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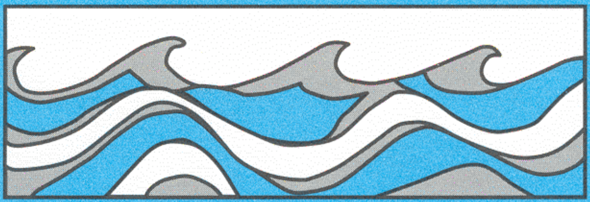
MULTIOBJECTIVE DECISION MAKING UNDER UNCERTAINTY: AN APPLICATION OF FUZZY SET THEORY

Douglas M. Johnson



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University of Washington

Abstract

MULTIOBJECTIVE DECISIONMAKING UNDER UNCERTAINTY: AN APPLICATION OF FUZZY SET THEORY

By Douglas Marshall Johnston

Chairperson of the Supervisory Committee:

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Department of Civil Engineering

Decision problems occur in settings in which objectives and constraints are only vaguely or imprecisely stated and in which the judgment of the decision maker forms an integral component of the problem's solution. This study evaluates the application of fuzzy set theory to problems of this nature. Fuzzy set operations are shown to support axioms of classical sets and probability. Probability is a special case of fuzzy set theory.

Three methods for aggregating judgments of preference in multiobjective problems are evaluated. They are: Saaty's hierarchical scaling method, Yager's fuzzy decision making method, and Tsukamoto and Terano's fuzzy logic method. Essential differences are in consistency requirements and propagation of uncertainty.

A questionnaire based experiment to evaluate the performance of the methods in predicting decision behavior is described. The questionnaire was administered to civil engineering students. Rankings of preference for alternatives generated by each method are compared with rankings obtained directly from test subjects. In this test setting all methods performed poorly. Factors which may contribute to these results include methodological limits to inconsistent judgment and the quality of responses elicited from test subjects.

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Facts all come with points of view,
Facts don't do what we want them to.

Talking Heads

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INTRODUCTION

The value of a rational, analytical approach to decision making has long been recognized within the engineering profession. Decision analysis is concerned with the formulation of a procedure for selecting an alternative within the context of a set of objectives and constraints. Any procedure for decision making must be consistent, and must at least in part, reflect human behavior. Many procedures for decision making have been developed for rather narrowly defined problems. For example, various methods are concerned with solving problems with single objectives and known constraints and alternatives. As an illustration, an objective of maximizing profit might be sought, conditioned by known production capacity, costs, and product demand. Situations as simple as these are rare in practice. More commonly, problems consist of multiple objectives, uncertain variables, and subjective judgment.

Many methods for decision making under conditions of complexity and uncertainty appear to be based upon prescriptions developed for more simple problems. For example, many methods considering uncertainty require assumptions permitting uncertainty to be described as random in nature. In many settings, these assumptions restrict the applicability of the methods.

Fuzzy set theory has been explicitly developed to consider non-random forms of uncertainty (Zadeh, 1973) and complexity introduced by necessary applications of approximation and subjective judgment. By examining decision making methods incorporating elements of fuzzy set theory, it may be possible to support the use of these methods for ill-defined problems. This study explores several methods using fuzzy set theory.

Study Hypothesis.

Decision problems occur in settings in which objectives and constraints may be only vaguely or imprecisely stated and in which the judgment or subjective beliefs of the decision maker form an integral component of a problem's solution. A procedure for decision making based upon idealizations of the decision problem and decision maker may not reflect actual behavior in problems of this nature.

This study examines the premise that a procedure for decision making must not only follow axioms of rationality but must reflect actual decision behavior. Assumptions examined in this study include that the model of the decision problem is complete, decision makers sampled in the study represent themselves and are not necessarily representative of the population, and solutions for one decision problem and decision maker are not generalizable. The study hypothesis is first that methods based upon fuzzy set theory support accepted axioms of rational decision making and second that rankings of alternatives generated by fuzzy set based techniques are positively correlated to ranks obtained directly from decision makers in problem settings comprising subjective judgment and uncertainty. The first part of the hypothesis will be examined through a comparison of the axioms of fuzzy logic with those of classical sets and probability theory. The second part will be examined with an experiment.

Framework.

Decision making in the context of this study is concerned with the selection of an alternative from a set of alternatives, employing a set of evaluation criteria that relates the attributes of the alternatives to the objectives of the decision makers. The components of a decision problem are a global objective, a set of evaluation criteria, and a set of alternatives. The structure of such a problem is illustrated through a hierarchical diagram (Figure 1).

Decision making is assumed to occur along a continuum between decisions arising from certain information and decisions arising from uncertain information (Baird, 1978). Within this continuum lie methods which vary in their information

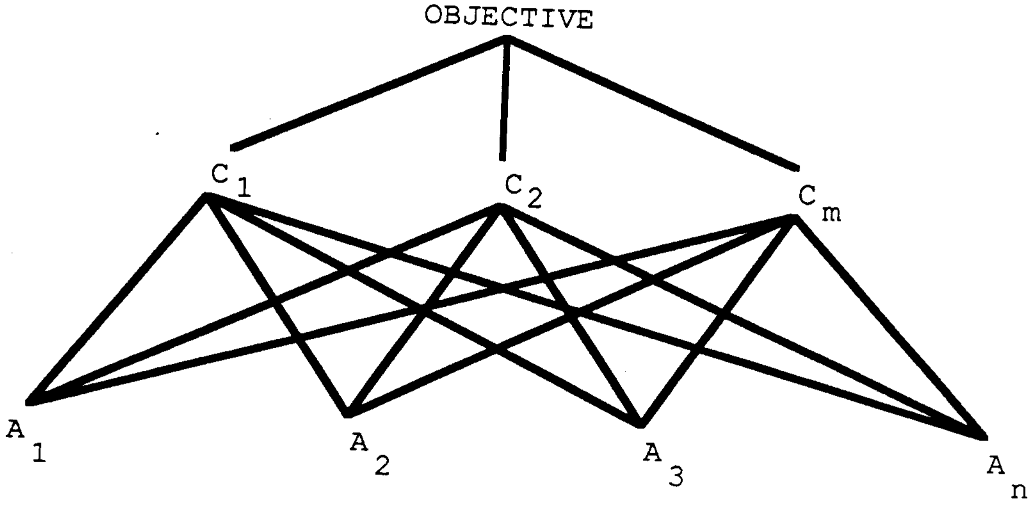

Figure 1. Hierarchical Structure of Decision Problem (Buckley, 1984)

requirements and quality of solution. Here quality is interpreted as a measure of the degree of certainty in the solution.

Uncertainty may assume many meanings within the context of decision analysis. Uncertainty may be defined as that which is characterized by the absence of sufficient information, or may be characterized by vagueness or doubt. A dictionary definition includes the characteristic that "uncertainty ranges in implication from a mere lack of absolute sureness to such vagueness as to preclude anything more than guesswork" (Websters, 1982). A more restricted definition frequently used in decision analysis focuses on uncertainty as the chance (usually measured by probability) that any one of *n* possible outcomes of a decision will occur (Raiffa, 1968). For the purposes of this study, the form of uncertainty of interest is generally that of vagueness or "lack of sureness" rather than the probabilistic definition. Both definitions are considered, however, in the discussion within Chapter One.

Each point along the continuum imposes varying requirements upon the decision maker and requires varying assumptions about the behavior of the decision maker. Such a structure is presented and information requirements and assumptions are discussed.

In a simple setting, the set of alternatives to be evaluated are known and the preferences of the decision maker with respect to the alternatives are also known. Let $A_1, A_2, \ldots, A_n$ represent the set of alternatives, while $e_1, e_2, \ldots, e_n$ represents the preferences associated with $A_1, A_2, \ldots, A_n$ respectively. This deterministic decision model is represented in Figure 2. A possible decision prescription (e.g., Baird, 1978) is to order the alternatives such that $e_i \geq e_{i+1}$, for all i's, to select the alternative with the largest preference:

$$D^* = \mathrm{MAX}_i \ (e_i).$$

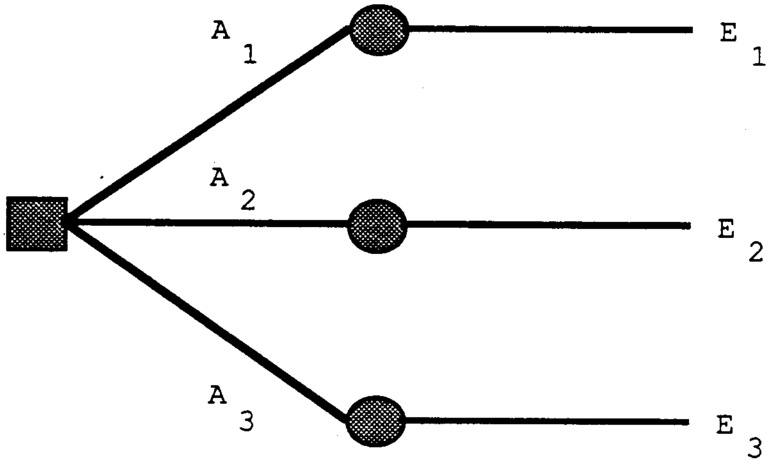

Figure 2. Decision Problem With Deterministic Outcomes

Implicit in this model are several informational and behavioral assumptions. First, it assumes all information is perfect information, that is, every time alternative A_i is selected, preference e_i will be obtained. Second, it assumes the decision model does not change. In this sense, the value to the decision maker of the preference associated with alternative A_i remains constant. Third, it assumes the decision model contains all the relevant information, that is, all the information that is required by the decision maker to select an alternative. Finally, it assumes the decision maker is "rational" and will always strive to maximize preference and will only consider the preferences explicitly incorporated into the decision model. These assumptions may severely restrict the applicability of a deterministic model.

Now assume that the preferences associated with an alternative are uncertain, that is, the possible outcomes arising from the selection of an alternative are uncertain and therefore the preferences for an alternative are uncertain. For example, if the information is not perfect information, then each time alternative A_i is selected it is possible that some preference other than preference e_i may be obtained. This outcome leaves many possible prescriptions for decision making. To illustrate, consider the hypothetical decision problem given in Figure 3. In this problem the selection of alternative one results in either of two outcomes with the outcomes not equally preferred. The same holds for the selection of alternative two. Using the decision rule from the deterministic example, the decision maker selects the alternative that contains the outcome that is most preferred in the expectation that it is possible to obtain that outcome. For example, given the preference relation

$$e_{11} > e_{21} > e_{22} > e_{12}$$

the decision would be to select alternative one because that alternative may result in the outcome that is most preferred. The decision rule may be stated as:

$$D^{\star} = MAX_i \ (MAX_j \ (e_{ij})) .$$

A potential problem with this decision rule is that while it is possible to obtain the outcome that is most preferred

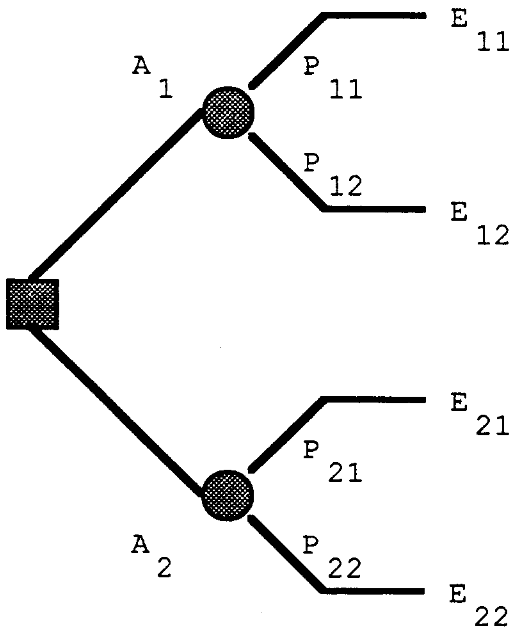

Figure 3. Decision Problem With Uncertain Outcomes

through the selection of alternative one (e_{11}), it is also possible to obtain the outcome that is least preferred (e_{12}). The decision maker might therefore be unwilling to select that alternative.

