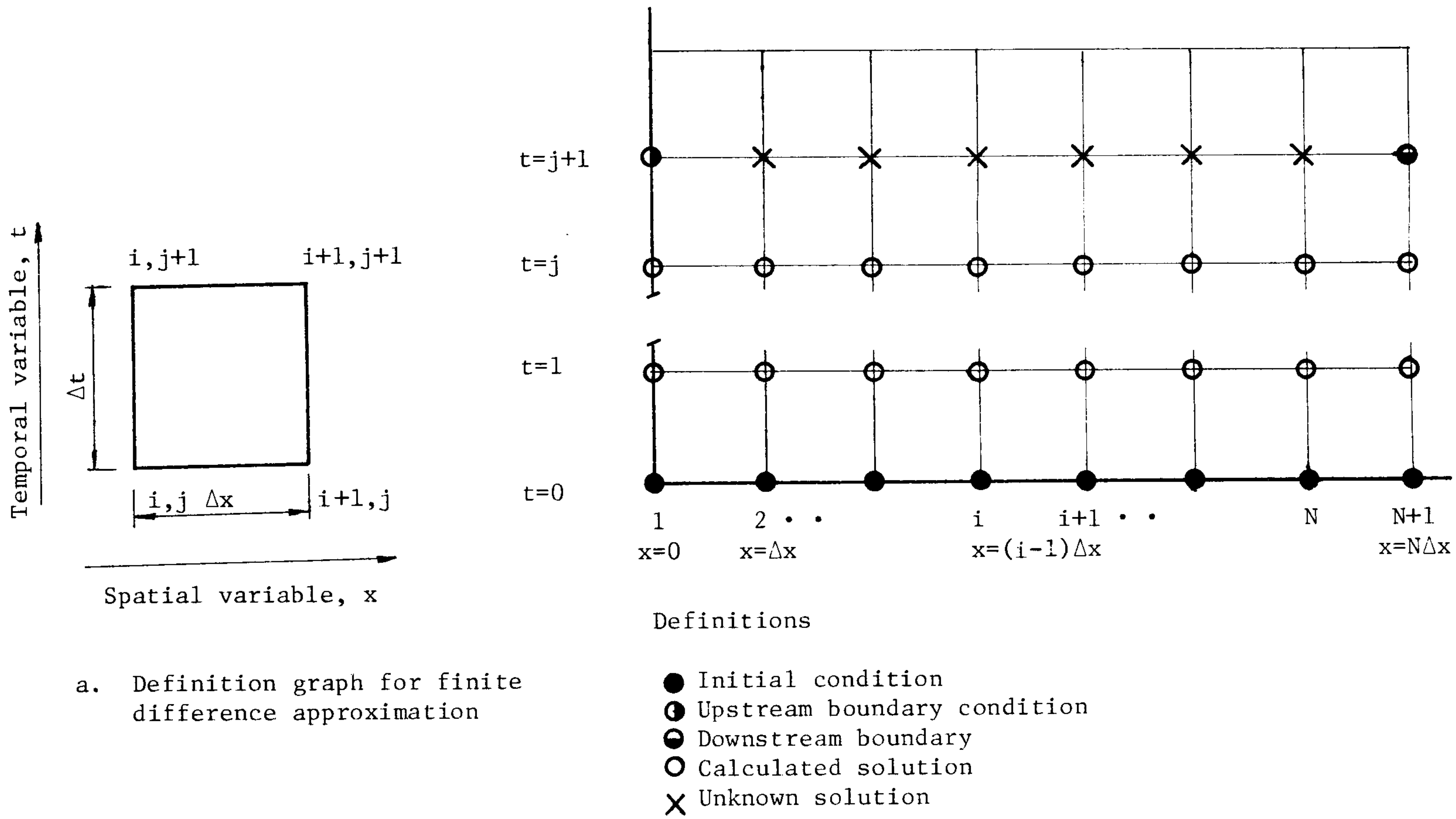
Discharge = 75 cfs
Wall roughness = .006 ft
Minimum upstream depth

Figure 3.8 Conduit Cost as Function of Velocity of Flow for a Constant Discharge



D	=	conduit diameter		
y	=	liquid depth	=	$D/2(1-\cos(\theta/2))$
θ	=	central angle	=	$2\cos^{-1}(1-2y/D)$
A_f	=	flow area	=	$D^2/8(\theta-\sin\theta)$
P	=	wetted perimeter	=	$D/2\theta$
B	=	surface width	=	$D\sin(\theta/2)$
R	=	hydraulic radius	=	$D/4\ (1-\sin\theta/\theta)$

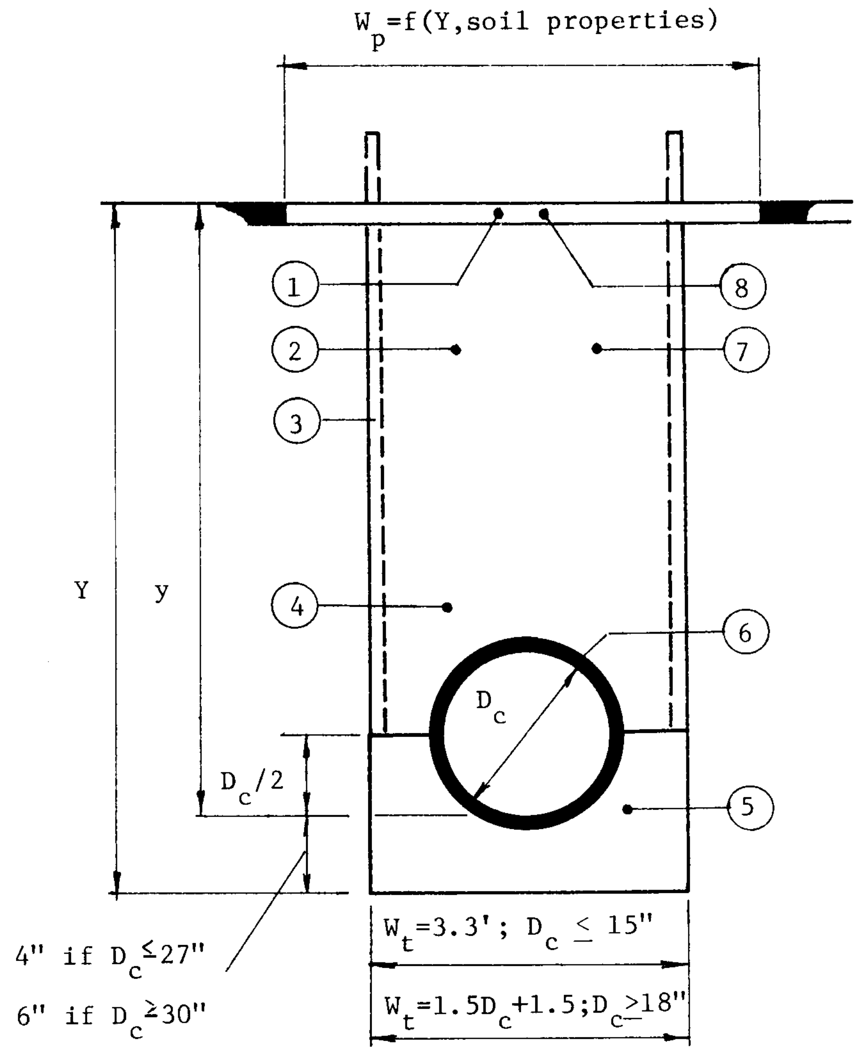
Figure 3.9 Definition Sketch and Geometric Parameters for a Partially Filled Circular Conduit



a. Definition graph for finite difference approximation

b. Point mesh on the x,t-plane

Figure 3.10 Finite Difference Scheme



Definition of variables

D_c = conduit diameter, inches

f( ) = function

W_p = width of pavement cut, ft

W_t = trench width, ft

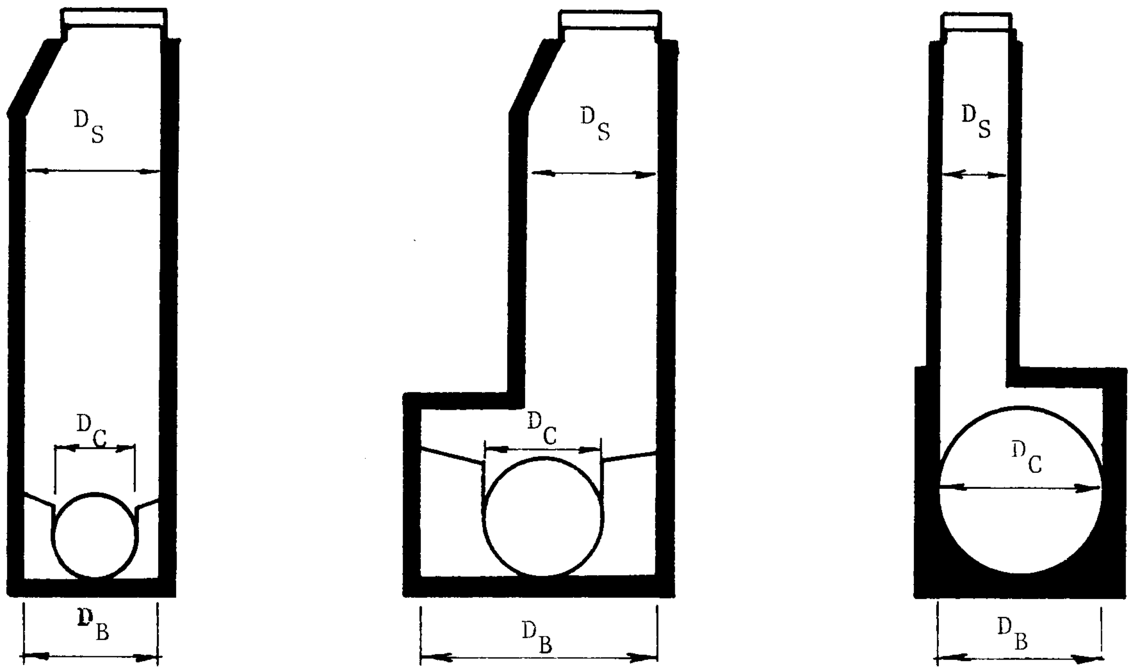
y = invert depth, ft

Y = trench depth, ft

Cost components

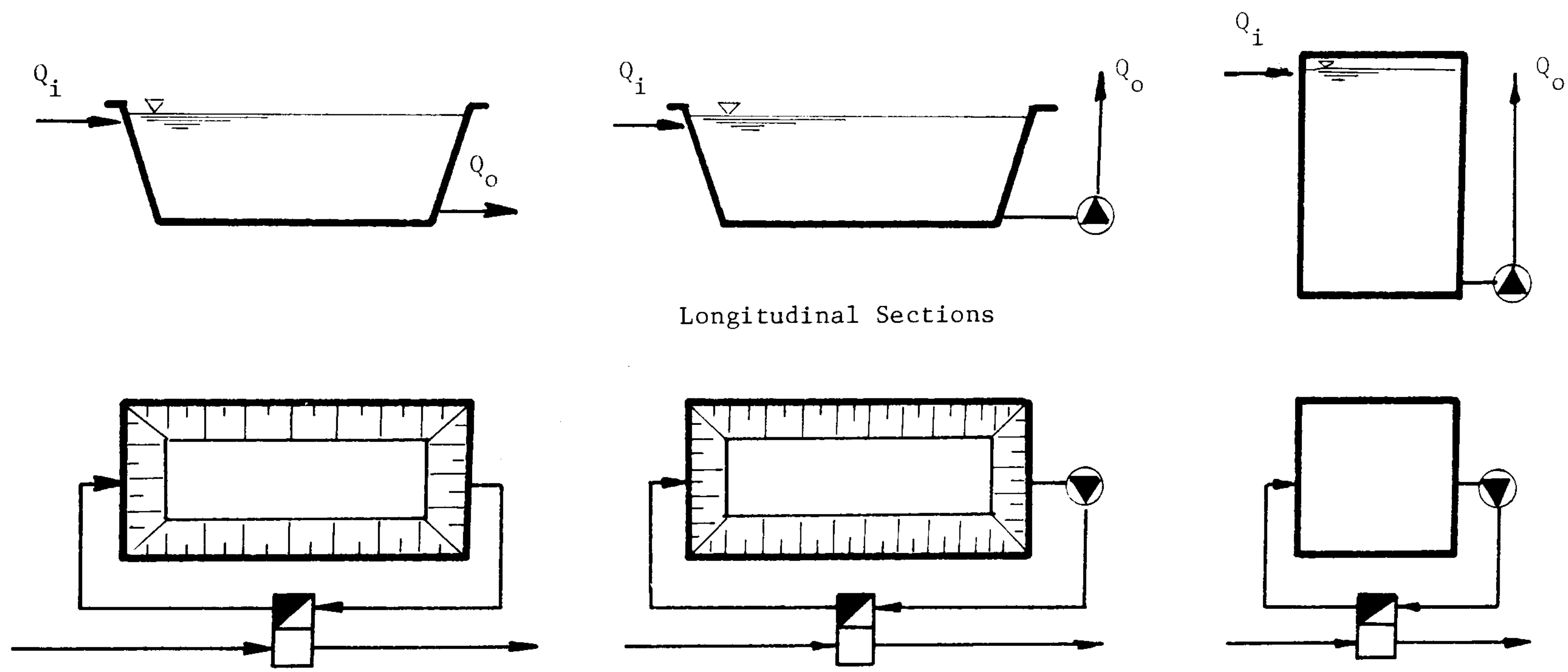
1. Pavement removal
2. Trench excavation
3. Trench shoring
4. Trench dewatering
5. Conduit bedding
6. Conduit, installed
7. Trench backfill
8. Pavement replacement

Figure 3.11 Conduit Cost Components



TYPE I	TYPE II	TYPE III
$D_c \leq 21''$	$24' \leq D_c \leq 42''$	$D_c \geq 48''$
$D_B = 48''$	$D_B = 72''$	$D_B = D_C$
$D_S = 48''$	$D_S = 36''$	$D_S = 36''$

Figure 3.12 General Conduit Junction Configurations



Plan Views

Type I

Shallow basin

Gravity inflow and outflow

Earthen Construction

Type II

Shallow basin

Gravity inflow and pumped outflow

Earthen construction

TYPE III

Deep basin

Gravity inflow and pumped outflow

Reinforced concrete construction

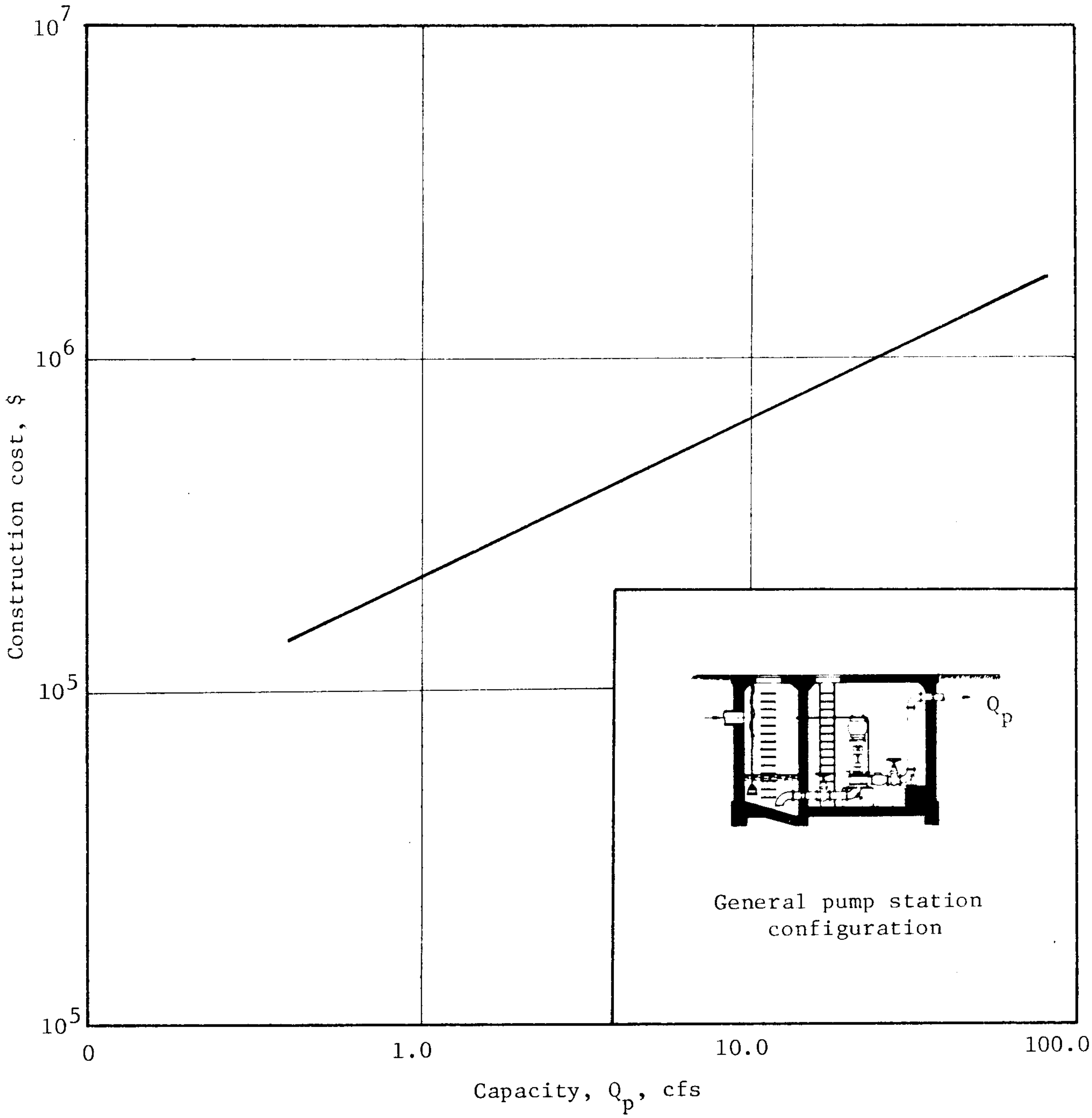
LEGEND & DEFINITIONS

- System node
- Diversion structure
- Pump Station

Q_i = inflow

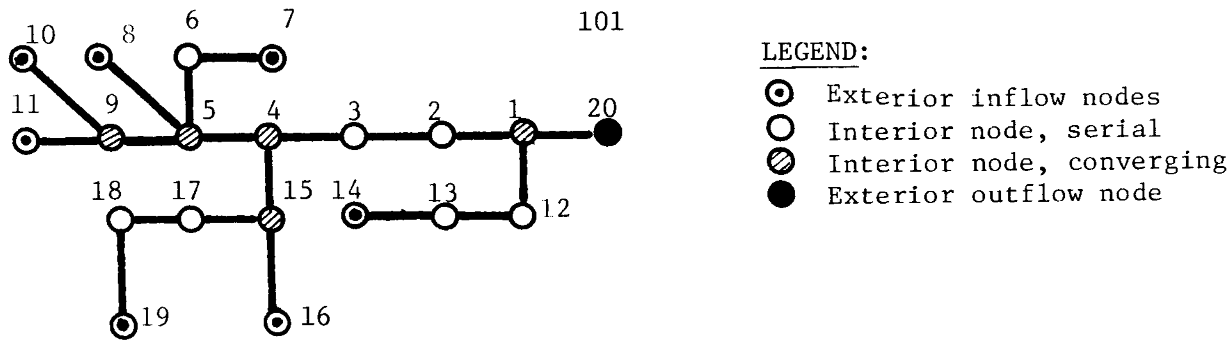
Q_o = outflow

Figure 3.13 Schematic Storage Basin Configurations

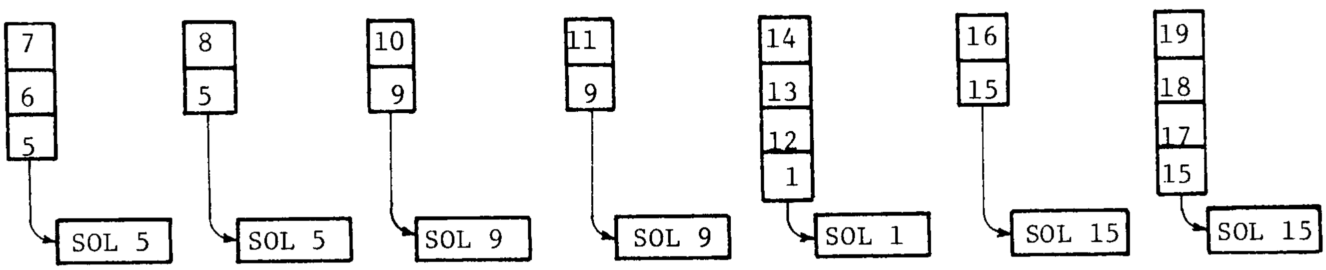


ENRCCI = 2140

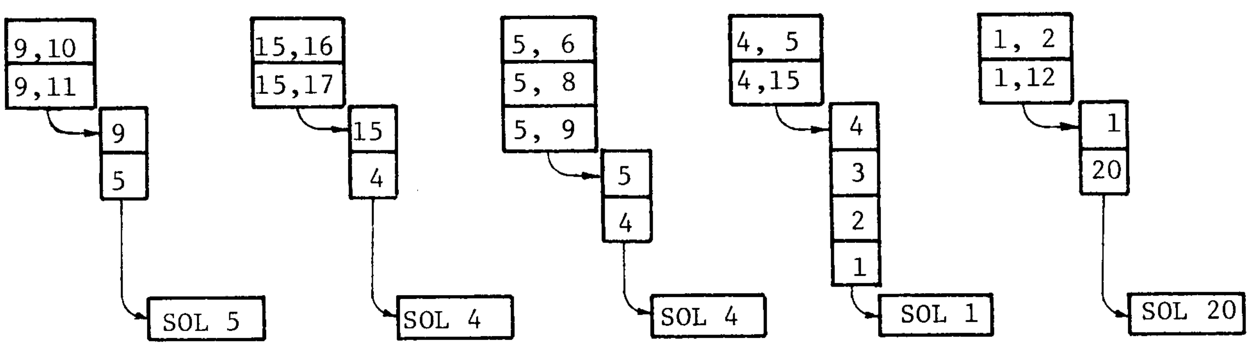
Figure 3.14 Pump Station Cost Function



a. Schematic System Layout



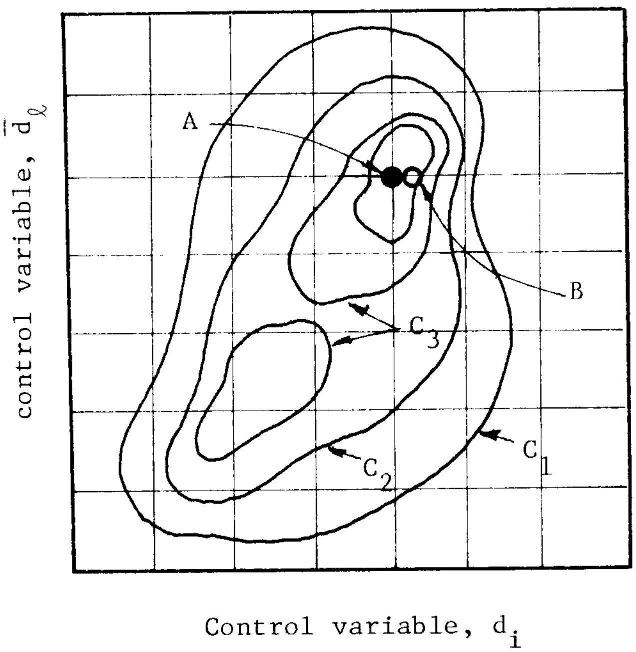
Serial system solutions starting on exterior nodes



Serial system solutions starting on interior nodes

b. Solution Order Sequence

Figure 3.15 Stage Variable Solution Order Sequence



A = quantized optimum

B = true optimum

$C_1 > C_2 > C_3$, etc. are equal cost contours

Figure 3.16 Solution Domain with Equal Cost Contours

CHAPTER IV

MODEL DESCRIPTION AND APPLICATION

> "I think we're over the chimneys and cracks," he said as I clambered over the edge and sat panting by his side. "If you look over there to the right, you will see that it is a bit broken up, and not quite vertical. I think we can pick our way up that face with a little luck." I had my doubts, but I held his rope while he inched out on the wall, and hold by hold we moved upwards ---

4.1 Chapter Introduction

The preceding chapter presented the development of an approach to determine least cost strategies in urban drainage network design. The goal of this chapter is to illustrate the implementation and the application of this technique as well as to demonstrate the general utility of the approach. The design approach that has been developed is implemented on a digital computer. The computer program described below can be purchased from the author.

Application of the model is demonstrated on a hypothetical drainage network. For the purpose of examining the effects of design constraints, network characteristics and hydrograph properties on optimal control, a series of solutions have been derived. The results and discussion of the findings are included in separate sections of this chapter.

The chapter is closed with a summary which discusses the utility and the limitations of the developed methodology.

4.2 Model Description

The optimization model is developed in a modular fashion, and consists of a main program and 30 subroutines. A schematic diagram of the overall model structure is given in Figure 4.1.

The model is programmed in FORTRAN IV and has been executed on the CDC 6400 at the University of Washington Computer Center. The program consists of approximately 2,500 FORTRAN statements and requires 116,000 octal words of high speed memory for networks consisting of 10 nodes, or stages; each stage having 10 state increments, and implementation of 10 discrete flow controls. As presently structured, the model uses

5 auxiliary magnetic discs for temporary storage of intermediate results.

A program listing for the model is given in Appendix B. Separate models are used to generate cost data for conduits and junction structures. These models, SEWRCO and JSTRUC, respectively, are not interfaced with the optimization model but generate relevant cost data which are used as input to the optimization scheme. This organization was selected for the purpose of minimizing high speed memory requirements.

Code names and general functions of each subroutine or module are summarized below.

<u>MAIN</u> controls the sequence of computational blocks to be executed

<u>READ</u> reads source data

<u>WRITE1</u> writes selected source data

<u>SEQUEN</u> decomposes branched networks into equivalent serial chains. The results from this subroutine are used to advance the solution over the stage variable and to recover the optimal policy from the dynamic programming calculations.

<u>EXECUT</u> is the primary optimization package which controls the dynamic programming calculations and the hydraulic simulation of the system elements.

<u>INITIA</u> specifies upstream system boundary conditions by generating state solution vectors for exterior inflow nodes.

<u>SHIFT</u> recovers initial boundary values for stages on serial structures from previous dynamic programming calculations.

<u>RELOAD</u> recovers boundary values for initial stages on decomposed serial chains from previous dynamic programming calculations.

<u>HYDRO1</u> implements the flow controls and calculates design flows, retention basin volumes and outflow hydrographs for exterior inflow nodes.

<u>HYDRO2</u> summarizes inflow and inlet hydrographs to interior nodes and calculates design flows, retention basin volumes and outflow hydrographs by implementing the flow controls.

<u>HYDESI</u> performs hydraulic design and cost estimation for the network elements.

<u>DNORM</u> calculates depths and velocities of steady uniform flows by Newton-Raphson iteration on the Darcy-Weisbach equation. Reynolds number and the Darcy-Weisbach friction coefficient are included in the iterative scheme.

<u>ANGLEC</u> finds the central angle for circular segments of a given conduit size by Newton-Raphson iteration.

<u>RETBAS</u> determines retention basin construction costs for specified basin types and volumes.

<u>JUNCTC</u> determines junction structure costs for specified depths and channel sizes.

<u>SHELSO</u> implements Shell's sorting algorithm to tag-sort one-dimensional arrays.

<u>STATE</u> determines optimal state transitions by implementing the recursive equation based on Bellman's principle of optimality.

<u>SOLUT1</u> stores optimal solutions for admissable states and for all stages on magnetic discs.

<u>SOLUT2</u> uses magnetic discs to store optimal solutions for admissable states on end stages to converging interior system nodes.

<u>ROUTE</u> controls calculations of the gradually varied, unsteady flow propogations through the network channels using one of the routing techniques described in Section 3.6. The technique selected for use in a particular case depends on the channel properties and is selected according to the decision criteria derived in Appendix A.

<u>HYDRO3</u> loads hydrographs temporarily stored on discs into high speed memory arrays.

<u>DYNEMA</u> calculates initial conditions and boundary values and controls the flow routing based on the dynamic equation set.

<u>IDYNAM</u> performs numerical integration of the dynamic wave equations where an implicit solution is found by N-dimensional Newton-Raphson iteration.

<u>KINEMA</u> calculates initial conditions and boundary values and controls the flood wave routing based on the kinematic equation set.

<u>IKINEM</u> performs numerical integration of the kinematic wave equations where an implicit solution is found by N-dimensional Newton-Raphson iteration.

<u>DARCYW</u> calculates Darcy-Weisbach friction factors for flows in partially and completely filled circular conduits by Blasius or von-Karman equations depending on the Reynolds number.

<u>FSLOPE</u> uses the Darcy-Weisbach equation to calculate friction slopes for flows in partially and completely filled circular conduits.

<u>INVR</u> inverts non-singular square matrixes by Jordan's elimination method.

<u>JUNCTN</u> determines state solution vectors for non-serial stages by implementation of the recursive equation based on Bellman's principle of optimality.

<u>OPTSOL</u> recovers the optimal trajectory from the results of the dynamic programming calculations.

<u>WRITE2</u> writes physical and hydraulic characteristics and cost of optimal controls.

4.3 Model Application

Model application was undertaken for a two-fold purpose. First, the utility of the model within the previously defined sequence of activities during the planning phase of urban drainage alternatives was examined, and secondly, in keeping with the study objectives, an attempt was made to determine the relationships between inlet hydrograph properties and drainage network costs. For the purpose of this discussion are test network, hydrographs, quantization of state and decision variables, and the system constraints described below.

Test Network Characteristics

A hypothetical drainage basin was chosen for model testing purposes. The layout of the drainage network alternative which was used is shown in Figure 4.2 and assumed to be the result of the activity labeled "Identify Alternatives to Achieve Goals" shown on the planning domain in Figure 3.2. The network data requirements are listed in Table 4.1.

The test network consists of seven nodes and six channels. The total channel length is 38,800 feet and the average ground slope between the nodes is approximately 0.005 ft/ft. The network can then, by using the average ground slope as a criterion, be classified as flat.

Control Costs

Conduit and junction structure costs were derived by submodels SEWRCO and JSTRUC, respectively. The results are printed in Tables 4.2 and 4.3. The cost function for lift stations is given by equation 3.63, and retention basin costs are given by equations 3.60 through 3.62 and shown in Figure 4.3.

All cost functions are considered to be representative for Seattle area price levels for the second quarter of 1975 and to represent total implementation costs. They do not, however, include overhead, profit and cost of land, where applicable. The cost values in Tables 4.2 and 4.3 are in good agreement with work by Merritt (1970), Meredith (1971) and cost values reported in the Dodge Guide (1974). The retention basin costs used in this study are higher than those used by Crawford (1973) for Santa Maria, California.

Quantization of State and Decision Variables

Stage, state and decision variables defined by continuous functions need to be quantized into discrete increments before inclusion in the dynamic programming problem. Thus, dynamic programming problems that include quantized variables might result in a solution which is in the neighborhood of the true optimum; especially if the quantization is too coarse. The solution can be improved by using smaller increments, but a finer resolution will increase the computational requirements.

The model which has been developed through this study requires the state variables to be quantized over the feasible channel trajectory corridor at each stage, and similarly, the decision variables need to be included as discrete sets. One of the decision variables, channel sizes, follows standard design practice by limiting the decisions to commercial dimensions. Since all conduit dimensions are considered, approximations are not introduced through this variable. The other decision variable, design flow rates, and the state variables are quantized into ten increments at each stage. As 30 feet was selected as the maximum trench depth, this resolution gives state increments of less than 2.5 feet in a typical solution, and the previously described flow control vector was selected as:

$$\bar{i} = [1.00, 1.10, 1.25, 1.50., 1.75, 2.00, 2.50, 3.00, 3.50, 4.00] \quad (4.1)$$

The design flows are determined by dividing the peak flows on the inflow hydrographs by the flow control vector elements. Thus, the model can be used without considering retention basins as a control opion simply by specifying $\bar{i} = 1.00$ in equation 4.1.