If, among other assumptions, it is assumed that the decision process may be repeated many times then it is possible to measure the likelihood of a particular outcome resulting from the selection of an alternative. In particular, it may be possible to assess probabilistic measures. Given this information, a possible prescription would be to select the alternative that will most likely result in the most preferred outcome. A composite measure of these considerations is expected value; the magnitude of the preference weighted by the probability of occurence. Assume a decision model as illustrated in Figure 3. The selection of alternative one will result in an outcome of preference e_{11} with probability p_{11} or e_{12} with probability p_{12} . The expected value from the selection of alternative i may be expressed as:

$$E(A_i) = \sum_{j=1}^{m} p_{ij} * e_{ij}.$$

A possible decision rule (Raiffa, 1968) might be to select the alternative with the largest expected value for its outcomes:

$$D^{\star} = MAX_i \; (E(A_i)) = MAX_i \; (\sum_{j=1}^{m} p_{ij} * e_{ij}).$$

This prescription for decision making requires several assumptions regarding the structure of the model and the behavior of the decision maker. One assumption, stated above, is that the decision problem is repeatable or is at least similar enough to other decision problems so that probabilistic measures of the occurence of outcomes may be assessed. In unrepeatable decisions, subjective probabilities may be assigned but it is arguable as to whether that would constitute a violation of the method's theoretical basis in that numerous repetitions of the experiment are required for the expectation to assume its value. Another difficulty in assessing probabilities is that all the outcomes may not be known. In this instance it is again strictly not possible to assign probabilities to unknown events (Blockley, 1985). Finally, the prescription assumes that probabilistic measures adequately represent the decision maker's perception of the likelihood of an outcome.

While the requirement for perfect information has been relaxed in this setting, the other assumptions attributed to deterministic settings remain. In addition, the possible outcomes from the selection of an alternative must now be known and the utilities associated with each outcome must also be assessed. Furthermore, the decision must be repeatable so that the probabilities associated with each alternative may be assessed.

It may be desirable (or necessary) to further relax some of the assumptions in the decision model. For example, if the decision may not be repeatable, then at best it is only possible to obtain subjective estimates of probabilistic measures of the likelihood of an outcome. Also, all possible outcomes arising from a decision may not be known nor may the magnitude of the utilities associated with the alternatives.

Another decision approach is one of examining the least preferred outcome of an alternative rather than the most preferred. In the example given, the worst outcome from alternative one has a preference of e_{12} while the worst outcome from alternative two has a preference of e_{22}. Outcome e_{22} is preferred to outcome e_{12} so a decision prescription might be to select alternative two as the worst outcome from that alternative is still better than the worst outcome from other alternatives. The decision rule may be expressed as

$$D^* = MAX_i \ (MIN_j \ (e_{ij}))$$

commonly known as the MAXIMIN prescription for decision making (Hillier and Leiberman, 1980). This rule demands relatively little information regarding preferences, but may require the decision maker to be conservative in the selection of an alternative.

An additional complexity introduced by multicriteria problems may be illustrated as follows. Consider the decision problem in which there are three alternatives and two evaluation criteria. The preferences may be expressed in the form of a preference relation

C_1: $e_{11} > e_{21} > e_{31}$

C_2: $e_{22} > e_{12} > e_{32}$.

Alternative one best satisfies criterion one (C_1) while alternative two best satisfies criterion two (C_2). Note that alternative three least satisfies both criteria. This alternative may be considered to be dominated by the other two alternatives, while alternatives one and two are non-dominated alternatives (Hillier and Leiberman, 1980). It is conceivable that either a maximizing or a maximin decision prescription could be applied to this problem. Another approach is to weight the criteria according to some measure, shifting the levels of preference. These weights would be similar to probabilities except that there exists no prescription as to how they guide a decision maker. In fact, the weights might be argued to be a descriptive element reflecting the decision maker's actual priorities rather than a prescription of priority that the decision maker must then follow.

Normative Versus Descriptive Models.

Normative (prescriptive) decision models are concerned with the construction of a model of the decision problem and also a model of an "ideal" decision maker to guide the real decision maker toward a solution to the real problem. This approach depends upon a "macroanalytic" view of the world in which the model is applicable to a generic decision maker and problem. The methods previously discussed describe several models. The decision rules in the methods are based upon the assumption of ideal behavior, that is, behavior that is consistent and follows certain axioms of rationality. For example, the decision rule of maximizing expected utility under uncertainty assumes the rational decision maker will seek alternatives that will return the greatest payoff, weighted by the probabilities of the possible outcomes. It is necessary to assume that the probability of an outcome is an accurate measure of the decision maker's perception (or belief) of the likelihood of an outcome.

Decision making under a normative model identifies inconsistencies in the analysis by the actual decision maker (French, 1984). The decision maker is then expected to modify his or her behavior toward that of the ideal decision maker thus reducing inconsistency. If a decision maker's perception of the likelihood of an event is not equivalent to the probability, then under the normative model, the decision maker is expected to reconsider and accept the probabilities.

A prescriptive model of a decision process contains an idealized decision problem. This model implies several factors of importance in decision analysis. First, it is implied that the model accurately, and as completely as necessary, represents the actual decision problem. For example, in the hierarchical decision model of Figure 1, a prescriptive model assumes that all relevant alternatives, criteria and objectives are contained in the model of the problem. It is further assumed that all necessary measures may be obtained from the model.

The application of a descriptive model is centered on the ability to predict how an actual decision maker will behave under a given set of circumstances. In this sense it requires a "microanalytic" approach. A descriptive theory is (initially) focused on making conjectures about the behavior of individual decision makers. If the theory is reinforced through empirical evidence then the theory may become accepted as a predictive model.

The perceptions and preferences of the decision maker are assumed to be given in descriptive analysis. The decision maker has no expectation of being required to modify perception to be consistent with some ideal. For example, if the perception of the likelihood of an event is not equivalent to the probability, then the model is expected to change to reflect the decision maker's perception.

Even a normative model must possess some descriptive capability. The ideal decision maker must possess beliefs similar to the actual decision maker if the normative model is to be perceived as desirable or useful, although the beliefs of the decision maker are expected to evolve toward those of the model. Questions (French, 1984) that might be asked of a normative model include:

1) Is the model decision problem a suitable representation of real problems?

2) Is the "ideal" behavior of the model decision maker desirable?

3) Is the construction of the model decision maker both feasible and informative?

Essentially what is being questioned is the success of the descriptive component of a normative approach and the usefulness of the resulting model with respect to its ability to guide the evolution of preferences.

Bellman and Zadeh (1970) maintain that

> much of the decision making in the real world takes place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely.

In this context Bellman and Zadeh contend that a method based on fuzzy mathematics better models both problem settings and decision makers. Thus a fuzzy normative process is founded upon a descriptive model that is maintained to better represent actual decision making processes. This assertion is examined and tested in this thesis.

A criticism of normative methods of fuzzy reasoning is that the uncertainty described and assessed in the model development stage is not maintained through the decision process (French, 1984). That is, the solution generated by fuzzy set methods based upon the decision model presented by Bellman and Zadeh (1970) result in decisions that are themselves certain. An alternative model based upon fuzzy sets and fuzzy logic is thus proposed as a means to convey or propagate uncertainty through the decision process.

Outline of Subsequent Chapters.

While it has been argued that fuzzy set theory is theoretically similar to more traditional methods based upon classical sets and probability, it has also been asserted that fuzzy sets provide fundamental differences with respect to the measurement of uncertainty that affect the applicability of fuzzy set based methods. Chapter One examines the relationship between fuzzy sets and classical sets and shows that classical sets is a subset of fuzzy sets. The similarities between logics based upon fuzzy and classical sets are also examined and it is shown that the axioms of probability are a special case of those of fuzzy logic. Finally, essential differences between the two methods are examined and necessary assumptions about the decision maker's behavior are identified.

Fuzzy set methods based upon the decision model proposed by Bellman and Zadeh (1970) are shown in Chapter Two to possess undesirable characteristics with respect to the propagation of uncertainty through the decision process. An alternative approach based upon fuzzy logic is proposed. The derivation of a method to solve decision problems from Tsukamoto and

Terano (1975) is given, along with the derivation of an extension to the method to resolve inconsistent solutions arising from insufficient information.

In Chapter Three an experiment is designed to determine the relative ability of several decision methods to describe or predict individual behavior under uncertainty and multiple objectives. The methods examined include the fuzzy multiobjective method of Yager (1978), the hierarchical scaling method of Saaty (1977) and the fuzzy logic method described in Chapter Two. Results from the experiment are presented in Chapter Four.

Chapter Five contains an evaluation of the experiment and further analysis of the decision methods, including group decision making. Chapter Six provides summaries and conclusions. Future research needs are also identified.

CHAPTER ONE

AXIOMATIC FRAMEWORK

In this chapter are reviewed basic concepts of classical set theory and probability as a measure on sets. Fuzzy set theory is introduced followed by a description of fuzzy logic as a measure on fuzzy sets. Classical and fuzzy sets and their respective logics are compared to identify requirements imposed upon decision makers using these approaches.

Interest in probability measures arises from its broad application in decision analysis techniques. While its best use may be within a rather carefully defined area of application, its success in characterizing some aspects of decision analysis has led to wide spread application in areas where it may be desired to measure less quantitative aspects of decision models, including elements of human decision behavior.

Fuzzy set methods, on the other hand, have been developed rather specifically to address less numerical representations used in decision analysis (Zadeh, 1973). Because fuzzy set theory may address more general aspects of symbolic representation, it is important to identify what restrictions are imposed on the types of problems to be considered.

Decision Example.

Both probability theory and fuzzy set theory arise from similar axiomatic origins and have been applied to similar types of decision problems. The two theories are claimed to be fundamentally different, however (Zadeh, 1975; Gaines, 1978). To illustrate the nature of problems addressed by the different approaches, consider the following problem setting (adapted from Hillier and Leiberman, 1980).