While finer state variable resolution could be desirable for actual design problems, ten increments were selected for the purpose of reducing computation time. This was considered justifiable since "optimal" solutions were not found to vary significantly with the number of state increments that were used. This observation is in agreement with the findings of Thielke (1973) and Mays and Yen (1975).

System Constraints

The numerical values of the system constraints that were used during the model testing phase are

Minimum conduit cover:	3.00 ft
Maximum depth of conduit invert:	30.00 ft
Minimum average velocity of flow:	3.00 ft/sec
Maximum average velocity of flow:	12.00 ft/sec

A pipe progression constraint requiring downstream conduit sizes to be at least as large as the upstream one was imposed on all dimensions equal to or less than 24 inches in diameter.

Test Hydrographs

The hydrographs selected for model testing purposes are shown on Figure 4.4. These hypothetical runoff hydrographs are assumed to be selected periods on simulated flow hydrographs resulting from known precipitation events on the drainage basin sub-catchments.

Two series of hydrographs were used. The first set, which is shown on Figure 4.4a, have identical peaks, 120 cfs, but have different volumes.

The purpose of using this series is to examine the impact on network cost resulting from varying runoff volumes at equal peaks. This would provide insight into the importance of accuracy in estimating overland flow supply or precipitation losses in urban runoff modeling.

The second hydrograph set has constant flow volumes but different peak flows, and is shown in Figure 4.4b. The purpose of using this series was to examine the cost impact of identical flow volumes and varying peak discharges. This would explore the importance of peak runoff rate simulation in urban runoff modeling.

Identical hydrographs were imposed on all exterior inflow and interior nodes[a] during all test runs. Thus, the drainage network carries flow from six identical input hydrographs for each test case.

4.4 Results

System Design

Table 4.4 is a sample model output. The table summarizes physical and hydraulic characteristics and implementation cost of the optimal design. The table on physical characteristics contains the information which, in addition to the network layout, is needed for detail design. Among these are sizes, invert elevations, depths and bedslopes for all conduits, pump station capacities and lift heights, and retention basin configurations and volumes. Pumping is not included among the set of controls in this particular solution.

The tabulated hydraulic characteristics summarize the performance information which would be of interest to the design engineer.

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[a]Node definitions are given on Figure 3.15.

The cost table lists the control costs over each stage and summarizes the total costs.

Figures 4.5 through 4.8 illustrate optimal system cost and outflow hydrograph sensitivity to changes in input hydrographs. In particular, Figure 4.5b shows the cost consequence of designing the test network to carry inlet hydrographs that have different volumes. As expected, the control costs increase as the runoff volumes increase, but this investigation has been too limited in scope to allow general conclusions to be drawn. Network design was carried out both with and without retention storage as a control option. Figure 4.5 shows that the inclusion of retention basins among the controls reduces the system cost approximately 12% on the average. The cost reduction obtainable through use of retention storages is expected to depend largely on the configuration of the drainage network and cannot be stated in general terms based on the limited model application to date.

Figure 4.6 shows the outflow hydrographs for the optimal designs summarized in Figure 4.5b. System designs which include retention storage are shown to have slower hydrograph rises and reduced peak flows. The results also show that systems designed without retention storages have transient pipe storage capacities which attenuate the flood peaks significantly.

The test network, when designed to carry the second hydrograph series, has costs as shown in Figure 4.7b. The corresponding hydrographs are shown in Figure 4.7a. All designs include storage as a control option. The outflow hydrographs for all design alternatives are shown in Figure 4.8.

The control costs are, as indicated by Figure 4.7b, sensitive to variations in inlet hydrograph peaks, but the model has not yet been used extensively enough to allow general conclusions to be made. However, the results suggest that runoff hydrograph peaks should be accurately simulated to serve the needs of urban drainage design.

Computational Aspects

The array dimensions used for the test problem have the following magnitudes.

Variable	Array Dimensions	Test Problem Dimensions
Stages	10	7
States	10	10
Flow Controls	10	1-10
Conduit Sizes	31	31
Hydrograph routing time increments	100	60

As previously reported, model implementation with these array dimensions required approximately 116,000 octal words of high speed memory on the University of Washington CDC 6400 digital computer.

The total computation time can be broken into two components: the optimization calculations, which include hydraulic design and control cost estimation; and hydraulic simulation or wave prop gation calculations. For the test cases, it was found that the total run times were approximately 155 and 180 seconds[a] per stage for 1 and 10 flow controls,

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[a] Seconds refer to "central processor" seconds used by the CDC 6500 computer.

respectively. Of this time, 55 seconds per stage, or approximately one-third, were used for hydraulic simulation and the remainder was used for the dynamic programming calculations.

4.5 Discussion

Examination of the Approach

Several comments can be made on the design approach which has been developed during this inquiry. One question that must be addressed is whether or not the approach and the optimal solution provided by the analysis is a useful approach to urban drainage network design. From a positive standpoint, it has been shown that the model determines the optimal solution among all feasible alternatives and that the control strategy problem is unique and can be optimized on a least cost basis. Furthermore, the model as developed gives all information necessary to carry out detail design together with itemized and total cost summaries for the optimal controls. Finally, the approach has reasonable data and computational requirements and since it is computer based it posesses the possibility of having several practical advantages. These could include benefits as a result of systemized procedures, redistribution of technical personnel and elimination of the need to undertake highly repetitive and routine operations, rapid execution of computations, standardization of report formats and reduction in cost of planning and design.

The cost savings that might result from applying the developed optimization technique are difficult to estimate but appear to be substantial. The cost-saving potential has two aspects:

1. Application of optimization techniques to determine least cost

combinations of drainage conduits have been found to substantially reduce total network costs. Merritt and Bogan (1973) estimated that overall cost savings of 10 to 20% would typically result from such an approach.

2. Inclusion of retention storage among the possible controls might result in additional overall savings over least cost conduit combinations. This increment is shown in Figure 4.5b where it amounts to approximately 12%.

Thus, the approach developed during this inquiry could, when compared to traditional approaches, reduce the implementation cost of urban drainage by as much as 30%, and should result in functionally improved designs. This cost saving potential can be assessed on the basis of information from the literature. Using estimates by Knapp and Rawls (1969) updated to 1975 price levels, urban drainage cost at least $1,750 per acre; thus, overall savings in excess of $500 per acre could be realized.

The proposed design approach would require more information than conventional methods do. The cost of collecting and processing such data would probably be subject to considerable economics of scale, but should not be associated with exhorbitant time or money requirements. Thus, Linsley (1971) suggested that the cost of data processing and computer simulation required to generate such information should be below $10 per acre, even for very small watersheds.

Since optimal control strategies are based on selecting least cost solutions, all control costs need to be adequately described. This requirement gives rise to a very basic epistemological question.

Identification of drainage network component costs as a function of size and excavation depths is not a trivial task. Presently, most drainage construction projects are broken down as to conduit size but not into depth increments for bidding and payment purposes. Rather, the estimates are made on an average cost per foot basis for the entire job, as designed. Attempts to correlate costs between jobs with depths are usually futile because of the large number of differences in addition to depth, and none of the numerous equations derived to describe these cost relationships is entirely satisfactory.

This situation led to the development of computerized cost estimating procedures for conduits and junction structures as described in Section 3.7. Thus, the solution chosen as the "best one" could fail to identify the true optimum if the control costs were erroneously estimated and the approach was sensitive to these costs.

Fortunately, the optimization procedure incorporated in the design algorithms only requires correct relative costs for the control options, and since the cost functions are estimated on the same unit cost basis, they are homogenous throughout their application domain and were found to agree closely with the work of other investigators (Merritt, 1970; Meredith, 1971). Furthermore, Holland (1966) used cost functions similar to those employed in this study to examine the sensitivity of optimal controls to changes in cost functions and found that major changes in the relative magnitudes of the cost functions were necessary to cause significant changes in the optimal solution. Based on this, the control cost relationships used in this inquiry could be deficient in reliably predicting total project costs for a variety of conditions but are considered adequate as a basis for selecting least cost control strategies.

The treatment of the hydrologic variables should be examined to identify model weaknesses. The model, as structured, determines the least cost solution to fixed network layouts for known drainage basin configurations, and relies upon using a "design period" as the hydrologic variable. The design period is a series of storm events for which the runoff hydrograph is derived by continuous simulation. Thus, the runoff hydrographs used in the model testing are taken to be the runoff events resulting from the chosen design periods. The optimization problem has then been conceptualized as a discrete decision process which is deterministic in the sense that state transitions and control costs are uniquely determined by the state variables, the decisions and the stages.

The design period concept, which has also been suggested by Crawford (1973), is essentially a trade-off between two conflicting interests. On one hand, there is an incentive to limit the simulation length since unsteady flow routing is time consuming, even on a digital computer, and the computation efforts are roughly proportional to the simulation length, if the time steps used in the numerical schemes are held constant. On the other hand, determination of flooding probabilities based on a short record might be quite misleading.

Related to the above, significant questions would be: is the true optimum identified and do the model outputs agree with actual occurrences? The answer to the question of model validity, or whether or not the true optimum has been identified cannot be verified by field experiments, or beyond the proof offered in Section 3.9. The second question, the one of adequately reproducing prototype behavior would quite obviously be impossible to know a priori, but could theoretically be answered through field experiments. Ideally this would consist of using the model for system design, and once designed and implemented, prototype behavior and simulated system behavior should be compared. In this vein, satisfactory results from a wide range of applications which hold up under close scrutiny would build confidence in the approach, although comparison of prototype behavior and model performance over the entire application domain of the model would be a prohibitive undertaking.

The limited experience with the design methodology and the limitations mentioned above serve to point out the preliminary nature of the present model structure. Nevertheless, the dynamic programming optimization approach to selective control strategy formulation as developed, appears to offer an attractive means of generating flexible and realistic alternatives at considerable cost savings.

Suggested Approach to Runoff Hydrograph Generation

In addition to the optimization and simulation models that were developed during this inquiry, a runoff hydrograph generator needs to be included in order to make the design package complete. The purpose of this discussion is to identify the methods or techniques that should be used and the steps that should be taken to model urban runoff hydrographs.

Efficient urban drainage design can only be based on knowledge of the peaks and volumes of runoff from the watershed. Only on this basis can economic evaluations and comparisons of drainage costs against the benefits from drainage be examined. In this context, incorrectly determined "return periods" might lead to overdesign at excessive cost or underdesign with nuisance flooding, or flooding damages occurring more frequently than expected.

Larson (1965) showed that the relationships between rainfall frequencies and flood peak frequencies is poorly defined. While it is felt by many that this correlation improves as the impervious portion of the watershed increases, there is little evidence to support this belief. This suggests that the general concept of a "design storm" is an unsound basis for design and that only continuous simulation which permits development of peak frequency at each point in the system on the basis of historic rainfall records can provide the data required for a design in which all elements in the system are designed for a desired probability level.

Theoretical considerations and experience with rural watersheds confirm that a continuous simulation model must involve a water balance with loss rates in some way related to the current moisture storage in the watershed. Arguments in support for such capability in urban runoff models can be discussed. Since antecedent rains create initial flows which increase peaks and runoff volumes throughout the system, soil moisture storage should be accounted for on pervious catchments. Thus, simulation of runoff from pervious areas should be based on the water balance (flow and moisture continuity) concept, while use of simple loss functions should be adequate on impervious areas. However, planning needs require application of the model to future assumed conditions in both cases, and the techniques that are used should be based on physical properties of the watershed such that the need for historical flow data is eliminated.

Another question revolves around the need for linear or non-linear flow routing capability. It can be shown that linear routing is equivalent to using the unit hydrograph principle, while non-linear techniques treat the flow in the watershed as a problem of applied fluid mechanics.

The major assumption of the linear systems techniques used in hydrology is that the system is linear and hence, the principle of superposition applies. While most hydrological systems are non-linear, Linsley (1971) claims that use of linear techniques does not introduce serious error in the analysis of moderate flows. Extrapolation substantially beyond the range of data from which the unit hydrograph is derived can, however, lead to large inaccuracies, but since data are not commonly available to check this extrapolation, no indication of likely errors or modeling inaccuracy is evident.

Perhaps in stronger support of a non-linear routing method is the absolute requirement that the runoff generation should be capable of operating in a forecast mode. This excludes linear routing methods since it was concluded during the literature study phase of this inquiry that use of "synthetic" unit hydrographs was an unproductive approach to urban runoff modeling. On the other hand, physically based routing is attractive, but subject to the difficulty of describing the geometry of the flow system.

Linsley (1971) also summarized the importance of the overland flow process in modeling urban runoff. He concluded that the detention in overland flow is a very significant part of the total system storage, and could for small systems exceed the storage effects in sewers or channels. As the basin size increases, however, the channel storage gains importance and eventually dominates the process such that overland flow effects are negligible. This suggests that the storage of the surface may be a variable of great importance in urban runoff studies, especially if small basin hydrographs are to be obtained.

Physically based routing could be based on either the dynamic or kinematic wave formulations, but the kinematic wave concept would be preferred because of its smaller computational requirements. Lighthill and Witham (1955) found the kinematic approximation to be satisfactory as long as the Froude number was less than two.

The experience with the developed optimization model and the previous discussion make it possible to specify the characteristics of the runoff hydrograph generator that should be included in the optimization package. These properties are summarized in Table 4.5, and it is clear that the model should be computer based if it is to meet the other specifications. The need to estimate probabilities of occurrence for events of various magnitudes, and planning needs requiring application of the model to assumed future conditions, together with the requirement of reasonable operating costs, form the basis for the general properties of the model. These properties necessarily require a continuous record of precipitation data. For large urban areas, spatial variations may be important, hence, the model must be able to utilize all available precipitation data to simulate the effects of areal variations.

Specific functional model constraints are difficult to list. Continuous simulation necessarily requires a continuous estimate of rainfall losses. Whether this simulation should be based on the soil moisture concept or on simple loss functions, can be determined through sensitivity analysis during the model development phase. In the same vein, selection of overland flow routing technique cannot be based on findings from the literature alone, thus it is felt that application domains for these techniques has to be established by numerical comparisons and abilities to reproduce field observations.

4.6 Chapter Summary

The dynamic programming analysis developed in this study has been implemented on a digital computer and applied to a hypothetical drainage network. The model application was carried out for a two-fold purpose; first, the utility of the model as a design tool was examined, and secondly, an attempt was made to examine the effects of hydrograph properties on network costs.

A detailed description of the model subroutines was given to indicate the level of computational complexity required by the approach used.

Application of the optimization requires quantization of state and decision variables and thereby introduces an approximation. The optimal solution, however, was found to be only modestly sensitive to this approximation.

Cost relationships for installed concrete conduits and junction structures were generated by separate models. Unit prices and cost relationships used as input to these models as well as to determine costs of the other controls were obtained from a variety of sources. Initial testing, comparisons with work by others, and the fact that optimal solutions have been found not to be sensitive to variations in the control costs led to the conclusion that the control cost functions were adequate as a basis for selecting least cost control strategies, but might be deficient in reliably predicting total project costs for a variety of conditions.

Model testing and network designs were based on a series of inlet hydrographs. The associated network costs were found to be sensitive both to variations in runoff peaks and volumes. These findings and information from the literature were used to identify a proposed urban runoff hydrograph generator. This method should base modeling of runoff on physical properties of the watershed such that the dependence upon historical flow data is diminished, and be able to perform continuous simulation of runoff quantities and qualities.

As postulated at the onset of this inquiry, inclusion of off-line storage capacity as a control option in urban drainage systems reduces the overall network costs. This reduction was found to be approximately 12% for the network that was analyzed. This cost reduction is based on a comparison with the least cost combinations for the remaining controls and use of calculation techniques that take transient conduit storages into consideration.

Although the limited experience and the possible limitations mentioned above serve to point out the preliminary nature of the present model structure, the optimization by dynamic programming, as developed, appears to be a very useful approach to urban drainage network design. The method requires limited amounts of data, has reasonable computational requirements and generates solutions with potential cost savings of up to approximately 30% when compared to conventional drainage network design methods.

Table 4.1 Test Network Data

NODE NO.	BASIN TYPE	GROUND ELEVATION FT	DOWNSTREAM NODE NO.	DOWNSTREAM CHANNEL LENGTH FT	UPSTREAM NODE NO.	UPSTREAM NODE NO.	UPSTREAM NODE NO.
1	1	210.00	3	4800.00	0	0	0
2	1	200.00	3	6400.00	0	0	0
3	2	175.00	4	7200.00	1	2	0
4	3	140.00	7	8000.00	3	5	6
5	1	175.00	4	6000.00	0	0	0
6	1	155.00	4	6400.00	0	0	0
7	0	100.00	0	.00	4	0	0

Table 4.2 Total costs of Installed Concrete Conduits - $/lin.ft.

DIA IN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	INVERT DEPTH FT																													
4	0	0	13	14	15	16	17	18	19	23	24	26	27	29	30	32	34	35	37	39	41	43	47	50	53	55	56	62	65	69
6	0	0	14	14	16	17	18	19	20	24	25	27	28	30	31	33	35	36	38	40	42	44	48	51	53	56	59	63	66	70
8	0	0	14	15	16	17	18	19	20	24	25	27	28	30	32	33	35	37	39	41	43	45	49	52	54	57	60	63	67	71
10	0	0	0	15	17	18	19	20	21	25	26	28	29	31	33	34	36	38	39	41	43	46	51	53	56	59	62	65	68	72
12	0	0	0	16	17	18	19	20	21	25	27	28	30	31	33	35	36	38	40	42	44	47	49	52	54	57	60	64	67	72
15	0	0	0	17	18	20	21	22	23	26	28	29	31	32	34	36	39	40	41	43	45	48	50	53	55	58	61	65	68	73
18	0	0	0	19	20	21	22	23	24	28	30	31	33	34	36	38	40	42	44	46	48	50	53	56	58	61	64	68	72	76
21	0	0	0	0	22	23	24	25	26	30	32	34	35	37	38	41	43	45	47	49	51	54	57	59	62	66	69	73	77	81
24	0	0	0	0	24	26	27	28	29	33	35	36	38	39	41	44	46	48	50	52	55	58	60	63	66	70	73	77	81	86
27	0	0	0	0	29	30	31	32	34	38	40	42	44	45	49	51	53	55	57	59	62	65	67	70	74	77	83	87	91	95
30	0	0	0	0	0	33	35	36	38	42	44	46	48	50	51	55	57	59	62	64	67	69	72	76	79	83	88	92	96	101
33	0	0	0	0	0	36	38	39	40	45	47	49	51	53	55	57	59	63	66	68	71	74	77	80	84	87	92	96	101	105
36	0	0	0	0	0	39	41	42	43	47	50	52	54	57	59	61	65	67	70	72	75	78	81	85	88	92	97	101	106	111
42	0	0	0	0	0	0	50	51	53	57	59	62	64	66	69	71	74	76	79	85	88	91	95	98	102	106	111	116	121	127
48	0	0	0	0	0	0	57	59	60	65	66	68	73	75	78	81	83	86	89	95	99	102	106	110	114	119	124	129	134	140
54	0	0	0	0	0	0	0	66	68	72	74	76	81	83	86	90	92	95	98	106	109	113	117	121	126	131	136	142	146	154
60	0	0	0	0	0	0	0	77	79	84	86	88	90	96	98	103	106	109	112	116	126	130	134	138	143	149	154	160	167	174
66	0	0	0	0	0	0	0	0	90	95	97	99	102	108	111	114	120	124	127	131	141	146	150	155	160	166	172	179	185	193
72	0	0	0	0	0	0	0	0	101	106	108	111	113	120	123	130	133	137	141	153	157	162	167	172	178	184	190	197	215	223
78	0	0	0	0	0	0	0	0	0	118	121	123	126	134	137	141	148	151	155	160	173	178	183	189	195	201	208	215	230	238
84	0	0	0	0	0	0	0	0	0	130	133	136	133	141	151	154	161	165	170	174	189	194	199	205	212	218	226	233	242	259
90	0	0	0	0	0	0	0	0	0	0	141	144	147	150	160	164	168	176	181	185	190	206	212	218	225	232	240	248	257	275
96	0	0	0	0	0	0	0	0	0	0	149	153	156	159	162	174	178	187	192	197	202	219	225	232	239	247	255	263	273	292
102	0	0	0	0	0	0	0	0	0	0	0	161	164	168	171	184	189	193	203	208	214	220	238	245	253	261	270	279	289	299
108	0	0	0	0	0	0	0	0	0	0	0	169	173	177	180	184	199	204	214	220	226	232	252	259	267	276	285	294	305	316
114	0	0	0	0	0	0	0	0	0	0	0	0	181	186	189	193	197	215	221	232	238	245	252	273	282	291	300	310	321	333
120	0	0	0	0	0	0	0	0	0	0	0	0	190	195	198	202	207	226	232	244	250	257	265	287	296	306	316	326	338	350
126	0	0	0	0	0	0	0	0	0	0	0	0	0	203	208	212	216	221	244	250	263	270	273	287	311	321	332	343	355	368
132	0	0	0	0	0	0	0	0	0	0	0	0	0	212	217	222	226	231	255	262	275	283	292	301	326	336	347	359	372	385
138	0	0	0	0	0	0	0	0	0	0	0	0	0	0	225	231	236	241	247	274	282	296	305	315	325	352	364	376	390	404
144	0	0	0	0	0	0	0	0	0	0	0	0	0	0	234	241	246	251	257	287	295	310	319	329	340	368	380	393	407	422

Table 4.3 Total Costs of Conduit Junctions, $

INVERT DEPTH (FT)	TYPE I DC≤21	TYPE II 24≤DC≤42	TYPE III 48≤DC≤78	DC≥84
1	.00	.00	.00	.00
2	.00	.00	.00	.00
3	313.54	.00	.00	.00
4	314.27	871.28	.00	.00
5	364.87	872.57	.00	.00
6	402.02	873.94	.00	.00
7	439.25	910.69	1398.20	.00
8	476.56	946.85	1434.55	.00
9	514.00	983.21	1470.90	.00
10	551.58	1019.82	1507.25	.00
11	589.33	1056.72	1543.60	3165.95
12	627.27	1093.94	1579.95	3165.95
13	665.42	1131.53	1616.30	3184.12
14	703.81	1169.52	1652.65	3220.47
15	742.46	1207.96	1689.00	3256.82
16	781.39	1246.88	1725.35	3293.17
17	820.63	1286.33	1761.70	3329.52
18	860.20	1326.34	1798.05	3365.87
19	900.11	1366.95	1834.40	3402.22
20	940.49	1408.20	1870.75	3438.57
21	981.03	1450.13	1907.10	3474.32
22	1022.19	1492.77	1943.45	3511.27
23	1063.73	1556.17	1979.80	3547.62
24	1105.74	1580.35	2016.15	3583.97
25	1148.22	1625.36	2052.50	3620.32
26	1191.21	1671.23	2088.85	3656.67
27	1234.72	1718.01	2125.20	3693.02
28	1278.78	1765.71	2161.55	3729.37
29	1323.41	1814.39	2197.90	3765.72
30	1368.62	1864.08	2234.25	3802.07

DC = Conduit diameter, inches

Table 4.4 Sample Output

PHYSICAL CHARACTERISTICS OF OPTIMAL DESIGN

UPSTREAM NODE NO.	DOWNSTREAM NODE NO.	CHANNEL CHARACTERISTICS LENGTH FT	DIAMETER FT	SLOPE FT/FT	UPSTREAM CONDITIONS GROUND ELEV. FT	INVERT ELEV. FT	DEPTH FT.	DOWNSTREAM CONDITIONS GROUND ELEV. FT	INVERT ELEV. FT	DEPTH FT	PUMP STATION CAP. CFS	LIFT HIGHT FT	RETENTION BASIN TYPE	VOLUME CF
1	3	4800	3.50	.00707	210.00	202.00	8.00	175.00	168.08	6.92	0	0	1	4862
2	3	6400	3.50	.00383	200.00	192.00	8.00	175.00	167.50	7.50	0	0	1	12007
5	4	6000	3.00	.00568	175.00	167.00	8.00	140.00	132.92	7.08	0	0	1	12007
6	4	6400	3.50	.00253	155.00	147.00	8.00	140.00	130.81	9.19	0	0	1	12007
3	4	7200	5.00	.00435	175.00	163.25	11.75	140.00	131.90	8.10	0	0	2	4875
4	7	8000	6.50	.00441	140.00	125.12	14.88	100.00	89.84	10.16	0	0	0	0

HYDRAULIC CHARACTERISTICS OF OPTIMAL DESIGN

UPSTREAM NODE NO.	DOWNSTREAM NODE NO.	CHANNEL CHARACTERISTICS LENGTH FT	DIAMETER FT	SLOPE FT/FT	CAPACITY CFS	MAX FLOW CFS	MAX VEL FPS	PUMP STATION CAP. CFS	LIFT HIGHT FT	RETENTION BASIN TYPE	VOLUME CF
1	3	4800	3.50	.00707	84.78	80.00	9.50	0	0	1	4862
2	3	6400	3.50	.00383	62.40	48.00	6.90	0	0	1	12007
5	4	6000	3.00	.00568	50.39	48.00	7.70	0	0	1	12007
6	4	6400	3.50	.00253	50.72	48.00	5.70	0	0	1	12007
3	4	7200	5.00	.00435	172.31	160.00	9.30	0	0	2	4875
4	7	8000	6.50	.00441	349.09	320.00	11.10	0	0	0	0

COST OF OPTIMAL DESIGN

UPSTREAM NODE NO.	DOWNSTREAM NODE NO.	CHANNEL COST $	JUNCTION COST $	PUMP STATION COST $	RETENTION BASIN COST $	STAGE COST $
1	3	177600	911	0	9770	188281
2	3	236800	947	0	23482	261229
5	4	210000	947	0	23482	234429
6	4	249600	1020	0	23482	274102
3	4	381600	1471	0	31906	414977
4	7	576000	1544	0	0	577544
TOTAL COST		1831600	6840	0	112122	1950562

Table 4.5 Properties of a Proposed Urban Runoff Hydrograph Generator

General Properties

Continuous simulation of runoff quantities and qualities at any specified point in the watershed.

Should be capable of operating in forecasting mode.

Should be computer based and have reasonable operating costs.

Input Constraints

Should use precipitation as basic input.

Should operate on any specified time interval.

Should accept inputs from several stations and for any spatial resolution.

Should provide for stochastic input.

Functional Constraints

Be based on physical phenomena and should not require judgement factors.

Base loss functions on physical phenomena.

Overland flow simulation based on non linear routing principles.

```
C. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C
C            *MAIN
C            *  CALLS
C            *         *READ
C            *         *WRITE1
C            *         *SEQUEN
C            *         *EXECUT
C            *         *  CALLS
C            *         *         *INITIA
C            *         *         *  CALLS
C            *         *         *         *JUNCTC
C            *         *         *SHIFT
C            *         *         *RELOAD
C            *         *         *HYDRO1
C            *         *         *HYDRO2
C            *         *         *HYDESI
C            *         *         *  CALLS
C            *         *         *         *DARCYW
C            *         *         *         *DNORM
C            *         *         *         *ANGLEC
C            *         *         *         *RETBAS
C            *         *         *         *JUNCTC
C            *         *         *SHELSO
C            *         *         *STATE
C            *         *         *SOLUT1
C            *         *         *SOLUT2
C            *         *         *ROUTE
C            *         *         *  CALLS
C            *         *         *         *HYDRO3
C            *         *         *         *DYNEMA
C            *         *         *         *  CALLS
C            *         *         *         *         *DNORM
C            *         *         *         *         *DARCYW
C            *         *         *         *         *FSLOPE
C            *         *         *         *         *IDYNAM
C            *         *         *         *         *  CALLS
C            *         *         *         *         *         *DARCYW
C            *         *         *         *         *         *FSLOPE
C            *         *         *         *         *         *INVR
C            *         *         *         *KINEMA
C            *         *         *         *  CALLS
C            *         *         *         *         *DNORM
C            *         *         *         *         *IKINEM
C            *         *         *         *         *  CALLS
C            *         *         *         *         *         *DNORM
C            *         *         *         *         *         *DARCYW
C            *         *         *         *         *         *INVR
C            *         *         *JUNCTN
C            *         *OPTSOL
C            *         *         *  CALLS
C            *         *         *         *DNORM
C            *         *WRITE2
C            *         *EXIT
C
C. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
```