An oil company owns land that may contain oil. The company does not know whether in fact the land contains oil or how much oil it is expected to produce, but the company assumes four outcomes of the action of drilling a well: no oil; a 50,000 barrel well; a 200,000 barrel well; and a 500,000 barrel well. The profit per barrel of oil is $1.50. The cost of drilling a dry well is $75,000, and the cost of drilling a producing well is $100,000. In addition to drilling, the company has the option of leasing the land for $45,000 or conditionally leasing the land for a $.50 per barrel fee on wells larger than 200,000 barrels. The possible profits to the company from these alternatives are given in Table 1.

Table 1 Profits (in dollars) for Oil Company

	No Oil	50,000 Barrels	200,000 Barrels	500,000 Barrels
Drill	-75,000	-25,000	200,000	650,000
Lease	45,000	45,000	45,000	45,000
Conditional Lease	0	0	100,000	250,000

Assuming this property is similar to other properties with which the company is familiar, the probabilities that the land will actually produce the given outcomes are given as

$p(\theta = \theta_1$, dry well) = .50

$p(\theta = \theta_2$, 50,000 bbl.) = .25

$p(\theta = \theta_3$, 200,000 bbl.) = .15

$p(\theta = \theta_4$, 500,000 bbl.) = .10

Using the criterion of maximum expected value, the company calculates the gains from each alternative $g(a_i)$ as

$$g(a_1) = E[\ g(a_1),\ \theta_1] = 650{,}000(.10) + 200{,}000(.15)$$

$$- 25{,}000(.25) - 75{,}000(.50)$$

$$= 51{,}250$$

$$g(a_2) = E[\ g(a_2),\ \theta_2] = 45{,}000(.10) + 45{,}000(.15) +$$

$$45{,}000(.25) +45{,}000(.50)$$

$$= 45{,}000$$

$$g(a_3) = E[\ g(a_3),\ \theta_3] = 250{,}000(.10) + 100{,}000(.15)$$

$$= 40{,}000$$

Therefore the oil company's alternative of choice would be a_1, drill for oil.

manager was unable to hire the preferred drilling crew. There is insufficient evidence to suggest that probabilistic measures would be accurate, but the company's geologists and engineers have a fair idea of the possible outcomes from drilling. They might be quite certain that the well would be dry and somewhat uncertain that the approximate yields from the wells would be obtained. They have not eliminated the possibility, however, that a much larger yield may be obtained. Decisions in this setting are a function of partial knowledge and the application of judgment and intuition, in addition to available quantitative measures.

The above example is for a single criterion problem, that of maximizing profit. It is of course possible to extend the problem setting to one of multiple criteria where complexity adds another dimension of difficulty to be addressed by analytic methods. More specifically, these additional criteria may require subjective judgment or other less quantitative aspects of decision making.

Rationale for Comparison.

This chapter examines in detail the foundations of probabilistic and fuzzy set based methods for decision making and indicates limitations imposed upon decision makers and decision problems by these foundations. It is shown that fuzzy set based methods are in theory less restrictive than

This example contains many assumptions that make the use of probabilistic methods feasible (Raiffa, 1968). The first assumption is that there exists only four possible outcomes. That is, because other lands have resulted in these outcomes, it is assumed that no other outcome will occur, for example a one million barrel well. Another assumption is that all the parameters (e.g. costs) are known with certainty, that is, the cost of drilling a producing well will be exactly $100,000. Clearly, there is sufficient variation in reality to question this assumption. Variations in the levels of these values may, in some problems, significantly affect the rankings of the alternatives. Third, it is assumed that there has been sufficient experimentation (i.e. other well fields) under controlled conditions, to make statistically significant statements about the probability of each possible outcome. Fuzzy set methods have been claimed to address the types of problems indicated in the last two assumptions stated above.

As an illustration of a problem in a fuzzy set context, consider again the oil well problem. In this case, the profit per barrel of oil is approximately $1.50, while the costs of drilling a dry well and a producing well are high and very high, respectively, because the geology of the site is likely to be different from previous sites and the field those of probability, and in fact, that probability is a more restrictive special case of fuzzy sets and fuzzy logic.

It is desired to test the argument that probability and classical set theory are special cases of fuzzy set theory. This is important to establish both a consistent and continuous line of reasoning in decision making problems, to identify the essential differences between the theories, and to describe some limitations to application imposed by these differences. Because the differences are embedded in basic axioms and assumptions, it is necessary to consider and expand upon discussion of the relationship between fuzzy and classical sets provided by Gaines (1978), and others (Deluca and Termini, 1972; Blockley, 1985).

<u>Classical Sets</u>.

A set may be defined as a collection of objects, where the objects (a_i) are said to be elements or members of a set (S) denoted

$$a_i \in S.$$

The negation of membership (an object is not a member of the set) is denoted

$$a_i \notin S.$$

In decision making, there may exist the set of alternatives (A) to be evaluated

$$A = \{a_1, a_2, \ldots, a_n\}, \; a_i \in A,$$

the set of evaluation criteria

$$C = \{c_1, c_2, \ldots, c_m\}, \; c_j \in C,$$

and perhaps sets of decision makers and preferences (or outcomes) relating the alternatives to the criteria.

Set Operations.

The union of two sets, denoted $A \cup B$ is the set that contains all elements of sets A and B:

$$A \cup B = \{x: x \in A, \text{ and/or } x \in B\}.$$

The intersection of two sets, denoted $A \cap B$, is the set that contains elements that are contained by both A and B:

$$A \cap B = \{x: x \in A, \text{ and } x \in B\}.$$

The complement of a set (A^c) is the set that contains all elements which are not contained by the set A:

$$A^c = \{x: x \in U, x \notin A\}.$$

Using these operations, it can be shown that the laws and identities of propositions (e.g. Larson, 1974) are satisfied, resulting in the laws of the algebra of sets (Table 2).

Fuzzy Sets.

In classical set theory, the membership of an element to a set is denoted by a binary relation:

$$a \in S \begin{cases} 1 \text{ if a is a member of the set S} \\ 0 \text{ otherwise.} \end{cases}$$

Table 2. Laws of Algebra of Sets

1. Reflexive Law:

$A \cup A = A$ $\qquad$ $A \cap A = A$

2. Associative Law:

$(A \cup B) \cup C = A \cup (B \cup C)$ $\qquad$ $(A \cap B) \cap C = A \cap (B \cap C)$

3. Commutative Law:

$A \cup B = B \cup A$ $\qquad$ $A \cap B = B \cap A$

4. Distributive Law:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $\qquad$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Identity Law:

$A \cup \emptyset = A$ $\qquad$ $A \cap S = A$

$A \cup S = S$ $\qquad$ $A \cap \emptyset = \emptyset$

6. Complement Law:

$A \cup A^c = S$ $\qquad$ $A \cap A^c = \emptyset$

$(A^c)^c = A$ $\qquad$ $S^c = \emptyset, \ \emptyset^c = S$

7. DeMorgan's Law:

$(A \cup B)^c = A^c \cap B^c$ $\qquad$ $(A \cap B)^c = A^c \cup B^c$

In fuzzy sets, membership is extended to include elements that only partially belong to a set. Thus the relation of membership would become:

$$a \in S \begin{cases} 1 \text{ if a is a member of the set S} \\ 0 \text{ if a is not a member of the set S} \\ \mu \text{ otherwise} \quad (0 < \mu < 1). \end{cases}$$

Sets in which elements may be only partially contained are said to be fuzzy sets (Zadeh, 1965). A fuzzy set is a set of ordered pairs

$\{x, \mu_A(x)\} \ \forall\, x \in S$

where

$\mu_A(x)$ is a membership characteristic function which denotes the degree of membership of x in A (A being a fuzzy subset of S).

For instance, using the oil well example (p. 21), the "very high cost" of drilling a producing well can be defined as:

A = "very high cost"

= [0|\$80k, .2|\$100k, .5|\$120k, .9|\$150k, 1.|\$170k].

To interpret, a cost of \$100,000 is given weak support for belonging to the set "very high cost" while \$120,000 is assigned stronger support. With a support of unity the cost \$170,000 is unambiguously a member of the set while \$80,000, with a support of zero, is unambiguously <u>not</u> a member of the set. The membership support should not be interpreted as a statement of the likelihood that any particular producing well <u>will</u> cost if it is drilled. Rather, it is a more general description, or mapping, of the dollar values associated with the concept of "very high cost".

To demonstrate that classical sets are a special case of fuzzy set theory, fuzzy set operations must be shown to contain the operations and propositions supported by classical sets. Similarly fuzzy logic must be shown to at least contain the propositions supported by probability logic.

In the next several sections, the relationship between fuzzy and classical sets, and between fuzzy logic and probability, are examined. It is shown that classical sets and probability are special cases of fuzzy set theory. An interpretation of the differences between these theories in the context of decision making is given.

Algebra of Fuzzy Sets

Before the relationships are considered, it is necessary to define permissible operations on fuzzy sets. The operations for union, intersection and complementation proposed by Zadeh (1965) are used here:

Union: $\mu(A \cup B) = MAX(\mu_A(x), \mu_B(x))$

Intersection: $\mu(A \cap B) = MIN(\mu_A(x), \mu_B(x))$

Complementation: $\mu_A{}^C(x) = 1 - \mu_A(x)$.

for example, let $A = \{.2|1, .5|2, .9|3\}$, and $B = \{.1|1, .4|2, 1|3\}$. Then the union of fuzzy sets A and B is equal to

$\{ MAX(.2, .1)|1, MAX(.5, .4)|2, MAX(.9, 1)|3\}$ or

$A \cup B = \{.2|1, .5|2, 1.|3\}$.

The intersection of fuzzy sets A and B is equal to

$A \cap B = \{.1|1, .4|2, .9|3\}$,

and the complement of A is equal to

$A^c = \{.8|1,\ .5|2,\ .1|3\}$.

The validity of these operations under some operational assumptions is proven by Bellman and Giertz (1973) in which they analyze operations for the compound statements of union and intersection, and seek to identify restrictions that must be observed when assigning supports to such compound statements.

Their analysis is initiated by two assumptions. The first assumption is that the supports assigned to a compound statement, $A \cap B$, or $A \cup B$, for example, depend only on the supports assigned to the individual elements of the compound statement. Letting $\mu(S)$ be the support for a statement then the support for a compound statement is

$\mu(S_1 \text{ and } S_2) = f[\mu(S_1), \mu(S_2)]$ or

$\mu(S_1 \text{ or } S_2) = g\ [\mu(S_1), \mu(S_2)]$.

The implication is that the support for a compound statement is soley a function of independent elements and cannot be obtained from joint statements (e.g. $\mu(S_1S_2)$). This assumption, in fact, reveals the "truth functionality" requirement of fuzzy set operations, which is discussed in a later section.

The second assumption stated by Bellman and Giertz generalizes the functions of the first assumption by assuming that compound forms of arbitrary statements S and T are governed by the same functions as in the first assumption:

$\mu(S \text{ and } T) = f[\mu(S), \mu(T)]$ or

$\mu(S \text{ or } T) = g\ [\mu(S), \mu(T)]$.