Figure 4.1 Overall Program Structure

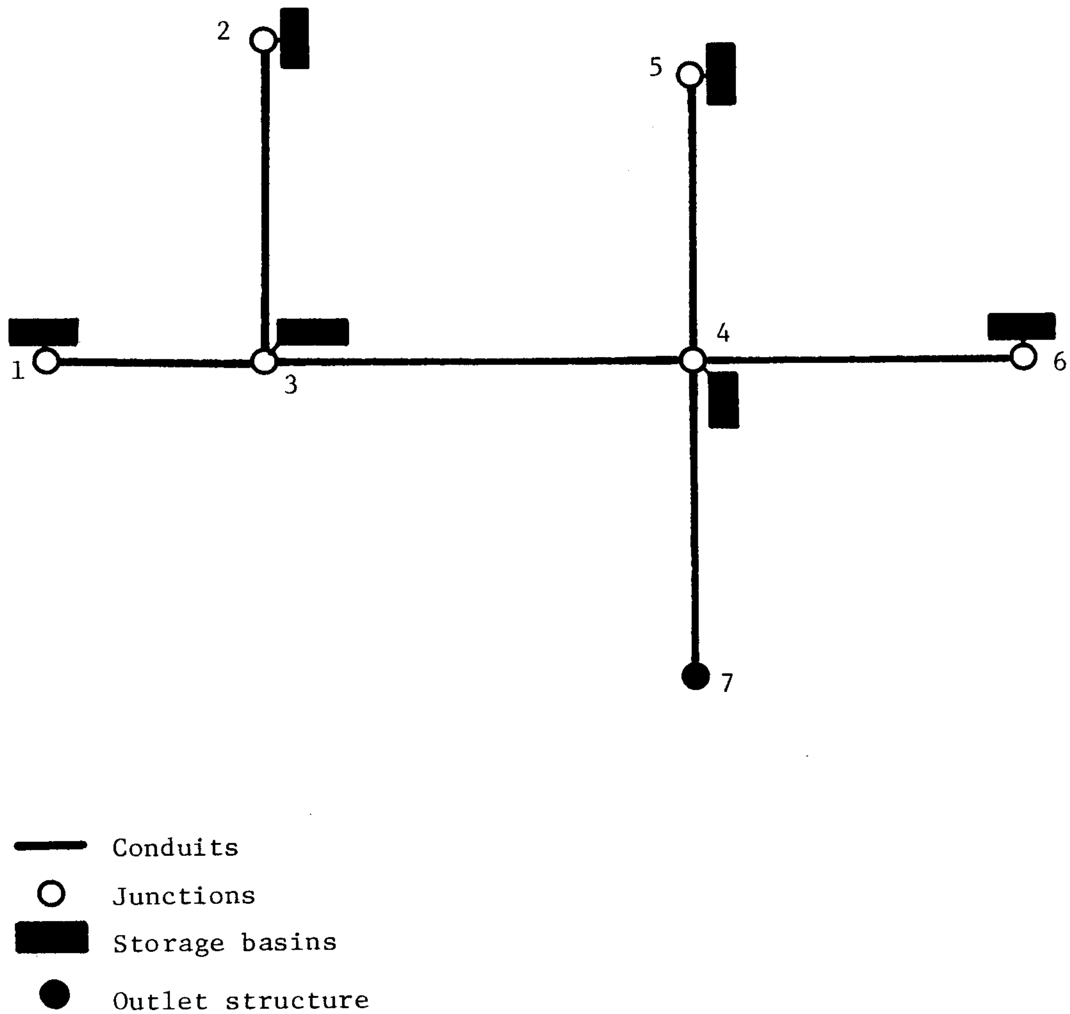


Figure 4.2 Test Network

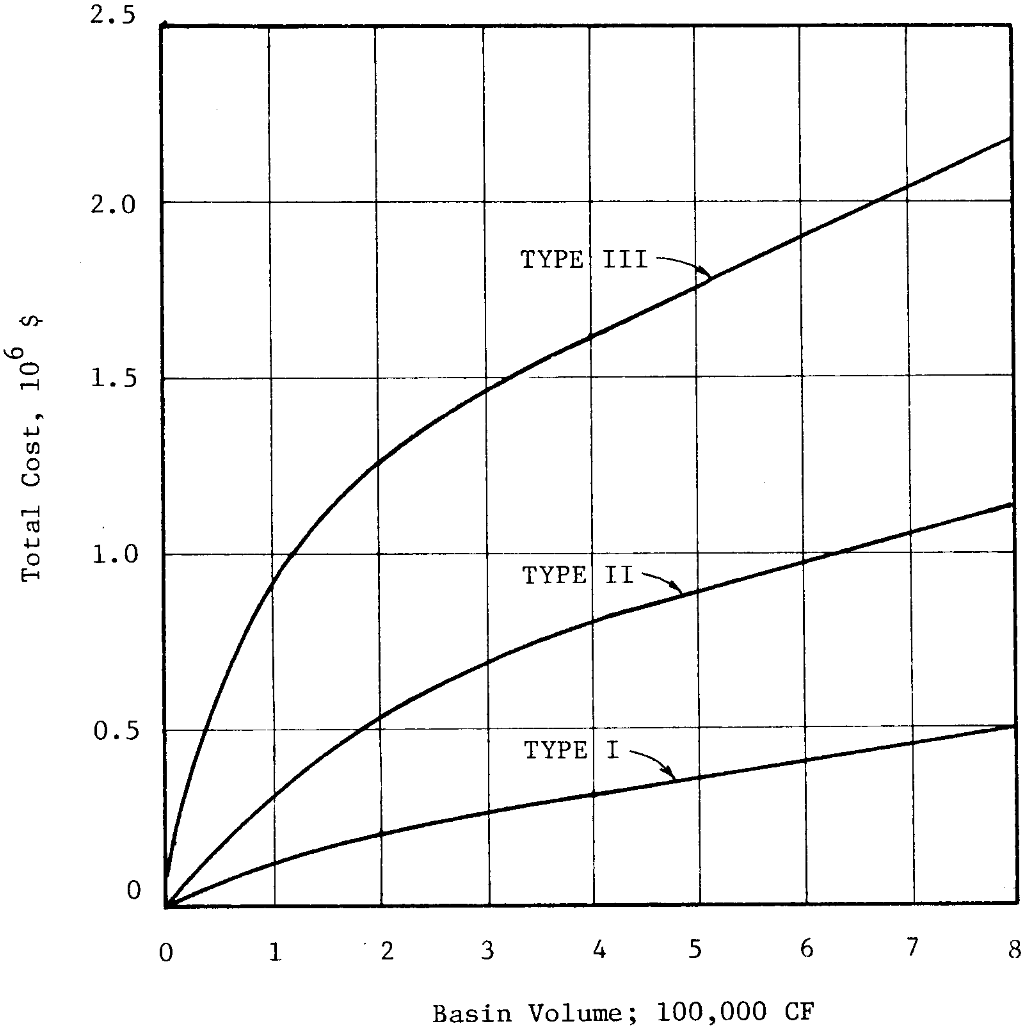
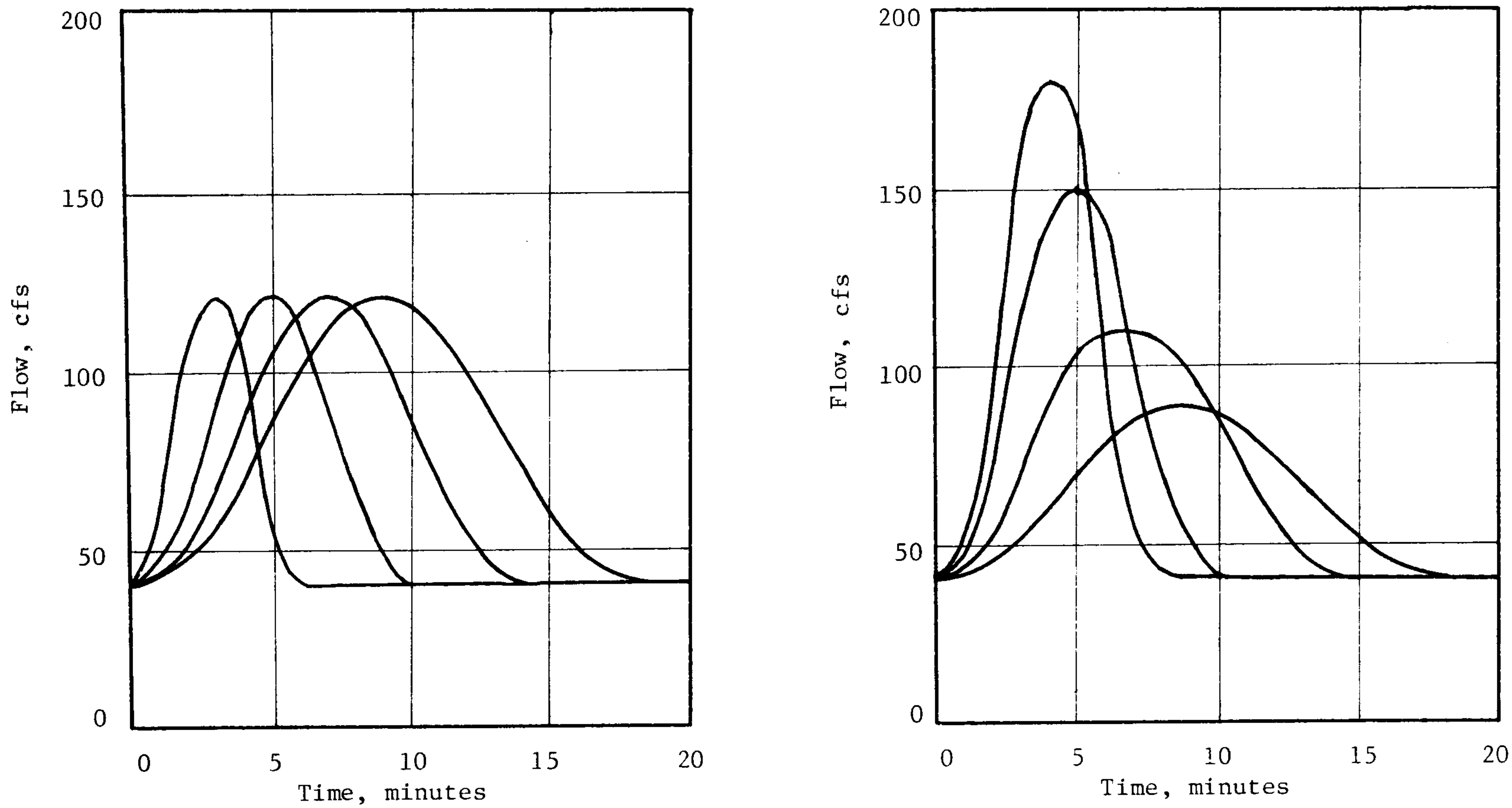


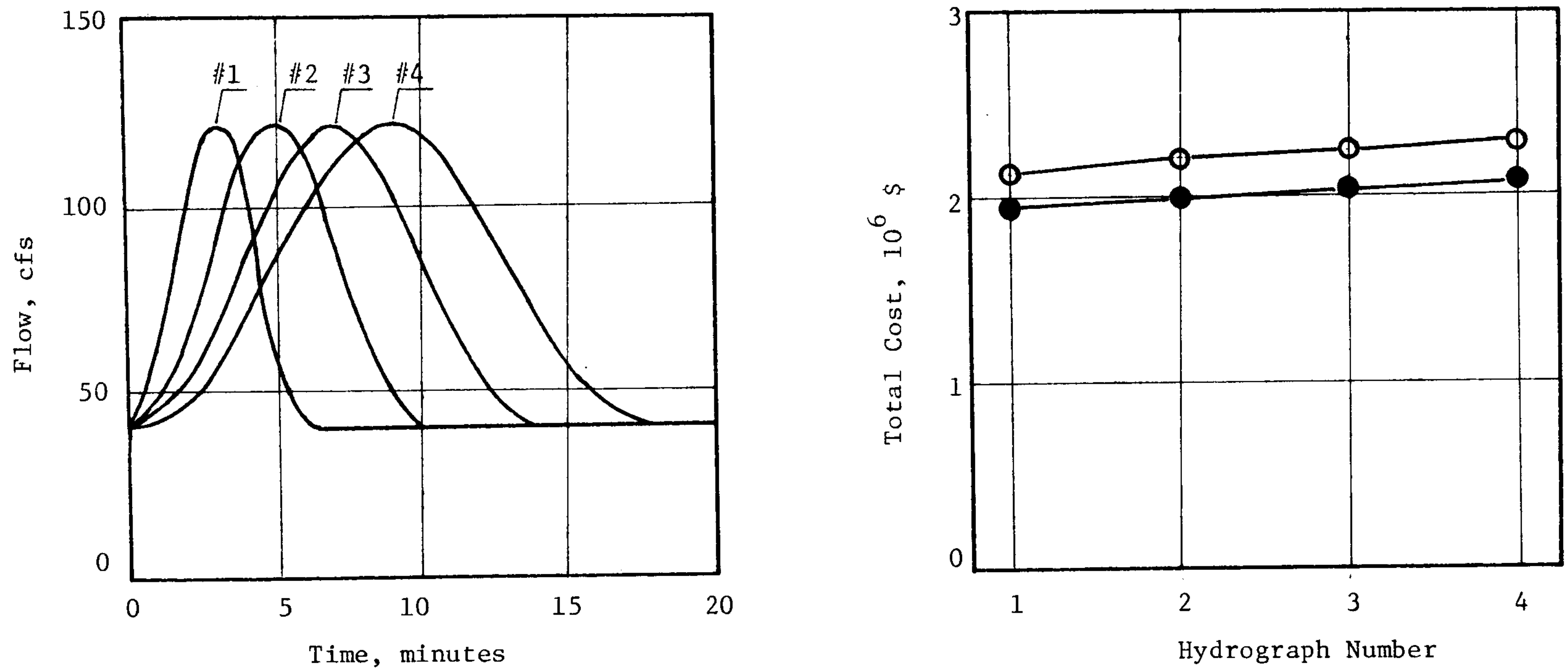
Figure 4.3 Storage Basin Construction Costs



(a) Inlet Hydrograph Series with Constant Peaks and Variable Volumes

(b) Inlet Hydrograph Series with Variable Peaks and Constant Volumes

Figure 4.4 Test Hydrographs



(a) Inlet Hydrographs

(b) Drainage Network Costs

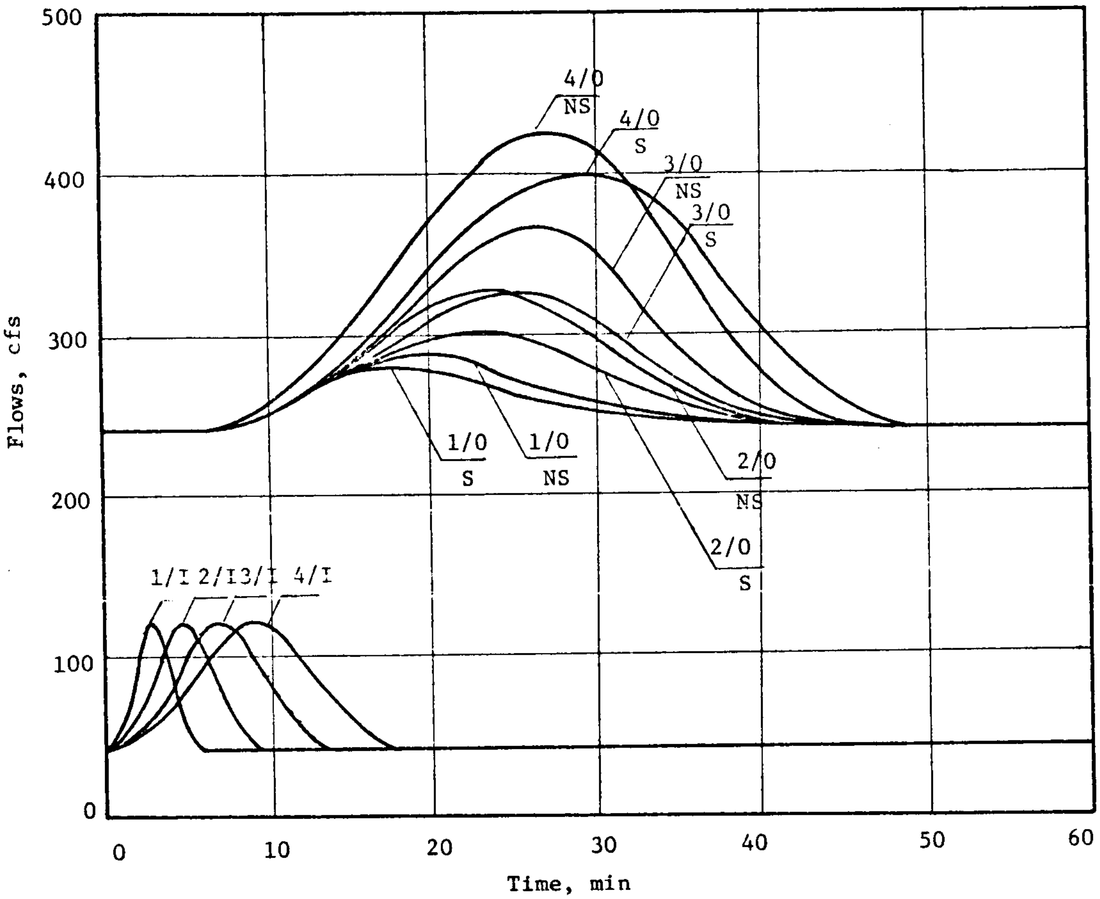
LEGEND:

○ Network design without storage basins

● Network design includes storage basins

Hydrograph Number

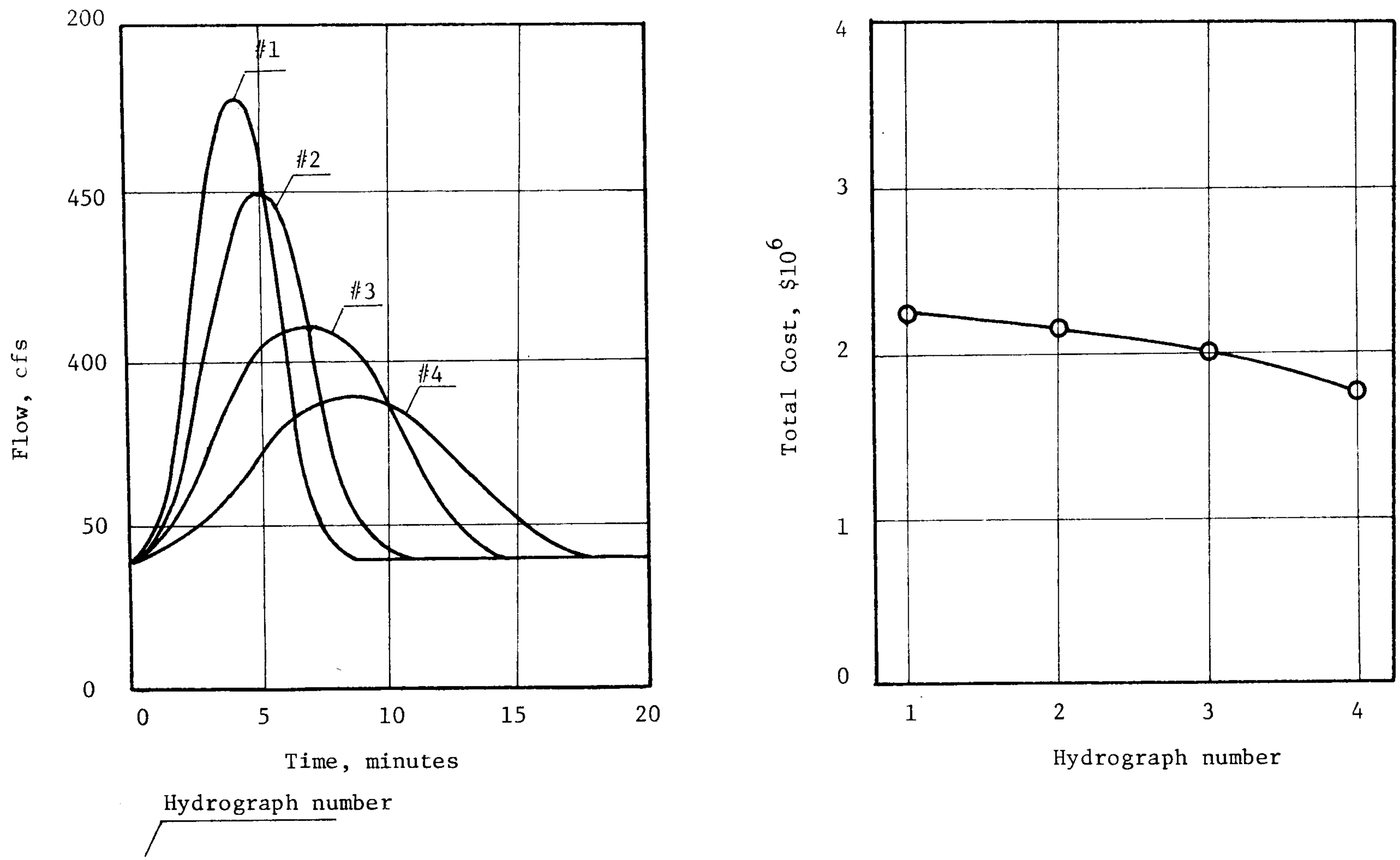
Figure 4.5 Variation in Total Drainage Network Costs with Variations in Inlet Hydrograph Volumes



Hydrograph Number — I = Inlet Hydrograph, O = Outflow Hydrograph

NS = Network design without storage basins
S = Network design includes storage basins

Figure 4.6 Outflow Hydrographs Resulting from Design Based on Hydrographs with Equal Peak Flows



(a) Inlet Hydrographs

(b) Drainage Network Cost

Figure 4.7 Variations in Total Drainage Network Costs with Variations in Inlet Hydrograph Peaks

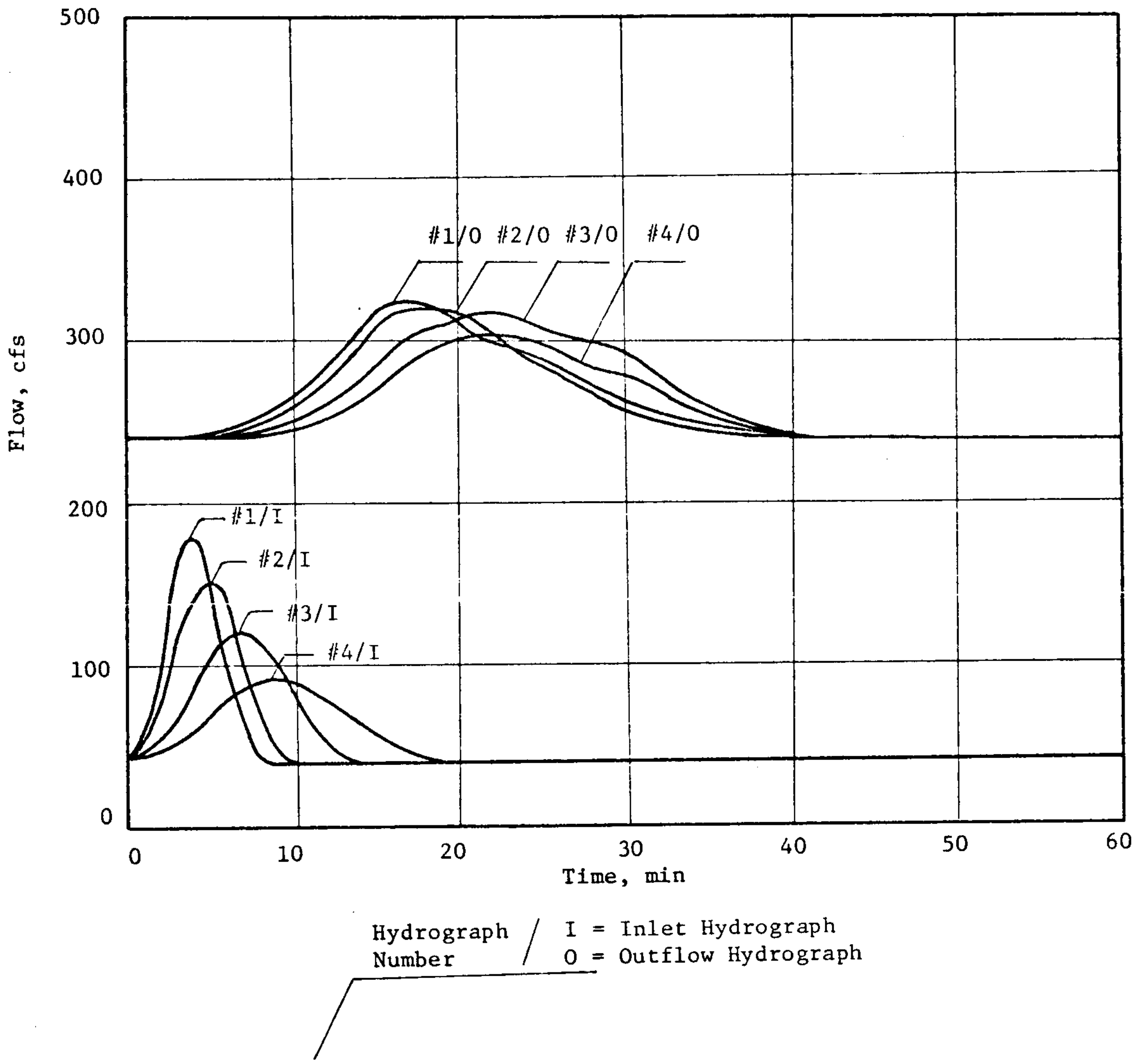


Figure 4.8 Outflow Hydrographs Resulting from Design Based on Inflow Hydrographs with Equal Volumes

CHAPTER V

SUMMARY AND CONCLUSIONS

> Now we sat on the highest pinacle. Patches of sunlight and clouds drifted over the lakes, the forested slopes and the valley beyond. Above us, shrouded in mist, was the ice and the rock of the main peak, but I ignored it in my satisfaction of reaching the summit of this little fragment of the main peak.
> (From Butler, 1964)

5.1 Summary

A dynamic programming approach to determine least cost strategies in urban drainage network design has been developed. The planning of these networks was shown to be a sequence of activities which identified specific alternatives for detailed study and design; it is to this latter step that this study is addressed.

Urban drainage networks were, for the purpose of this study, defined as combinations of conveyance and storage elements. The conveyance elements are circular cross-section conduits, junction and diversion structures and pump stations. The storage elements are ponds and basins of different configurations.

With the layouts of the network alternatives presumed to be known from the preceding planning activities, it was postulated that there exists an unique least cost solution to each identified alternative. Specifically, the study consisted of identifying and developing simulation models and algorithms to represent the various functional relationships that are involved and to make the decisions and calculations leading to "optimal design". As used in this study, optimization means finding the best solution among feasible alternatives based on least cost.

The optimization problem was cast as an N-stage sequential process for which dynamic programming was used to obtain the least cost or optimal solution. In dynamic programming terminology, network nodes or junctions are treated as the stage variable, conduit invert elevations at the junctions as the state variable, and maximum flow rates and conduit sizes as decision variables. Associated with the state variable is a state description which includes conduit slopes, storage basin and pump station requirements, cumulative cost, and control costs over the stage. This multi-decision dynamic programming approach, which is coupled with hydraulic simulation and cost models and structured to handle serial and non-serial networks was shown to converge to a global optimum.

The hydraulic model consists of two modules; a design module and a simulation module. Hydraulic design determines the geometric dimensions of the network elements and rests on the assumption of uniform flow for a controlled inflow. Hydraulic simulation calculates the flow propagation through the network elements and can use either dynamic or kinematic solutions to the gradually varied unsteady flow equations. The solution domains for these methods were established by a comparative analysis, and the method which is used in a particular situation depends on the bed slope of the conduit.

Using these techniques of dynamic programming and unsteady flow routing, the optimization problem is solved by a stagewise double pass over the network. First, all feasible solutions and the state transformations as defined by the iterative equation based on Bellman's principle of optimality are determined; followed by a second pass over the same stage for the purpose of routing the inflow hydrographs over the stage variable.

The optimization approach has been programmed in FORTRAN IV and implemented on the University of Washington CDC 6400 digital computer. A separate subroutine has been developed to control the stage variable solution order such that computer memory and calculation requirements become a minimum.

The utility of the model was examined through design of an hypothetical drainage network. Based on this analysis and the findings of other investigators as reported in the literature, the properties of a proposed urban runoff hydrograph generator were identified.

5.2 Conclusions

The following conclusions can be drawn from this study:

1. The approach to determine least cost strategies in urban drainage network design which was developed in this study and used on an hypothetical test network design demonstrated the feasibility of the methodology. The method converges to global optima, requires small amounts of data and is associated with reasonable computation requirements.
2. Compared to conventional design methods, use of the developed approach could achieve overall drainage network cost reductions amounting to 30%. Of this saving, 40% is attributed to use of storage elements as a control option, and the remaining 60% achieved by optimal combinations of conduit sizes and use of advanced hydraulic simulation techniques.
3. Quantitative comparisons were carried out to examine the relative accuracies and computational efficiencies of dynamic and kinematic wave simulations in closed circular conduits. Using implicit numerical solutions for both simulation models, it was shown that the kinematic solution accuracy is a function of conduit bed slopes and generally requires less than 15% of the computational time needed for dynamic simulations. As used in the model, kinematic simulation is used on all conduits with invert slopes greater than 5×10^{-4} ft/ft and for all conduits with diameter less than 48". This approach is computationally

efficient and introduces negligible errors in flow simulation.

4. Network costs were found to be sensitive to changes in hydrograph peaks and volumes, but the model has not yet been used extensively enough to allow general conclusions to be made. Nevertheless, the findings conclude that runoff hydrograph simulation should be based on advanced techniques if they are to serve the needs of the developed methodology.
5. A runoff hydrograph simulation technique to be coupled with the developed optimization model at a later date should be capable of carrying out continuous simulation of runoff quantities and qualities at specified points in the watershed, be based on physical phenomena such that the need for using judgement factors is eliminated, and should base overland flow simulation on non-linear routing principles.
6. The optimization model, as presently structured, should be further verified and extended to increase its usefulness. Specifically:

   a) The model should be used in a variety of design applications and modified as required upon examination of the model output.
   b) A runoff hydrograph simulation model should be developed and coupled with the optimization model. The runoff simulation model should satisfy the requirements developed in this study.
   c) Necessary forecasting and operation strategy models to control and determine optimal utilization of ambient and auxiliary storages in installed urban drainage networks should be developed.

CHAPTER VI

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APPENDIX A

QUANTITATIVE COMPARISON OF NUMERICAL SOLUTIONS TO THE GRADUALLY VARIED UNSTEADY FREE-SURFACE FLOW PROBLEM IN CLOSED CIRCULAR CONDUITS

Introduction

The developed model can simulate propagation of gradually varied and unsteady free surface floods through circular closed channels by both dynamic and kinematic routing techniques, and with the numerical problems solved by the implicit techniques which were developed in Chapter 3. Selection of a routing technique for a particular network channel is done by subroutine ROUTE; the decision is based on the channel's characteristics.

Kinematic routing is preferred for reasons of computational economy, but is generally recognized to give poor prototype simulation where "the channel bed slope is not sufficiently steep." While it has been concluded that movement of flood waves in natural channels is usually kinematic "except in the very flat ones" (Seddon, 1900), Metcalf & Eddy et al. (1971) found poor hydrograph reproduction for closed circular channels" with invert slopes on the order of 0.001 ft/ft or less." However, quantitative information on the relative merits of dynamic and kinematic flood wave routing in closed circular channels is not available from the literature. This situation led to the undertaking of a quantitative comparison for the purpose of establishing solution domains for these two methods in the developed drainage network design model. This appendix, which summarizes this work, starts with the problem formulation and a brief discussion of the solution techniques that have been employed. Test cases and the rationale behind selection of test procedures are reviewed, test results are summarized, and conclusions drawn.

Problem Formulation and Solution Techniques

Dynamic waves are described by the equations of continuity and momentum. In contrast, kinematic waves are defined principally by the properties of the continuity equation alone, with the discharges being functions only of the wave depths. The latter property of the kinematic wave implies that the friction slope is equal to the channel bed slope, and presupposes the other slope terms in the momentum equation to be negligible.

Numerical descriptions of the dynamic and kinematic flood wave routing problems were reviewed in Chapter 2, and the solution techniques that have been implemented were developed in Chapter 3. Table A1 summarizes governing equations, finite difference approximations, and the assumptions underlaying the implementation of the solution techniques.

Test Cases

The relative merits of different approaches to flow routing in open channels may be examined in numerous ways, depending on the quantities that are considered important. Usually, these comparisons are made on the basis of stability, convergence or some kind of an accuracy test, as well as the computational efficiency of the method. From a practical point of view, however, it is not necessary or worthwhile to carry out a computational scheme beyond the required engineering accuracy, or in this case, beyond the approximations associated with the one-dimensional gradually varied unsteady flow equations.

Of economical consequence in drainage network design are peak flows (which determine the required hydraulic capacities), flow velocities (which determine channel storages and the shape of downstream hydrographs in converging drainage networks), and the routing schemes capability of maintaining flow volume balances (which are of importance when storage volumes are involved). Differing computational efficiencies make alternative numerical schemes more or less attractive.

Examinations of the relative merits of the dynamic and kinematic simulation methods and the influence of the problem variables were attempted through an evaluation of numerical results derived for the test cases shown in Figure A1. The problem variables that were tested were channel diameters, channel bed slopes, and increments of spatial and temporal variables used in the numerical schemes. All channels had the same length, 5,200 feet, or approximately one mile. This particular length was chosen because it is a typical distance between successive inflow points on larger sizes of trunk sewer lines. Selection of shorter test lengths would give a false impression of high simulation accuracy while longer channels than those commonly used in urban drainage design have limited interests for these tests.

Each test case included use of the dynamic wave simulation for one combination of the temporal and spatial variable as shown in Figure A1 c. The dynamic simulation results were taken to be the baseline solution, and kinematic solutions were generated for comparison at all intersection points of the Δx-Δt grids in the x-t plane shown on Figure A1 c. All wave propagations were simulated over 90 minute periods.

The inflow hydrographs which were used in the test calculations were described by:

$$q_i(t) = Q_{base} + Q_{max}/2\ (1-\cos(2\pi t/T)) \qquad \text{(A-1)}$$

where

Q_{base} = base flow, cfs

Q_{max} = peak flow, cfs

$q_i(t)$= hydrograph ordinate at time t, cfs

T = wave duration, min

t = time, min

Numerical comparisons were based on the following four tests:

1. Flood volume balance, $V_r = \frac{\sum_t q_k(t)\Delta t}{\sum_t q_i(t)\Delta t}$ (A-2)

2. Peak ratio, $Q_r = \frac{Q_{k,max}}{Q_{d,max}}$ (A-3)

3. Crest time ratio $T_r = \frac{T_k}{T_d}$ (A-4)

4. Computation time ratio, $C_r = \frac{t_{c,k}}{t_{c,d}}$ (A-4)

where

$Q_{d,max}$ = peak outflow, dynamic simulation, cfs

$Q_{k,max}$ = peak outflow, kinematic simulation,cfs

$T_{c,d}$ = computation time, dynamic simulation, sec

$T_{c,k}$ = computation time, kinematic simulation, sec

T_d = time to crest, dynamic simulation, min

T_k = time to crest, kinematic simulation, min

$q_i(t)$ = inflow hydrograph, cfs

$q_k(t)$ = outflow hydrograph, kinematic solution, cfs

t = time, sec

$t_{c,d}$ = calculation time, dynamic solution, sec

$t_{c,k}$ = calculation time, kinematic solution, sec

Further definitions of the variables are given on Figure A1 b. The four tests were expressed as percentages.