In addition to these assumptions, Bellman and Giertz impose the following restrictions on the functions f and g:

(1) f and g are non-decreasing and continuous in both variables,

(2) f and g are symmetric, i.e. $f(x,y) = f(y,x)$ and $g(x,y) = g(y,x)$,

(3) $f(x,x)$ and $g(x,x)$ are strictly increasing in x,

(4) $f(x,y) \leq \min\{x,y\}$ and $g(x,y) \geq \max\{x,y\}$,

that is, f and g are constrained to be at least minimal and maximal functions respectively (they are not averaging operations),

(5) $f(1,1) = 1$ and $g(0,0) = 0$,

that is, the end points of possible membership functions are an exception to (4), and

(6) logically equivalent statements have equal supports.

Using $\cap$ to represent intersection, $\cup$ to represent union, and the commutative, associative, and distributive axioms identified in the next section, along with the restrictions:

$x \cap y$ and $x \cup y$ are continuous and non-decreasing,

$x \cap x$ and $x \cup x$ are strictly increasing in x,

$x \cap y \leq \min\{x,y\}$, and $x \cup y \geq \max\{x,y\}$,

$1 \cap 1 = 1$, and $0 \cup 0 = 0$,

obtained from the above assumptions, Bellman and Giertz prove that

$$x \cap y = \min\{x,y\} \text{ and } x \cup y = \max\{x,y\}.$$

Their proof proceeds as follows. Let $h(x) = x \cap x$. From (1) and (3), h maps on the interval [0,1]. If $h(x) = a$ then

$$x \cap x = a \leq a \cup (a \cap a) = (a \cup a) \cap (a \cup a)$$

so that $x \leq a \cup a$. Also,

$$x \geq x \cap (x \cup x) = (x \cap x) \cup (x \cap x) = a \cup a$$

so x must equal $a \cup a$. Thus

$$x \cap x = a \leftrightarrow x = a \cup a.$$

Therefore, it follows that

$$x \cap (x \cup x) = x \cup (x \cap x) = x \qquad x \in [0,1]$$

Substituting $x = a \cup a$ in $x = x \cup (x \cap x)$ gives

$$a \cup a = (a \cup a) \cup [(a \cup a) \cap (a \cup a)] = (a \cup a) \cup a,$$

or using the associative axiom,

$$a \cup a = a.$$

Also, $a \cap a = a$ which further implies that

$$a \cup (a \cap b) = a \cap (a \cup b) = a \quad a,b \in [0,1]$$

Now assume a and b are given in the interval [0,1] with $a \geq b$. Since $a \cap a = a$, then according to Bellman and Giertz, there must be some c that satisfies $a \cap c = b$. Use of the latter equation results in

$$a \cup b = a \cup (a \cap c) = a = \max\{a,b\} \text{ and}$$

$$a \cap b = a \cap (a \cap c) = (a \cap a) \cap c = a \cap c = b = \min\{a,b\}$$

thus providing proof that the maximum and minimum operators are the only valid operators for fuzzy union and intersection under the given assumptions.

The importance of this conclusion is that it constrains the range of operations that may be performed on fuzzy sets, and begins to define what is and is not a fuzzy set or operation. From the results of Bellman and Giertz important similarities and differences between fuzzy sets and probability logics are shown in the next several sections. This contributes to the determination of what constraints are to be placed on the use of these logics.

Fuzzy Sets Restricted to the Unit Interval.

The operations of fuzzy sets, when the fuzzy supports are constrained to {0,1} (i.e. elements are unambiguously members or non-members of the set), are equivalent to classical sets. This is demonstrated by showing that the restricted operations support the algebra of classical sets as illustrated in Table 2. The demonstrations are given in Table 3. First, for each law of algebra of classical sets, the classical formulation is stated, for example $A \cup A = A$. Second, the equivalent fuzzy set formulation is given. This is obtained by substituting fuzzy set operations for classical set operations. From the results of Bellman and Giertz, the union operator $\cup$ must be replaced by the fuzzy operator MAX (maximum). In the example the formulation becomes MAX(A,A) = A. Third, because the fuzzy set operations are temporarily restricted, the fuzzy set may assume two values, 0 or 1. To illustrate the fuzzy operations under all cases, tables similar to truth tables are presented. For the example, the table:

a	MAX(a,a)	=	a
1	1		1
0	0		0

shows that under both cases the formulation is true, confirming the support of this law of algebra of classical sets by the constrained form of fuzzy sets. To simplify the notation in these tables, let 'a' denote the fuzzy support $\mu_A(a)$, etc.

Thus it is shown here that under the stated restriction, fuzzy sets are equivalent to classical sets, with respect to their satisfaction of the algebra of classical sets.

The behavior of fuzzy sets when not restricted and allowed to range on the full interval [0, 1] is now examined. The membership function of a set A is denoted

$$A = [0, b, a, 1]$$

where 0, a, b, and 1 are supports for membership in the set A. The cases of a or b equal to one and zero were examined above. Other possible valuations are now considered here with algebraic demonstrations given in Table 4.

Table 3. Algebraic Properties of Fuzzy Sets Constrained to {0,1}

1. Reflexive Law.

$A \cup A = A$

MAX(a,a) = a

a	MAX(a,a)	a
1	1	1
0	0	0

$A \cap A = A$

MIN(a,a) = a

a	MIN(a,a)	a
1	1	1
0	0	0

2. Associative Law.

$(A \cup B) \cup C = A \cup (B \cup C)$

MAX(MAX(a,b), c) = MAX(a, MAX(b,c))

a	b	c	MAX(MAX(a,b), c)	MAX(a, MAX(b,
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
0	0	1	1	1
0	1	0	1	1
1	0	0	1	1
0	0	0	0	0

$(A \cap B) \cap C = A \cap (B \cap C)$

MIN(MIN(a,b), c) = MIN(a, MIN(b,c))

a	b	b	MIN(MIN(a,b), c)	MIN(a, MIN(b,c))
1	1	1	1	1
1	1	0	0	0
1	0	1	0	0
0	1	1	0	0
0	0	1	0	0
0	1	0	0	0
1	0	0	0	0
0	0	0	0	0

Table 3. continued.

3. <u>Commutative Law</u>.

$A \cup B = B \cup A$

MAX(a,b) = MAX(b,a)

a	b	MAX(a,b)	MAX(b,a)
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

$A \cap B = B \cap A$

MIN(a,b) = MIN(b,a)

a	b	MIN(a,b)	MIN(b,a)
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

4. <u>Distributive Law</u>.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

MAX[a, MIN(b,c)] = MIN [MAX(a,b), MAX(b,c)]
(1) (2) (3) (4) (5) (6)

a	b	c	MIN(b,c)	MAX(a,b)	MAX(a,c)	MAX((1),(4))	MIN((5),(6))
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
0	1	0	0	1	0	0	0
0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

MIN[a, MAX(b,c)] = MAX [MIN(a,b), MIN(b,c)]
(1) (2) (3) (4) (5) (6)

a	b	c	MAX(b,c)	MIN(a,b)	MIN(a,c)	MIN((1),(4))	MAX((5),(6))
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Table 3. continued.

5. Identity Laws.

$A \cup \emptyset = A$

MAX(a,0) = a

a	0	MAX(a,0)	a
1	0	1	1
0	0	0	0

$A \cap \emptyset = \emptyset$

MIN(a,0) = 0

a	0	MIN(a,0)
1	0	0
0	0	0

$A \cap U = A$

MIN(a,1) = a

a	1	MIN(a,1)	a
1	1	1	1
0	1	0	0

$A \cup S = S$

MAX(a,1) = 1

a	1	MAX(a,1)
1	1	1
0	1	1

6. Complement Laws

$A \cup A^{C} = U$

MAX(a, 1-a) = 1

a	1-a	MAX(a, 1-a)
1	0	1
0	1	1

$(A^{C})^{C} = A$

$$1 - (1 - a) = a$$
$$a = a$$

$A \cap A^{C} = \emptyset$

MIN(a, 1-a) = 0

a	1-a	MIN(a,1-a)
1	0	0
0	1	0

7. DeMorgan's Law.

$(A \cup B)^{C} = A^{C} \cap B^{C}$

1- MAX[a,b] = MIN[1-a, 1-b]
(1) (2) (3) (4)

a	b	1-a	1-b	1- MAX((1),(2))	MIN((3),(4))
1	1	0	0	0	0
1	0	0	1	0	0
0	1	1	0	0	0
0	0	1	1	1	1

Table 3. continued.

$(A \cap B)^c = A^c \cup B^c$

1- MIN[a,b] = MAX[1-a, 1-b]
(1) (2) (3) (4)

a	b	1-a	1-b	1- MIN((1),(2))	MAX((3),(4))
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	1	1	1	1

The operations of fuzzy sets are shown to support the laws of classical sets with the exception of complementation, that is, a fuzzy set and its complement are not mutually exclusive. Some implications of this result are discussed in a later section. Intuitively, this result states that if an element only partially belongs to a set ($\mu_A(x) < 1$), then it cannot be said that the element does not belong to the set.

Probability.

Probability is a measure on sets. It may be interpreted as a measure of the occurence of an outcome or a set of outcomes. Let S (the sample space) denote the set of all possible outcomes of some event. An event (a) is a set of some possible outcomes ($a \in S$).

Table 4. Algebraic Properties of Unconstrained Fuzzy Sets

1. <u>Reflexive Law</u>.

$A \cup A = A$

MAX(a,a) = a

a	MAX(a,a)	a
a	a	a

$A \cap A = A$

MIN(a,a) = a

a	MIN(a,a)	a
a	a	a

2. <u>Associative Law</u>.

$(A \cup B) \cup C = A \cup (B \cup C)$

MAX(MAX(a,b), c) = MAX(a, MAX(b,c))

a	b	c	MAX(MAX(a,b), c)	MAX(a, MAX(b,c)
a	=b	=c	a,b,c	a,b,c
a	>b	=c	a	a
a	=b	>c	a,b	a,b
a	>b	>c	a	a

$(A \cap B) \cap C = A \cap (B \cap C)$

MIN(MIN(a,b), c) = MIN(a, MIN(b,c))

a	b	b	MIN(MIN(a,b), c)	MIN(a, MIN(b,c))
a	=b	=c	a,b,c	a,b,c
a	>b	=c	b,c	b,c
a	=b	>c	c	c
a	>b	>c	c	c

3. <u>Commutative Law</u>.

$A \cup B = B \cup A$

MAX(a,b) = MAX(b,a)

a	b	MAX(a,b)	MAX(b,a)
a	=b	a,b	a,b
a	>b	a	a
a	<b	b	b

$A \cap B = B \cap A$

MIN(a,b) = MIN(b,a)

a	b	MIN(a,b)	MIN(b,a)
a	=b	a,b	a,b
a	>b	b	b
a	<b	a	a

Table 4. continued.