Results

The ability of both simulation schemes to reproduce "observed" flood-waves was tested during the development stage of the numerical solution schemes. The dynamic scheme as structured appeares to have very good reproduction capabilities; kinematic simulation generally resulted in lesser peak attenuations and delayed peaks when compared to observed wave movements on flat slopes. The reproduction capabilities of the dynamic method was therefore judged to be satisfactory for use as baseline solutions during these tests.

The dynamic method appears to be unconditionally stable but convergence problems arise when long time steps are used in simulations of flood movements on steep slopes. This situation can be circumvented by reducing the length of the time step used in the simulation. The kinematic method appears to converge readily but shows some oscillations on the rising limb under extreme hydrograph rise rates. Both of these problems are of limited significance to this study, since they appear under circumstances which are of limited interest in urban drainage design.

The simulation results for flat and steep slopes are summarized in Figures A2 and A3, respectively. The conservation of mass, as examined by the ratio V_r, and defined by equation A2 shows that the kinematic solution maintains the flow volume balance under all conditions that were tested. Similar results were found for the dynamic simulation method, but these results are not reported herein.

The outflow peak ratio, as examined by the quantity Q_r and defined by equation A3, shows different results for flat and steep slopes. Implemented on flat slopes, the relative differences in peak ratios are less than 20% for the 3 and 9 feet diameter conduits, while the simulation results for the 6 feet diameter conduit show up to a 40% difference. This relatively large "error" for the 6 feet diameter conduit appears to be caused by the differently proprotioned inflow hydrograph that was used in this case. While the ratio between base and peak flows for the 3 and 9 feet diameter conduits were 2.0 and 2.5, respectively, this ratio is considerably larger for the 6 feet diameter conduit, viz. 4.5. While the inflow hydrograph changes from 20 cfs (base flow) to 90 cfs (peak flow) in 10 minutes, under the assumption of uniform flow the depth of flow changes from 2.10 feet to 5.05 feet during the same time period. This rise rate is unreasonably large for actual situations and magnifies the lesser peak attenuations obtained by kinematic simulation, and could lead to false impressions of simulation inaccuracy.

The crest time ratio, as expressed by T_r and defined by equation A4, generally shows poor agreements for simulations on flat slopes and generally good ones on steep slopes. As shown on Figure A1 b, the inflow hydrograph rise starts at time zero. Thus, the time base for comparison is the flow time for 1 mile of channel, i.e. between 15 and 45 minutes, depending on the channel bed slope. Thus, a "30% error", which could be no more than 5 minutes, could convey the false impression of poor accuracy while a 5 minute disagreement between simulated peaks throughout simulation periods of one day or more would be a very minor disagreement. The crest time ratios show good agreement for steep slopes.

From Figures A2 and A3 there seems to be a clear relationship between channel bed slopes and kinematic simulation accuracy; poorer kinematic simulation accuracy results for flat slopes. This result is expected. Also as expected, channel sizes do not appear to influence the results.

A more diffuse relationship appears to exist between the simulation results and sizes of the temporal and spatial increments used in the numerical schemes. An inquiry into this relationship was attempted by the plotting of Figure A4, where Figures A4 a and A4 b show the relative simulation differences in % as defined by equations A3 and A4, respectively. In both plots, a strong correlation between channel bed slopes and relative differences between dynamic and kinematic simulations is present. Kinematic simulation appears to give "errors" in peak simulation on the order of less than 10 to 15% for simulations on channels with bed slopes steeper than 5×10^{-4} ft/ft. Beyond this slope the relative difference in the time ratio estimate is on the order of 15 to 20%. Unlike the peak flow ratio, Q_r, the accuracy of the time ratio simulation, T_r, appears to improve slightly as the Δx-Δt ratio increases.

The simulation differences for peak flow ratios and time ratios are replotted as functions of bed slopes only in Figure A5. This figure shows clearly that the kinematic simulation accuracy decreases sharply as the channel bed slopes become as flat as 5×10^{-4}ft/ft.

Conclusions

The following conclusions are drawn from the comparative analysis between dynamic and kinematic flood wave simulations in closed circular conduits:

1. The kinematic simulation accuracy relative to the dynamic scheme decreases as the channel bed slope decreases.

2. Kinematic simulations generally require less than 15% of the computational time of dynamic simulations.

3. The overall consequences of hydraulic simulation methods can only be known after careful application and critical interpretation of the results obtained with the optimization model.

Figure A6 shows the proposed application domains when dynamic and kinematic simulations are used to calculate flood wave propagations in urban drainage networks. Kinematic simulation is used in urban drainage network design models where the channel bed slopes are larger than 5×10^{-4} ft/ft, and for all channel sizes less than 48" diameter, regardless of bed slopes. The latter conclusion is based on the limited economic consequences of hydraulic simulation for the smaller channel sizes.

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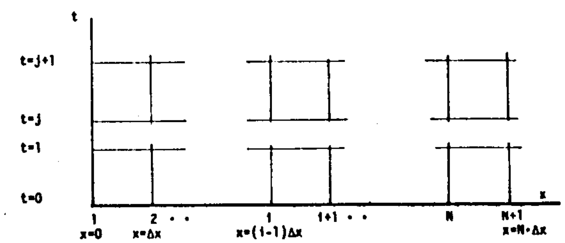
Table A.1. Numerical Formulation of the Gradually Varied, Unsteady Flow Problem

PROBLEM COMPONENTS AND GOVERNING EQUATIONS	NUMERICAL APPROXIMATIONS	COMMENTS
Dynamic wave: $Y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} + \frac{\partial y}{\partial x} = 0$ $v \frac{\partial v}{\partial x} + y \frac{\partial y}{\partial x} + \frac{\partial v}{\partial t} - g (S_o - S_f) = 0$	$\frac{1}{\Delta x} (Y_i^{j+1} + Y_{i+1}^{j+1}) (V_{i+1}^{j+1} - V_i^{j+1}) + \frac{1}{\Delta x} (V_i^{j+1} + V_{i+1}^{j+1}) (Y_{k+1}^{j+1} - y_i^{j+1})$ $+ \frac{1}{\Delta t} (y_{i+1}^{j+1} + y_i^{j+1} - y_{i+1}^{j} - y_i^{j}) = 0$ $\frac{1}{\Delta t} (v_i^{j+1} + v_{i+1}^{j+1} - v_i^{j} - v_{i+1}^{j}) + \frac{1}{2\Delta x} (V_{i+1}^{j+1} V_{i+1}^{j+1} - V_i^{j+1} V_i^{j+1})$ $+ \frac{g}{\Delta x} (y_{i+1}^{j+1} - y_i^{j+1}) + \frac{g}{2} (S_{f_i}^{j+1} - S_{f_{i+1}}^{j+1}) - g S_o = 0$	Problem variables are written in terms of their finite difference approximations by a four point non-central scheme given by eq. 3.40-3.42. Implicit solutions are based on a N-dimensional Newton-Raphson iteration procedure as described in Chapter 3.
Kinematic wave: $Y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0$ $S_o = f V^2/8gR$	$\frac{1}{\Delta x} (Y_i^{j+1} + Y_{i+1}^{j+1}) (V_{i+1}^{j+1} - V_i^{j+1}) + \frac{1}{\Delta x} (V_i^{j+1} + V_{i+1}^{j+1}) (Y_{i+1}^{j+1} - y_i^{j+1})$ $+ \frac{1}{\Delta t} (y_{i+1}^{j+1} + y_i^{j+1} - y_{i+1}^{j} - y_i^{j}) = 0$ $v_i^{j+1} = (2\ g\ D\ S_o/f_i(1 - \sin(\theta_i/\theta_i)))^{.5}$ $v_{i+1}^{j+1} = (2\ g\ D\ S_o/f_{i+1} (1 - \sin(\theta_{i+1}/\theta_{i+1})))^{.5}$	Partial differential equation variables are written in terms of the finite difference approximations given by the four point non-centra scheme by eq. 3.40-3.42. Velocities are determined by the Darcy-Weisbach uniform flow formula where friction factor and central angle of circular flow segment are found by iterative schemes, and with the friction factor calculated by Blassines or von-Karman equation depending on Reynolds number. Implicit solutions to the flood wave problem are found by a N-dimensional Newton-Raphson iteration procedure as described in Chapter 3.
Initial conditions: $Q - A \cdot V = 0$ $S_o = f V^2/8gR$	$v_i^j = (8\ g\ R_i^j\ S_o/f_i^j)^{.5}$ $y_i^j = D/2 (1 - \cos(\theta_i^j/2))$	Initial conditions are normal depth and velocities for specified steady base flow on all channel reaches. The problem solution is based Newton-Raphson iteration on the Darcy-Weisbach equation with Reynolds number and friction coefficient included in the iterature scheme.
Upstream boundary conditions: $Q - A \cdot V = 0$ $S_o = f V^2/8gR$	$Q_1^{j+1} - v_1^{j+1}\ D/2 (\theta_1^{j+1} - \sin \theta_1^{j+1}) = 0$ $v_1^{j+1} - (8\ g\ R_1^{j+1}\ S_o/f_1^{j+1})^{.5} = 0$	Upstream boundary conditions are given by the inflow hydrograph and the approximating assumption of uniform flow. The Darcy-Weisbach equation solved by Newton-Raphson interation is used to generate depths and velocities.
Downstream boundary conditions $S_o = f V^2/8gR$	$v_{N+1}^{j+1} - (8g\ R_{N+1}^{j+1}\ S_o/f_{N+1}^{j+1})^{.5} = 0$	Downstream boundary conditions are derived under the assumption of uniform flow and based on the Darcy-Weisbach uniform flow formula.

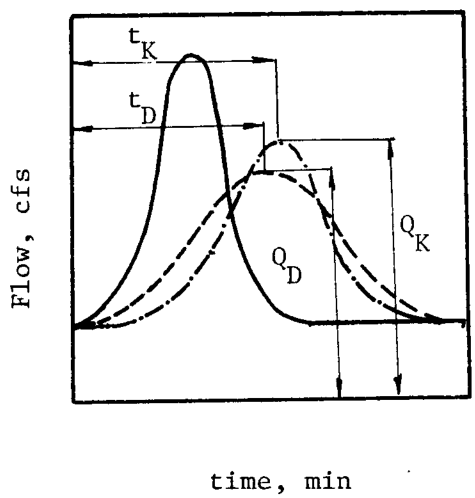
Definition of Variables:

- A = flow area, ft^2
- D = channel diameter, ft
- N = number of internal channel reaches
- Q = discharge, cfs
- R = hydraulic radius, ft
- S_o = channel bed slope, ft/ft
- S_f = friction slope, ft/ft
- V = average velocity, fps
- Δt = time step, sec
- Δx = length step, ft
- θ = central angle of circular segment, rad
- f = friction factor
- g = acceleration of gravity, fps
- i = subscript, spatial variable
- j = superscript, temporal variable
- t = time, sec
- v = average velocity of flow, fps
- x = distance along conduit, ft
- y = flow depth, ft
- Y = hydraulic depth (flow area/flow surface width), ft



Test Case No.	Channel Diameter ft	Channel Length ft	Channel Slope ft/ft	Wall Roughness ft	Kinematic Viscosity ft^2/sec	Inflow Hydrograph Q_{base} cfs	Inflow Hydrograph Q_{max} cfs
1	3	5,200	.0010	.006	$1.59x10^{-5}$	5	10
			.0060	.006	$1.59x10^{-5}$	22	40
2	6	5,200	.0005	.006	$1.59x10^{-5}$	20	90
			.0055	.006	$1.59x10^{-5}$	175	275
3	9	5,200	.0005	.006	$1.59x10^{-5}$	100	250
			.0035	.006	$1.59x10^{-5}$	300	700

(a) Summary of Test Cases



LEGEND:

—— Inflow hydrograph
– – – Outflow hydrograph, dynamic simul.
–·–·– Outflow hydrograph, kinematic simulation

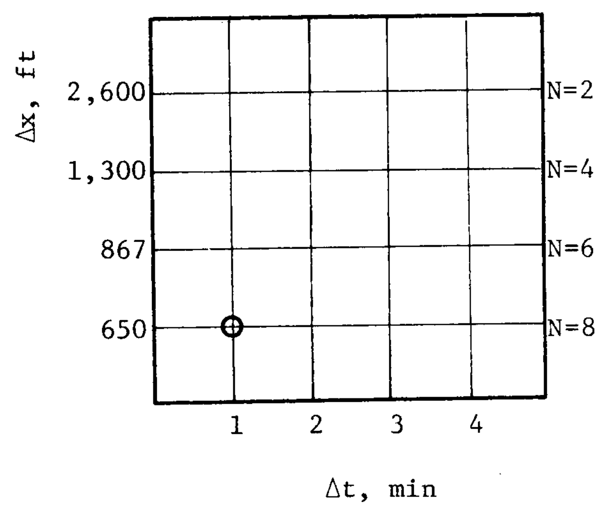
Q_D = peak discharge, dynamic wave, cfs

Q_K = peak discharge, kinetmatic wave, cfs

t_D = time to crest, dynamic wave, cfs

t_K = time to crest, kinematic wave, cfs

(b) Test Hydrographs

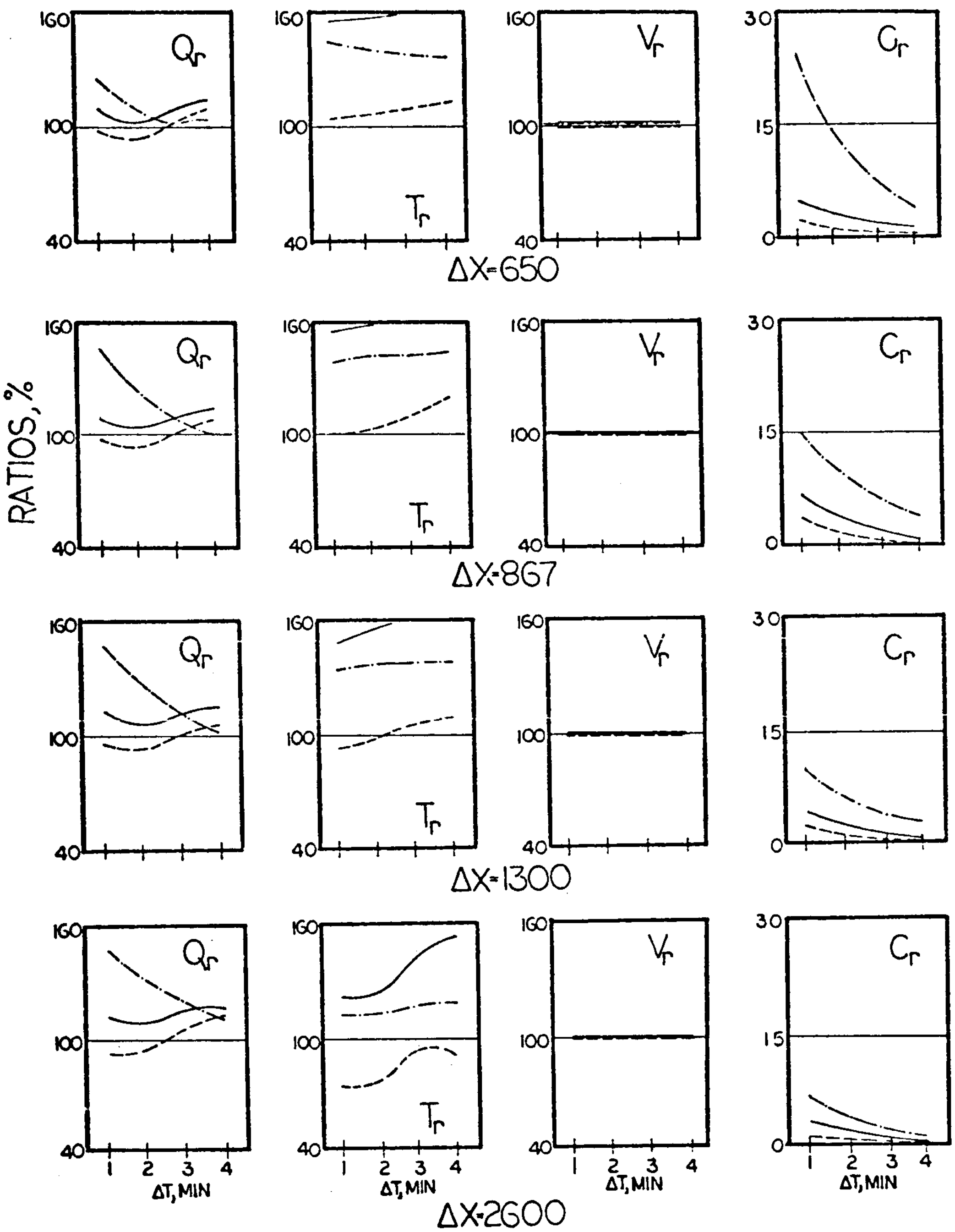


O = Intersection point on Δx - Δt grid used for dynamic simulation

N = number of internal reaches

(c) Combinations of Independent Variables used for Test Cases

Figure A1. Test Cases



____CONDUIT DIA=3.0 FT ; S_o = .0010 FT/FT
_._._CONDUIT DIA=6.0 FT ; S_o = .0005 FT/FT
_____CONDUIT DIA=9.0 FT ; S_o = .0005 FT/FT