4. Distributive Law.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

MAX[a, MIN(b,c)] = MIN [MAX(a,b), MAX(b,c)]

(1)	(2)	(3)	(4)	(5)	(6)		
a	b	c	MIN(b,c)	MAX(a,b)	MAX(a,c)	MAX((1),(4))	MIN((5),(6))
a	=b	=c	b,c	a,b	a,c	a,b,c	a,b,c
a	>b	=c	b,c	a	a	a	a
a	=b	>c	c	a,b	a	a	a(b)
a	>b	>c	c	a	a	a	a

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

MIN[a, MAX(b,c)] = MAX [MIN(a,b), MIN(b,c)]

(1)	(2)	(3)	(4)	(5)	(6)		
a	b	c	MAX(b,c)	MIN(a,b)	MIN(a,c)	MIN((1),(4))	MAX((5),(6))
a	=b	=c	b,c	a,b	a,c	a,b,c	a,b,c
a	>b	=c	b,c	b	c	b,c	b,c
a	=b	>c	b	a,b	c	a,b	a,b
a	>b	>c	b	b	c	b	b

5. Identity Laws

$A \cup \emptyset = A$

MAX(a,0) = a

a	0	MAX(a,0)	a
a	0	a	a

$A \cap \emptyset = \emptyset$

MIN(a,0) = 0

a	0	MIN(a,0)
a	0	0

$A \cap U = A$

MIN(a,1) = a

a	1	MIN(a,1)	a
a	1	a	a

$A \cup S = S$

MAX(a,1) = 1

a	1	MAX(a,1)
a	1	1

6. Complement Laws

$A \cup A^c = U$

MAX(a, 1-a) = 1

$A \cap A^c = \emptyset$

MIN(a, 1-a) = 0

Table 4. continued.

a	1-a	MAX(a, 1-a)	1	a	1-a	MIN(a,1-a)	0
$0<a<1$	$1>a^c>0$	$0<MAX(a,a^c)<1$	1	$0<a<1$	$1>a^c>0$	$0<MIN(a,a^c)<1$	0

Therefore, $MAX(a, 1-a) \neq 1$, and $MIN(a, 1-a) \neq 0$.

$(A^c)^c = A$

$1 - (1 - a) = a$
$a = a$

7. <u>DeMorgan's Law</u>

$(A \cup B)^c = A^c \cap B^c$

$1- MAX[a,b] = MIN[1-a, 1-b]$

(1)	(2)	(3)	(4)		
a	b	1-a	1-b	1- MAX((1),(2))	MIN((3),(4))
a	=b	1-a	1-b	1-a, 1-b	1-a, 1-b
a	>b	1-a	1-b	1-a	1-a
a	<b	1-a	1-b	1-b	1-b

$(A \cap B)^c = A^c \cup B^c$

$1- MIN[a,b] = MAX[1-a, 1-b]$

(1)	(2)	(3)	(4)		
a	b	1-a	1-b	1- MIN((1),(2))	MAX((3),(4))
a	=b	1-a	1-b	1-a, 1-b	1-a, 1-b
a	>b	1-a	1-b	1-b	1-b
a	<b	1-a	1-b	1-a	1-a

Probability Axioms.

Letting S be a finite sample space: $S = \{a_1, a_2, \ldots a_n\}$ and assigning to each point $a_i \in S$ a real number p_i called the probability of a_i with the properties (Rescher, 1969; Larson, 1974):

i) $p_i \geq 0$

ii $\sum_{i=1}^{n} p_i = 1$

then the axioms of probability may be given as:

(7) $P(S) = 1$

(8) $P(A) \geq 0$

(9) $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$

$\text{iff } A_{i \cap j} = 0, i \neq j$

with the resulting theorems:

(10) $P(\emptyset) = 0$

(11) $P(A^c) = 1 - P(A)$

(12) $P(A^c \cup B) = P(B) - P(A \cap B)$

(13) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

The complement theorem (11) is derived from the premise that the members of A^c are the members in the sample space S that are not contained in A, that is, $A \cup A^c = S$, or:

$$P(A \cup A^c) = P(S)$$

$$P(A) + P(A^c) = 1 \quad \text{(from A1 and A3)}$$

$$P(A^c) = 1 - P(A).$$

Fuzzy Logic.

Fuzzy logic may be interpreted as a measure on fuzzy sets, that is, it is concerned with defining consistent measures of the truth of fuzzy propositions. Gaines (1976) presents three different definitions for fuzzy logic as it is used in the literature:

1. "A basis for reasoning with vague or imprecise statements." In this very general definition its use is primarily restricted to the description of sets that have no well defined borders and includes hedges on these sets.
2. "A basis for reasoning with imprecise statements using fuzzy sets theory for the fuzzification of logical structures." Gaines feels that this definition comes closest to that implied by Zadeh and while it implies a somewhat independent logic structure, it is still related to classical logics.
3. "A multivalued logic in which truth values are in the interval [0,1], and the valuation of a disjunction is the maximum of those of the disjuncts, and that of a conjunction is the minimum of those of the conjuncts." This very restricted definition assumes union and intersection operators, but does not necessarily assume its derivation from fuzzy set theory.

The second definition may be the most appropriate in that it is more specific than the first and incorporates the implications of the third, without assuming or requiring specific operators. The second definition also provides a basis for comparing fuzzy logics to probability logic. It has been argued (e.g. Blockley et al., 1983) that fuzzy sets is a generalization of classical sets and that fuzzy logic would then be implied to be a generalization of probability logic. The second definition includes this implication.

Fuzzy Logic Axioms.

The multivalued logic widely used for fuzzy reasoning is that of Lukasiewicz' infinitely valued logic. Lukasiewicz (1930) extended the classical two-valued logic to include a third, indeterminate truth value "I". The motivation for introducing the indeterminate value was to provide a truth value for propositions of an essentially unknown state. That is, a proposition may be formulated, but it is not known at the present whether the outcome will be true or false. The truth values assigned to these three states became v(T) = 1, v(I) = .5, v(F) = 0, and the truth tables associated with the introduction of this third value are shown in Table 5 (Rescher, 1969).

Table 5. Truth Tables for Lukasiewicz' Three-Valued Logic

		q	$p \cap q$			$p \cup q$			$p \to q$			$p \leftrightarrow q$		
p	p^c	$p \backslash$	T	I	F	T	I	F	T	I	F	T	I	F
T	F	T	T	I	F	T	T	T	T	I	F	T	I	F
I	I	I	I	I	F	T	I	I	T	T	I	I	T	I
F	T	F	F	F	F	T	I	F	T	T	T	F	I	T

Subsequent to the introduction of the three valued logic, Lukasiewicz extended the idea to n-valued, and ultimately infinitely-valued (continuous) logic. Dividing the interval from 0 to 1 into n values results in the general form given in Table 6 (Rescher, 1969).

Table 6. General Form of Lukasiewicz' Multi-Valued Logic

n	Truth Values	
2	1 | 1, 0 | 1	(1, 0)
3	2 | 2, 1 | 2, 0 | 2	(1, 1/2, 0)
4	3 | 3, 2 | 3, 1 | 3, 0 | 3	(1, 2/3, 1/3, 0)
.		
.		
n	$\frac{n-1}{n-1}, \frac{n-2}{n-1}, \ldots, \frac{2}{n-1}, \frac{1}{n-1}, \frac{0}{n-1}$	

The truth tables for the three-valued, the n-valued, and the infinitely-valued logic may be obtained from the algebraic propositions (Rescher, 1969):

$$p^c = 1 - p$$

$$p \cup q = \max(p,q)$$

$$p \cap q = \min(p,q)$$

$$p \rightarrow q = \begin{cases} 1 \text{ if } p \leq q \\ 1 - p + q \quad \text{if } p > q \end{cases}$$

$$p \leftrightarrow q = (p \rightarrow q) \cap (q \rightarrow p) = 1 - p - q.$$

The result of these extensions of classical logic is a continuous range of truth values that may be attributed to uncertain (or indeterminate) propositions.

Zadeh (1975) adopts the logic system of Lukasiewicz as the base logic of fuzzy sets. Zadeh is concerned primarily with representing linguistic expressions of truthfulness such as true, very true, not true, false, etc. He maintains the essential differences between the two logics are that fuzzy logic is a countable set of truth values while Lukasiewicz's is a continuum (although, as shown above, Lukasiewicz's logic is also based upon a discrete set of truth values); that fuzzy logic is more efficient in that it represents propositions using a smaller number of truth-values; and that fuzzy logic addresses propositions which are themselves fuzzy rather than precise e.g. 'Vera is "highly intelligent"', 'Berkeley is "close" to San Francisco' (Zadeh, 1975).

Logical connectives between propositions (e.g. x *and* y) are defined in terms of truth or belief rules (Gaines, 1978). A statement, or proposition of the event x, has a truth value $\mu(x)$. The axioms which fuzzy logic support (Gaines, 1978) are:

$$0 \leq \mu(x) \leq 1 \tag{14}$$

$$\mu(x^c) = 1 - \mu(x) \tag{15}$$

$$\mu(x \cap y) = \mathrm{MIN}(\mu(x), \mu(y)) \tag{16}$$

$$\mu(x \cup y) = \mathrm{MAX}(\mu(x), \mu(y)) \tag{17}$$

$$\mu(x \Rightarrow y) = \mathrm{MIN}(1, 1 - \mu(x) + \mu(y)) \tag{18}$$

$$\mu(x = y) = \mathrm{MIN}(1 - \mu(x) + \mu(y), 1 + \mu(x) - \mu(y)). \tag{19}$$

For (14) a truth value of 0 denotes there is no truth to the statement (the statement is false) while a truth value of 1 indicates that there is complete truth in the statement (the statement is true).

The second axiom (Eq. 15) implies that because an event A is defined by a membership function $\{x, \mu_A(x)\}$ $x \in A$, the complement of an event A^c is defined by $\{x, 1 - \mu_A(x)\}$ $x \in A$.

The third and fourth axioms (Eq. 16 and 17) state that the truth value of the conjunction of two events is no stronger than the weakest truth value of an event and the truth value of the disjunction of an event is no less than that which may be obtained from the individual events.

Gaines' Standard Uncertainty Logic.

Gaines (1978) examines the relationship between fuzzy and probability logics. Gaines concludes that there are significant differences between the two but that they share a set of common axioms which he has designated a standard uncertainty logic. He demonstrates that the addition of a single axiom - the law of the excluded middle $p(x \cup x^c) = 1$, (or the law of contradiction) results in a standard probability logic. The addition of a different axiom - a truth functionality requirement, results in a Lukasiewicz or fuzzy logic. The fifteen axioms constituting the standard uncertainty logic posited by Gaines are:

(20) $x \cup x = x \cap x = x$:Reflexive

(21) $x \cup y = y \cup x$; $x \cap y = y \cap x$:Commutative

(22) $x \cup (y \cup z) = (x \cup y) \cup z$:Associative

$x \cap (y \cap z) = (x \cap y) \cap z$

(23) $x \cup (x \cap y) = x;\ x \cap (x \cup y) = x$:Absorptive

(24) $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$:Distributive

(25) $x \cup T = T;\ x \cap T = x$:Identities

$x \cup F = x;\ x \cap F = F$ (Definition of max, min elements)

(26) $y \geq x$ iff $\exists\, z \mid y = x \cup z$:Ordering criterion

In the previous sections, axioms 20 through 24 were shown to hold true for both fuzzy and probability logics. The others are also shown to hold under both logics.