Figure A2. Simulation Results for Flat Slopes

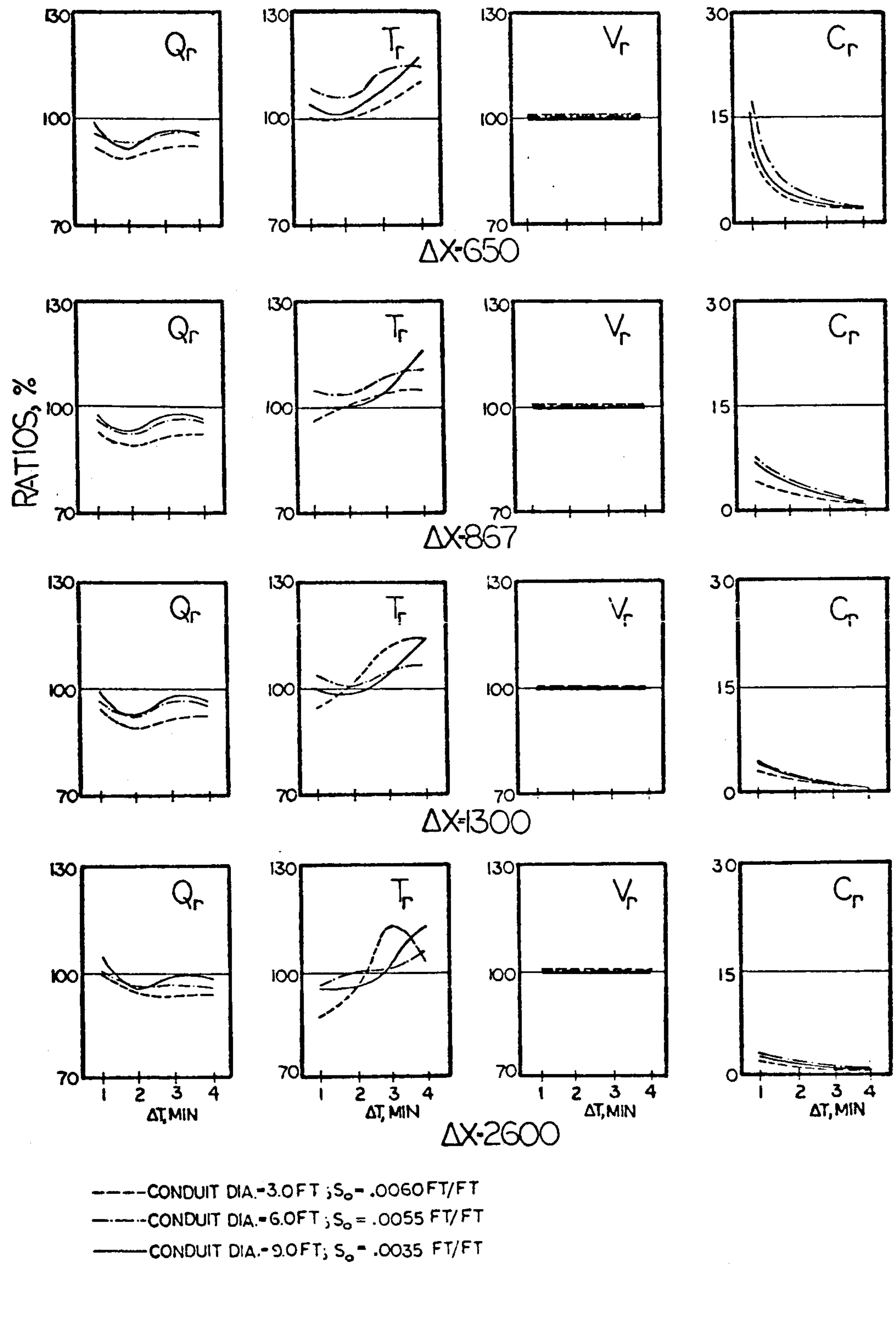
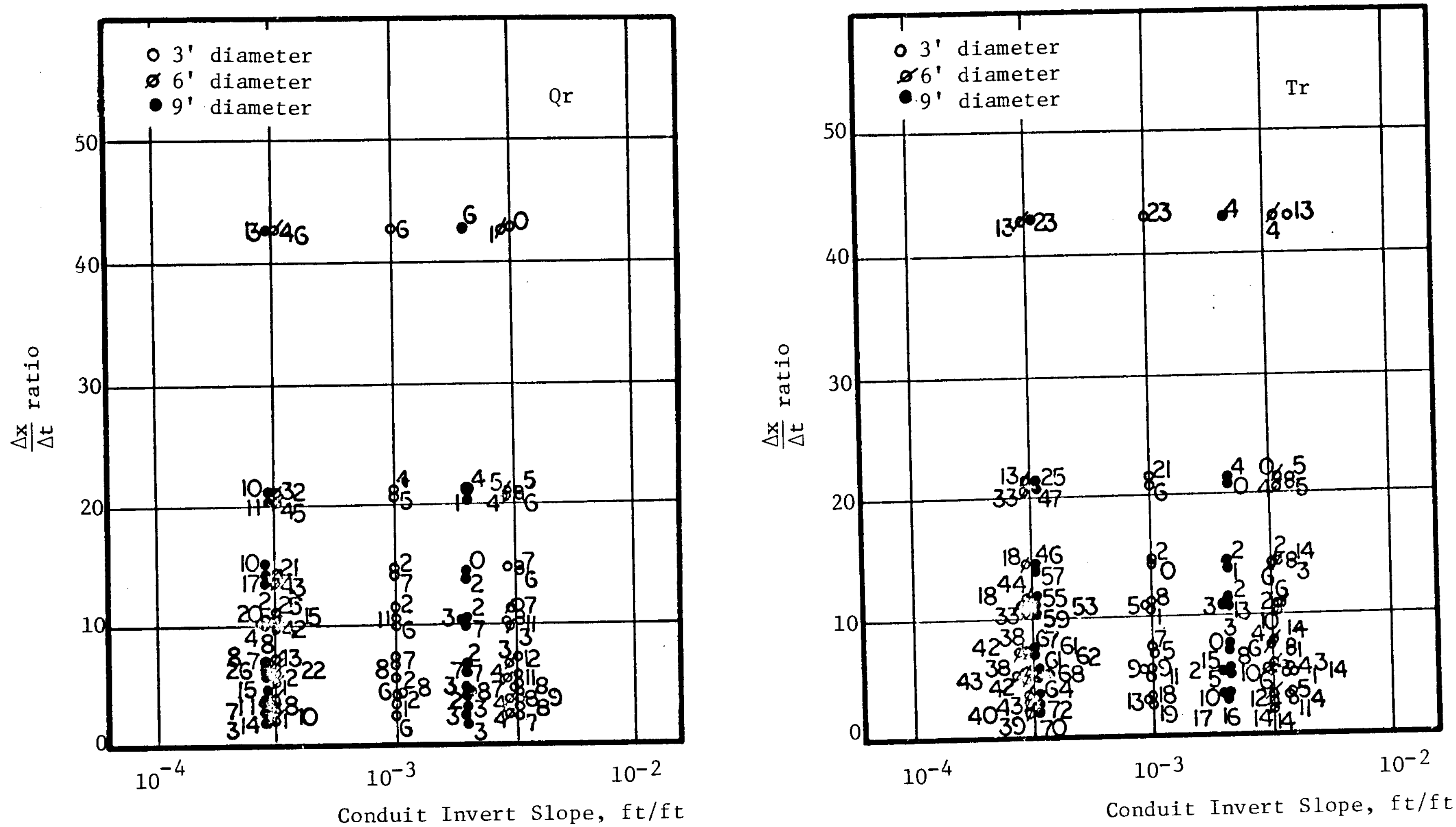


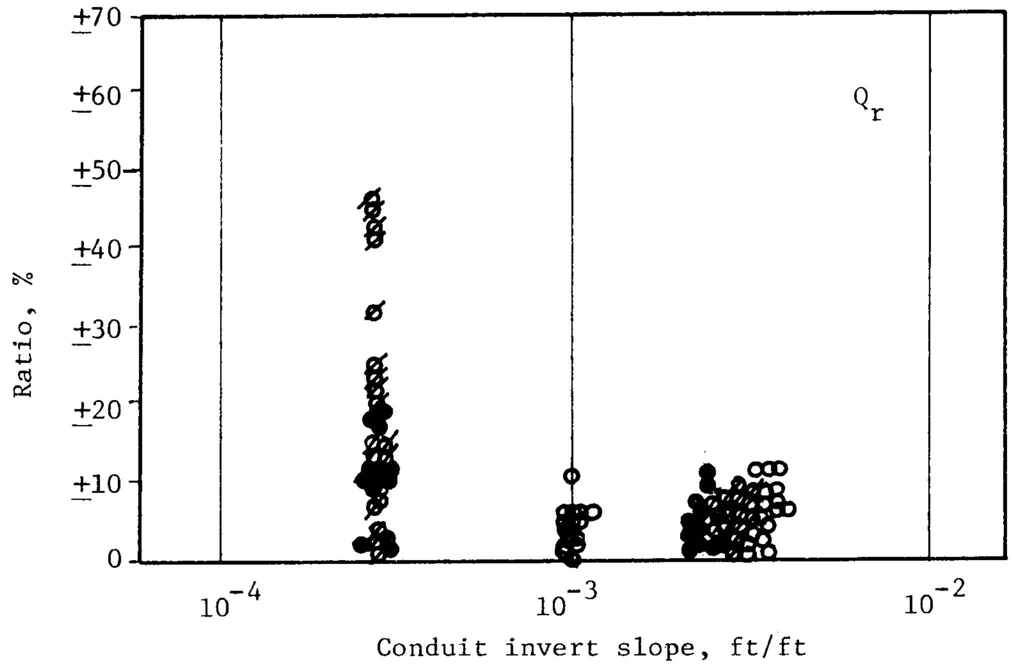
Figure A3. Simulation Results for Steep Slopes



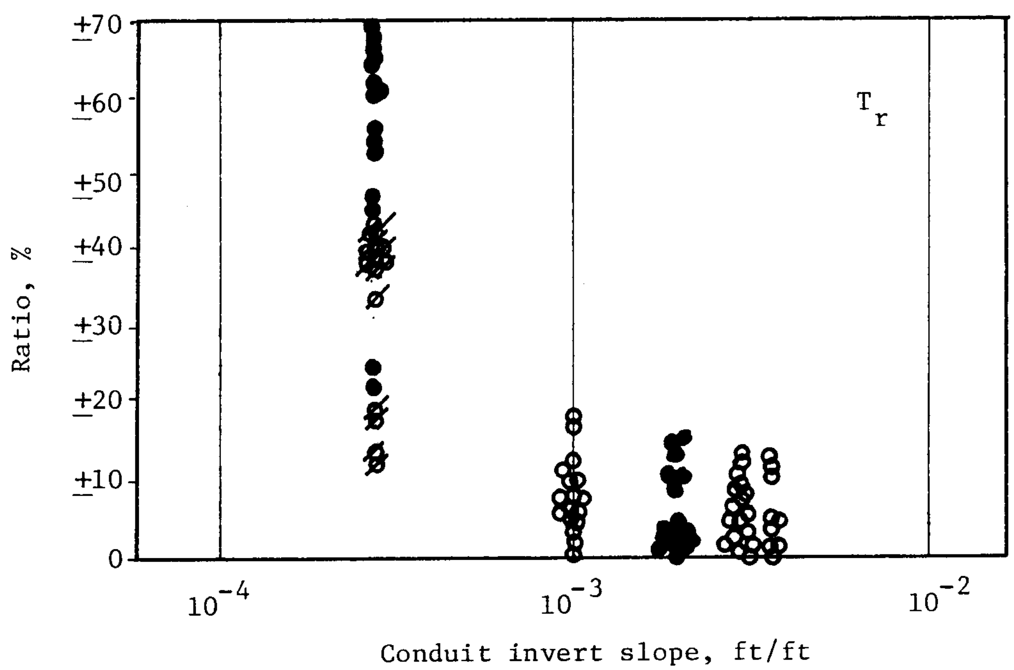
a) Peak ratio simulation

b) Time ratio simulation difference

Figure A4. Relative Simulation Accuracy as Function of Bed Slope



(a) Peak ratio



(b) Crest time ratio

Legend:
- o 3' diameter conduit
- ø 6' diameter conduit
- ● 9' diameter conduit

Figure A5. Relative Simulation Accuracy as a Function of Invert Slope

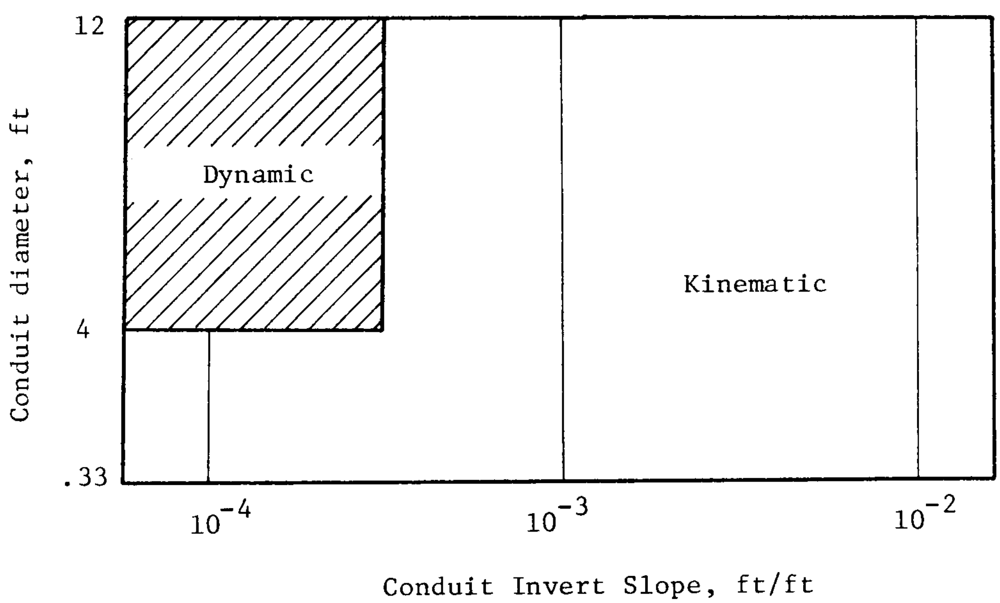


Figure A6. Application Domains for Dynamic and Kinematic Simulations used in the Developed Model.