Further mapping functions are defined by Gaines:

(27) $p(F) = 0;\ p(T) = 1$ (unambiguous cases define the endpoints of allowable range of supports)

(28) $y \geq x$ implies $p(y) \geq p(x)$

(29) $p(x \cap y) + p(x \cup y) = p(x) + p(y)$

(30) $x = y$ iff $p(x \cap y) = p(x \cup y)$

(31) $d(x, y) = p(x \cup y) - p(x \cap y)$:Metric on distance between sets

(32) $p(x = y) = 1 - d(x,y) = 1 - p(x \cup y) + p(x \cap y)$

(33) $p(x \Rightarrow y) = 1 - p(x) + p(x \cap y)$

$= 1 + p(y) - p(x \cup y)$

(34) $p(x^c) = 1 - p(x)$.

An additional formula may be obtained from Equations 29 and 34. Let $y = x^c$. Then (29) is

$$p(x \cap x^c) + p(x \cup x^c) = p(x) + p(x^c)$$

But from (34) $p(x^c) = 1 - p(x)$. Therefore,

$$p(x \cap x^c) + p(x \cup x^c) = p(x) + 1 - p(x)$$

(35) $p(x \cap x^c) + p(x \cup x^c) = 1$.

From Equation 35 the law of the excluded middle ($p(x \cup x^c) = 1$) is not a tautology and must therefore constitute an assumption when it used in probability logic. From this result Gaines shows that the addition of this law to the standard uncertainty logic will result in the probability logic defined previously.

Gaines further shows that the addition of the postulate

$$p(x \Rightarrow y) = 1 \text{ OR } p(y \Rightarrow x) = 1$$

to the standard uncertainty logic yields the Lukasiewicz infinitely valued logic which has been taken by Zadeh (1975) as the fuzzy logic identified in the previous section. The connection between the standard uncertainty logic and fuzzy logic is shown by Gaines as:

Given the postulate: $p(x \Rightarrow y) = 1$ OR $p(y \Rightarrow x) = 1$,

and $p(x \Rightarrow y) = 1 - p(x) + p(x \cap y)$

then, $p(x \Rightarrow y) = 1 - p(x) + p(x \cap y)$ OR

$$p(y \Rightarrow x) = 1 - p(y) + p(x \cap y).$$

But $p(x \Rightarrow y) = 1$ OR $p(y \Rightarrow x) = 1$, therefore

$$1 = 1 - p(x) + p(x \cap y) \text{ OR } 1 = 1 - p(x) + p(x \cap y),$$

$$p(x) = p(x \cap y) \text{ OR } p(y) = p(x \cap y)$$

But $p(x) = p(x \cap y)$ implies that $p(x) \leq p(y)$ and

$p(y) = p(x \cap y)$ implies that $p(y) \leq p(x)$.

In other words

$$p(x \cap y) = p(x) \text{ if } p(x) \leq p(y) \text{ and}$$

$$p(x \cap y) = p(y) \text{ if } p(y) \leq p(x).$$

Therefore, $p(x \cap y)$ implicitly equals $\mathrm{MIN}(p(x), p(y))$ and Gaines argues that the remaining axioms follow.

The results of Gaines' work show that fuzzy and probability logics are very similar. It is the difference between the two logics that is important in the context of this study. From the differences identified by Gaines, it is possible to identify characteristics of decision problems that limit the applicability of either of the two approaches. In the next sections, examinations of the similarities and differences between the two logics are expanded leading to the conclusion that probability logic is not a parallel logic, but is a subset of fuzzy logic.

Relationship Between Fuzzy Logic and Probability.

As an extension to Gaines' work, further examination of the relationship between fuzzy and probability logics is now developed. In a manner similar to the relationship between fuzzy sets and classical sets, the following is stated: the axioms of fuzzy logic will support the theorems of probability when the values of the characteristic function are constrained to {0,1}.

The validity of this statement is demonstrated by the following:

Axioms 1, 2, 4, and 5 (Eq. 7, 8, 10, and 11) follow directly from the definition of fuzzy logic axioms. Next, the sixth axiom is evaluated:

$$P(A^c \cap B) = P(B) - P(A \cap B).$$

Representation in fuzzy logic format:

$$\mu(A^c \cap B) = \mu(B) - \mu(A \cap B)$$

$$\mathrm{MIN}(1-a,\ b) = b - \mathrm{MIN}(a,\ b)$$

a	b	1-a	MIN((3),(2))	(2) - MIN((1),(2))
1	1	0	0	0
1	0	0	0	0
0	1	1	1	1
0	0	1	0	0

The seventh axiom,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

is represented in fuzzy logic as

$$\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B), \text{ or}$$

$$\mathrm{MAX}(a,\ b) = a + b - \mathrm{MIN}(a,\ b)$$

with proof in the truth table:

a	b	MAX((a),(b))	(a)+(b)-MIN((a),(b))
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

Finally, the third axiom of additivity is evaluated:

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots \quad \text{iff } A_{i \cap j} = \emptyset,\ i \neq j.$$

First, examine the constraint $A_{i \cap j} = \emptyset$, $i \neq j$. The fuzzy logic equivalent is

$$\mu(A_i \cap A_j) = 0, \text{ or}$$

$$\text{MIN}(A_i, A_j) = 0.$$

This implies that if any $A_i = 1$, then all others are equal to 0. This result follows from the definition of inclusion in fuzzy logic:

If two events have full support ($\mu(x) = 1$, $\mu(y) = 1$) then each is fully included in the other:

$$\begin{aligned}\mu(x \Rightarrow y) &= \text{MIN}(1, 1-\mu(x)+\mu(y)) \\ &= 1\end{aligned}$$

$$\begin{aligned}\mu(y \Rightarrow x) &= \text{MIN}(1, 1-\mu(y)+\mu(x)) \\ &= 1,\end{aligned}$$

and would violate the constraint for Axiom 3.

Therefore, the fuzzy equivalent of Axiom 3 is:

$$\mu(A_1 \cup A_2 \cup A_3 \cup \ldots) = \mu(A_1) + \mu(A_2) + \mu(A_3) + \ldots \quad \text{iff } \mu(A_i) \cap \mu(A_j) = \emptyset,\ i \neq j$$

$$\text{MAX}(\mu(A_1), \mu(A_2), \mu(A_3) \ldots) = \mu(A_1) + \mu(A_2) + \mu(A_3) + \ldots \quad \text{iff MIN}(A_i, A_j) = \emptyset,\ i \neq j.$$

From the constraint, only two cases are possible:

$$\text{MAX}(\mu(A_1), \mu(A_2), \mu(A_3), \ldots) = \begin{cases} 1 \text{ iff } \exists!\ \mu(A_i) = 1 \\ 0 \text{ iff } \mu(A_i) = 0,\ \forall i \in A \end{cases}$$

Therefore,

$$\underset{\substack{1\\0}}{\underline{\text{MAX}(\mu(A_1),\ \mu(A_2),\ \mu(A_3),\ \ldots)}} \quad \underset{\substack{1\\0}}{\underline{\mu(A_1)\ +\ \mu(A_2)\ +\ \mu(A_3)\ +\ \ldots}}$$

and Axiom 3 is shown to be supported by fuzzy logic. To arrive at this conclusion, the operation of fuzzy inclusion was used. It should be shown to be equivalent to the probability definition of inclusion so that it is seen to not be an exception to the above comparisons. The probability defintion of inclusion is:

$$p(A \Rightarrow B) = p(A^c) + p(A \cap B).$$

The fuzzy set equivalent is:

$$1-\mu(a) + \text{MIN}(\mu(a), \mu(b)).$$

Inclusion in fuzzy sets is:

$$\mu(A \Rightarrow B) = \text{MIN}(1, 1 - \mu(a) + \mu(b)).$$

Evaluating the two forms:

$$\text{MIN}(1, 1 - \mu(a) + \mu(b)) \overset{?}{=} 1- \mu(a) + \text{MIN}(\mu(a), \mu(b))$$

results in the comparison:

(1) x	(2) y	(3) 1-x	MIN(1, (3)+(2))	(3) + MIN((1), (2))
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

where it is shown that the constrained fuzzy version of inclusion is equivalent to the probabilistic definition of inclusion.

Interpretation of Differences/Similarities.

It was shown that fuzzy set operations do not support complement-ation in probability logic. A derivation of complementation, the law of the excluded middle, may be interpreted as requiring an individual to vote false for A^c whenever the individual votes true for A. In a decision making context, assume a set of alternatives A = $\{a_1, a_2, a_3, a_4\}$. If the decision maker assigns a probability p that a_1 is the preferred alternative, then the decision maker must also assign a probability of 1-p that a_1 is not preferred (or that $\{a_2, a_3, a_4\}$ includes the preferred alternative), and if the preference for a_1 changes, then the preference for $\{a_2, a_3, a_4\}$ must change proportionately.

Rather than examine the formal requirements imposed upon the decision maker, it is useful to consider conditions arising in decision making problems in which the law of the excluded middle may not be supported. Two cases will be considered: borderline decisions and intransitivity.

In the case of borderline decisions, a decision maker is unable to state strict preference. For instance, a decision maker stating that alternative A is preferred "somewhat" to alternative B indicates that the individual is unwilling to state that B is not preferred to A. Probability logic requires that if the truth value of the proposition "A is preferred to B" is greater than zero, then the truth value of the proposition "B is preferred to A" must equal zero.

In the instance of intransitivity, a decision maker may prefer alternative A to B and alternative B to C, but then state that alternative C is equally or more preferred than A. To see that this violates probability logic, the following proposition is stated:

$(A > B)$ and $(B > C)$ implies $(C \geq A)$, or

$$P[((A>B) \cap (B>C)) \Rightarrow (C \geq A)] = P[(A>B) \cap (B>C)] +$$
$$P[((A>B) \cap (B>C)) \cap (C \geq A)]$$

$$1 = P[((A>B) \cap (B>C)) \cap (C \geq A)]$$

$$1 \neq P[(A>C) \cap (C \geq A)]$$

therefore, $A > C$.

Having shown that probability logic supports neither borderline cases nor intransitivity, it remains for fuzzy set operations to support these conditions. For the borderline case, consider again the condition $A > B$ indicating that alternative A is preferred to alternative B. In probability, if $A > B$ then $B > A$. To examine fuzzy set operations, let $\mu(x)$ equal the proposition

$$\mu(A > B) > 0$$

that is, A is preferred to B to some degree greater than zero, and let $\mu(y)$ equal the proposition

$$\mu(A \leq B) \geq 0$$

that is, A is less or equally preferred to B to some degree greater than or equal to zero. Of interest then is the examination of the proposition that

$$\mu(x) \text{ implies } \mu(y)$$

or, if $(A > B)$ is true to some degree then $(A \leq B)$ may be true to some degree. Using the fuzzy implication operation, $\mu(x)$ implies $\mu(y)$ is rewritten as

$$\mu(x \text{ implies } y) = \min(1,\ 1 - \mu(x) + \mu(y)).$$

To test if borderline cases are permitted under fuzzy sets, it is necessary to when when $\mu(x$ implies $y)$ is false, that is, $\mu(x$ implies $y) = 0$:

$$0 = \min(1,\ 1 - \mu(x) + \mu(y)).$$

Since $0 \neq 1$, the cases are restricted to

$$0 = 1 - \mu(x) + \mu(y) \text{ or,}$$

$$\mu(x) = 1 + \mu(y).$$

There exists only one case where this equality will hold: when $\mu(x) = 1$ and $\mu(y) = 0$. This condition is a special case of fuzzy sets and is equivalent to classical two-valued statements. The interpretation is that only when it is completely certain that A is preferred to B $(\mu(x) = 1)$ can it be said that B is not preferred to A to some degree $(\mu(y) = 0)$. In all other cases, if there is some uncertainty regarding the preference of A, then B may be preferred to A to some degree. Thus fuzzy logic supports borderline cases.

For fuzzy sets to permit intransitivity it is necessary to examine comparisons between fuzzy numbers. Dubois and Prade (1980) define the degree of possibility of M ≥ N where M and N are two fuzzy numbers as

$$v(M \geq N) = \sup_{x,y:\ x \geq y} \min(\mu_N(x),\ \mu_M(y))$$

To illustrate this definition, consider the fuzzy numbers defined in Figure 4. Fuzzy number A might be interpreted as "approximately 3", number B "approximately 4", etc. The question of "which is the greater, A or B?" is answered in the following manner. The degree of possibility that B ≥ A is equal to unity as the largest support for A and B is unity where y ≥ x. The next question is whether it may be said

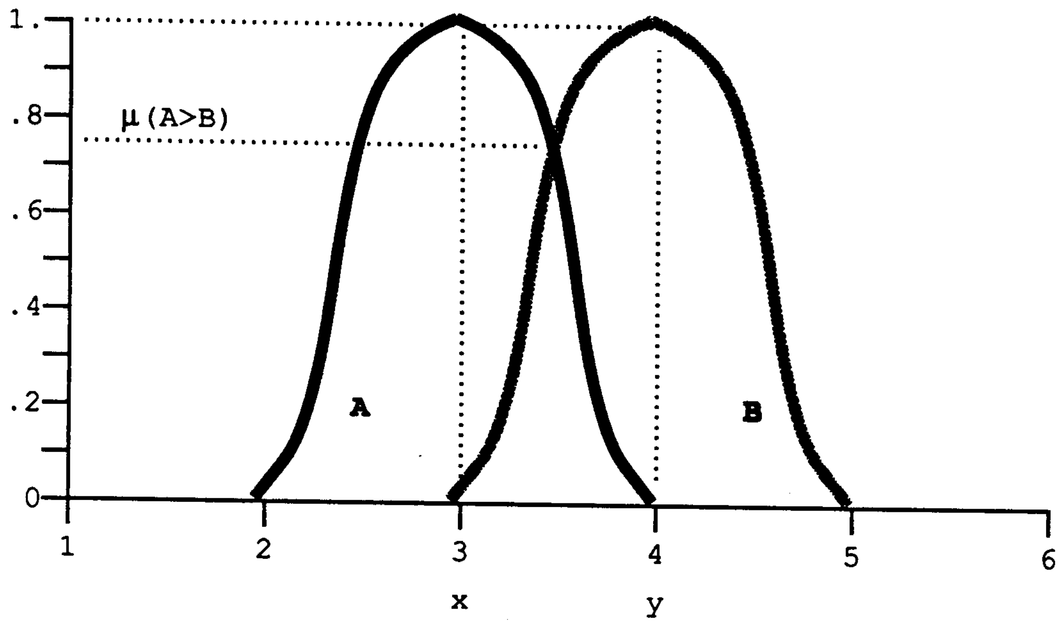

Figure 4. Intersection of Fuzzy Numbers A and B

that some possibility exists that A > B. If v(A>B) can be shown to be greater than zero for any case then it must be concluded that fuzzy set operations allow some degree of intransitivity. The degree of possibility that A > B is defined as

$$v(A>B) = hgt(A \cap B)$$

which in Figure 4 is the support of the intersection of A and B, which is greater than zero.

Restrictions on Truth Functionality.

It was shown that unconstrained fuzzy sets do not support the law of the excluded middle ($P(x \cup x^c) = 1$). It is not yet demonstrated that probability logic will support fuzzy logic, that is, the truth functionality requirement. It may be inferred from the derivation of fuzzy logic using the postulate $p(x \Rightarrow y) = 1$ OR $p(y \Rightarrow x) = 1$ that connectives ($p(x \cap y)$ or $p(x \cup y)$) in fuzzy logic are a function of individual truth values. It was shown that $p(x \cap y) = p(x)$ or $p(y)$. For probability logic to support fuzzy logic, it must be able to define connectives with respect to individual valuations on the elements (x, y). Taking the law of the excluded middle as the probability defining axiom, it can be shown to be inconsistent with truth functionality:

$$P(x \cup x^c) = 1$$

but, $P(x \cap x^c) = p(x) + p(x^c) - p(x \cap x^c)$

therefore,

$$p(x) + p(x^c) - p(x \cap x^c) = 1.$$

But, $p(x^c) = 1 - p(x)$

therefore,

$$p(x) + 1 - p(x) - p(x \cap x^c) = 1$$

$$p(x \cap x^c) = 0,$$

which is the law of contradiction.

The law of the excluded middle thus cannot be expressed as a function of only p(x) and p(y) and is therefore inconsistent with the truth functionality requirement of fuzzy logic. This implies that fuzzy sets can support all the axioms of probability while probability is constrained from supporting all axioms of fuzzy logic. Rather than being similar, but independent, logics these results show that probability is a special case of fuzzy logic.

There may exist conditions under which truth functionality is violated. The derivation of fuzzy logic by Gaines showed the operators for conjunction and disjunction implied to be minimum and maximum, respectively. That minimum and maximum are the only possible operators under truth functionality restrictions is shown by Bellman and Giertz (1973) and discussed earlier.

Conclusions.

This chapter introduced the operations of fuzzy sets and fuzzy logic that are pertinent to the development of a method for decision analysis and evaluated these operations relative to classical set operations and probability logic. It was shown that fuzzy sets are related to classical sets in that classical sets (and probability) are special cases of fuzzy sets. This has several implications. First, if probabilistic techniques are too restrictive for the problem to be solved, then fuzzy sets becomes a logical candidate for the solution of the problem. Because probability is only a special case, there is no guarantee that other logics besides fuzzy logic may not be applied. Second, while the theory of fuzzy sets is less restrictive in its assumptions, it still places demands upon the decision maker that may or may not reflect human behavior. This issue will be examined in Chapter Three. The third implication is that while the theory may be appropriate and desirable, it is important to test the theory through application. Several methods of application will be examined in the next chapter. The first contends to incorporate uncertainty in a manner similar to fuzzy sets. The second method is based directly on fuzzy sets and fuzzy set operations. The third method is proposed as a possible candidate for decision making problems. Based upon fuzzy logic, its development is given in the next chapter.

CHAPTER TWO

METHODOLOGICAL FRAMEWORK

To apply fuzzy set operations to decision making problems, it is first necessary to establish a methodology. In this chapter, several possible methods are considered. In addition to approaches based upon the Bellman-Zadeh (1970) model of decision making, the method proposed by Tsukamoto and Terano (1975) is described in detail. Tsukamoto and Terano's approach uses fuzzy logic operations introduced in the previous chapter. The method, based upon the multivalued logic of Lukasiewicz, is possibly of importance for several reasons. First, several investigators (Zadeh, 1975; Gaines, 1978) have maintained that methods based upon fuzzy implication may more closely model human decision behavior than traditional techniques of decision theory including probability theory. Second, the method developed by Tsukamoto and Terano has the feature of providing bounds on the uncertainty of a conclusion where other methods provide a single value. These other methods are shown in a later section to appear less sensitive to uncertainty in a decision problem.

The next portion of this chapter will discuss fuzzy set based and related methods. The operations of fuzzy implication are then examined in greater detail with respect to both classical and fuzzy logics. Later sections discuss desired characteristics of a fuzzy logic algorithm in the context of decision making, the solution algorithm by Tsukamoto and Terano is presented, and the derivation of a proposed extension of the algorithm to resolve inconsistencies is given.

Saaty's Method.

The method for decision analysis proposed by Saaty (1977) and in the context of fuzzy sets (Saaty, 1978) is based on the use of scaling ratios to determine relative priorities of elements of a decision problem evaluated within a hierarchical structure of objectives, criteria, and alternatives. The relative priorities constitute a matrix of pairwise comparisons indicating the relative dominance of one element over another with respect to a criterion against which they are compared.

The elements may be denoted $A_1, \ldots, A_n$ and their weights $w_1, \ldots, w_n$. The pairwise comparisons form a matrix (Figure 5). If the matrix is multiplied by the transpose of the vector $w^T = (w_1, \ldots, w_n)$ the vector nw is obtained. The problem may be expressed as:

$$Aw = nw.$$

In application the matrix A exists a priori and it is desired to find w, requiring the solution of the system $(A-nI)w = 0$. The system has a nonzero solution only when n is an eigenvalue of A. In this case the matrix A has a rank of one and all eigenvalues except one equal zero. The remaining eigenvalue, denoted λ_{max} equals n. If the matrix A is consistent then $a_{ij}a_{jk} = a_{ik}$. If the matrix is not consistent then $\lambda_{max} > n$.

The construction of a consistent matrix is important to the validity of results obtained from Saaty's method. While perfect consistency between pairwise comparisons (which is equivalent to transitivity among preferences) is not an absolute requirement, large perturbations from a consistent matrix yield unreliable results (Saaty, 1977).

$$A = \begin{array}{ccccc} & A_1 & A_2 & \cdots & A_n \\ \hline A_1 & \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ A_2 & \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \vdots & & \vdots \\ A_n & \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{array}$$

Figure 5. Pairwise Comparison Matrix

To derive the ratio scales used in defining the pairwise comparison matrix A, Saaty uses a scale valued on the range 1 to 9 representing judgments between elements in a particular level of the heirarchy with respect to an element in a higher level of the hierarchy. The suggested scale is:

1: element i is equally important as element j,
3: element i is weakly more important than element j,
5: element i is strongly more important than element j,
7: element i is demonstrably more important than j,
9: element i is absolutely more important than j.

Even numbers (2, 4, 6, 8) represent intermediate values if necessary. Inverse values (e.g. 1/3) reflect a valuing of j greater than i.

The problem of combining priority vectors (Saaty, 1978) is one of obtaining a single vector for the highest level in the hierarchy from the cumulative effects in the lower levels of the hierarchy, through successive weighting and aggregation. Let L_k denote the kth level in the hierarchy with h levels. L_{k+1} denotes the level below L_k, and L_{k-1} the level above. Assume that $Y = \{y_1, y_2, \ldots, y_{mk}\} \in L_k$ and $X = \{x_1, x_2, \ldots, x_{mk+1}\} \in L_{k+1}$. Assume further that there exists an element $z \in L_{k-1}$ which ensures that the vector Y does not represent the top level of the hierarchy. Priority functions are defined as:

w_z: $Y \rightarrow [0,1]$ and w_y: $X \rightarrow [0,1]$ $j = 1,2,\ldots, m_k$.

The priority function of the elements in X with respect to z, denoted w,w: $X \rightarrow [0,1]$ is defined by

$$w(x_i) = \sum_{j=1}^{m_k} w_{yj}(x_i) w_z(y_i) \quad i = 1, 2, \ldots, m_{k+1}.$$

The influence of y on the priority of x is weighted by multiplying it by the importance of y with respect to z. Letting $B = b_{ij} = w_{yj}(x_i)$, $W_i = w(x_i)$, and $w_j' = w_z(y_i)$ then the above equation becomes

$$W_i = \sum_{j=1}^{m_k} b_{ij} W_j' \quad i = 1, 2, \ldots, m_{k+1}.$$

In this manner, the priority vector of the lowest level with respect to the highest element b is given by

$$W = B_n B_{n-1} \ldots B_2 W'.$$

A simple three level hierarchy could be defined to determine the priority vector for a set of alternatives given a set of criteria in the upper level and the sets of weights obtained by evaluating the relative dominance of the alternatives relative to each criterion and the relative priority of each criterion.

The method for obtaining the priority of a set of alternatives using Saaty's technique is fundamentally different from methods of fuzzy sets. Where in fuzzy set methods the supports for an alternative are determined by extreme values in the sets defining performance, the supports in Saaty's method are a multiplicative weighting.

Saaty's method is applied to hierarchical decision problems of the nature illustrated in Figure 1. For each evaluation criterion the decision maker is required to rate the relative dominance of one alternative over another using a scale such as the one given earlier. It is not required that this particular scale be used exclusively (several variants are used for different problems in Saaty (1977)). Saaty does suggest that the scale be able to "represent as much as possible all distinct shades of feeling that people have" subject to human limits to simultaneous comparison, and that there exist a unit difference between successive scale values (Saaty, 1977).

The use of Saaty's method in the experiment described in Chapters 3 and 4 is illustrated through the following example. Given a decision setting in which four alternative solutions have been identified it is desired to determine the best alternative by comparing their performance with respect to a set of evaluation criteria. Saaty's method dictates that the decision maker make pairwise evaluations of alternatives with respect to each evaluation criterion, and make pairwise comparisons of each criterion with respect to the overall goal of choosing the best alternative available to resolve the decision problem.

Each group of pairwise comparisons form a matrix. For example, for the first criterion the matrix of judgments about the performance of the alternatives elicited from a decision maker is:

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} & 2 & 4 & 1/3 \\ & & 5 & 1/3 \\ & & & 1/5 \\ & & & \end{array}\right]. \end{array}$$

Comparing alternative 1 with alternative 2 for this criterion the decision maker has assigned a value of 2 which is interpreted as alternative 1 may be preferred to alternative 2. In comparing alternatives 3 and 4 the decision maker assigned a value 1/5 indicating alternative 4 is probably preferred to 3. The particular scale used in the experiment is given in Appendix 1.

The matrix is completed as described before resulting in:

$$\begin{array}{cc} C1: & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 1 & 2 & 4 & 1/3 \\ 1/2 & 1 & 5 & 1/3 \\ 1/4 & 1/5 & 1 & 1/5 \\ 3 & 3 & 5 & 1 \end{array}\right]. \end{array}$$

The remaining matrices are:

$$\begin{array}{cc} \text{C2:} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1/7 & 3 \\ 1 & 1 & 1/5 & 1 \\ 1/4 & 5 & 1 & 1/6 \\ 1/3 & 1 & 6 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \text{C3:} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 4 & 1/4 & 5 \\ 1/4 & 1 & 1/5 & 1/5 \\ 4 & 5 & 1 & 6 \\ 1/5 & 5 & 1/6 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \text{C4:} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1/7 & 4 \\ 1 & 1 & 1/4 & 1 \\ 7 & 4 & 1 & 1/8 \\ 1/4 & 1 & 8 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \text{Criteria:} & \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1/8 & 4 \\ 1 & 1 & 1/6 & 4 \\ 8 & 6 & 1 & 1/7 \\ 1/4 & 1/4 & 7 & 1 \end{bmatrix}. \end{array}$$

Computing the eigenvector of the maximum eigenvalue for each of these matrices yields:

C1: [.2468, .1877, .0623, .5032]

C2: [.2038, .1043, .3326, .3593]

C3: [.2632, .0558, .5633, .1177]

C4: [.2336, .0980, .2996, .3689]

Criteria: [.1824, .1839, .3298, .3039]

The final vector of weights for the preference of the alternatives with respect to the criteria and the overall objective is given by:

$$W(a_i) = \sum_{j=1}^{4} c_{ij}w_j.$$

For alternative one the final weight is:

$$A_1 = .2468(.1824)+.2038(.1839)+.2632(.3298)+.2336(.3039)$$
$$= .2403.$$

Likewise:

$A_2 = .10$
$A_3 = .35$
$A_4 = .31.$

The calculations to obtain the eigenvectors and final weights in the decision experiment were performed by the computer program CHOICE described in Mar et al.(1983).

Fuzzy Decision Making.

Many techniques for decision making using fuzzy sets are an outgrowth of Bellman and Zadeh (1970). The authors define decision making in a fuzzy environment as a process in which the goals and/or constraints are fuzzy. That is, the exact formulation of goals and constraints are uncertain or flexible.

Bellman and Zadeh define a decision as a fuzzy set resulting from the intersection of the goals and constraints. The source of this definition arises from the definition of a decision as a solution that satisfies both the set of goals and the set of constraints. It is claimed by the authors that the goal and constraint sets may be mapped to the same space. Thus a decision, in the sense of defining a feasible space, is that which satisfies both the goals (or performance criteria) <u>and</u> the constraints:

$$D = G_1, \text{ and } G_2, \text{ and } \ldots G_n, \text{ and } C_1, \text{ and } C_2, \text{ and } \ldots C_m$$

from which is derived the definition of the decision space as the (fuzzy) intersection of goals and constraints (where D equals the optimal decision, G_i = set of goals, and C_i = set of constraints).

The goal and constraint sets may be mapped to the same decision space when the goals are fuzzy in much the same manner as goal programming formulations. To illustrate, consider a water quality management problem adapted from Loucks et al. (1981). A stream with three monitoring sites receives wastes at sites 1 and 2. Let Q_i represent the post-treatment water quality at site i, and q_i the pre-treatment water quality. W_i represents the unit waste load at site i, and a_{ij} the improvement in quality at site i per unit waste removed at site j. Finally, x_i represents the fraction of waste removed at site i at a cost of c_i per unit x_i.

Assuming there is a mandatory water quality level at site 3, and the objective is to meet the quality level at minimum cost, then the problem is to find the level of treatment at sites one and two that meet the requirement at least cost. The problem may be formulated as a linear program as

$$\text{MINIMIZE } Z = c_1x_1 + c_2x_2$$

Subject To

$$q_2 + a_{21}w_1x_1 \geq Q_2$$

$$q_3 + a_{31}w_1x_1 + a_{32}w_2x_2 \geq Q_3$$

The bounds of waste removal operations may be added as:

$$x_1,\ x_2 \geq 0.3$$

$$x_1,\ x_2 \leq 0.95$$

Now assume an additional objective is added to the problem. If the water quality targets are replaced by a budget (b) then the objective may become one of finding the level of treatment at sites one and two that maximizes the water quality at sites two and three, within the budget limits.

There now exists a problem with two objectives (G_1, and G_2) and a set of constraints (C_1, C_2, ..., C_n). It may be that these goals are somewhat flexible or fuzzy. The problem may be expressed as a goal programming problem by defining levels of acceptable deviation from the stated goals (targets). In general, the goal programming model is given as

$$\text{MIN} \sum_{i=1}^{p} | F_i(x) - T_i |$$

ST

$x \in X$

where

T_i = a target or goal identified for the ith objective

$F_i(x)$ = feasible region for i defining range of x

p = the number of objectives.

The equivalent linear program may be expressed as:

$$\text{MIN } Z = \sum_{i=1}^{p} (dp_i + dn_i)$$

ST

$F_i(x) - dp_i + dn_i = T_i$

$dp_i, dn_i \geq 0$

where

dp_i = positive deviation from the goal, ith objective

dn_i = negative deviation from the goal, ith objective.

Referring to the example problem, the goal programming formulation for the simultaneous solution of the two objectives may be obtained

$$\text{MIN } dp_{11} + dn_{11} + dp_{12} + dn_{12} + dp_2 + dn_2$$

ST

$Q_2 - dp_{11} + dn_{11} = T_{11}$

$Q_3 - dp_{12} + dn_{12} = T_{12}$

$c_1x_1 + c_2x_2 - dp_2 + dn_2 = T_2$

(plus remaining original constraints)

The important feature to note is that the original objectives are now formulated as constraints, with the objectives being flexible (or uncertain). It is in much the same manner that Bellman and Zadeh state that fuzzy goals may be mapped to the same space as constraints in a decision problem. To complete the illustration, the fuzzy programming formulation of Zimmerman (1975) may be considered.

Suppose a given target (T_i) of an objective is desired to allow some latitude, expressed as $T_i^* \approx T_i$ where T_i^* is the fuzzy target and $\approx$ is interpreted as "approximately equal to". In a non-fuzzy formulation the lack of latitude may be expressed as

$$\lambda(T_i^* = T_i) = \begin{cases} 1 \text{ if } T_i^* = T_i \\ 0 \text{ if } T_i^* \neq T_i \end{cases}$$

where $\lambda(T_i^* = T_i)$ is interpreted as a measure of the degree to which the statement is satisfied.

In a fuzzy model $\lambda(T_i^* \approx T_i)$ is allowed to assume a continuous range of values returning unity if $\lambda(T_i^* = T_i)$ and returning less than unity if $\lambda(T_i^* \neq T_i)$. Assuming a linear relationship between the target and deviations from the target, $\lambda(T_i^* \approx T_i)$ might be defined as:

$$\lambda(T_i^* \approx T_i) = \begin{cases} 1 & \text{if } T_i^* = T_i \\ 1 - \dfrac{|T_i^* - T_i|}{d} & \text{if } T_i - d \le T_i^* \le T_i + d \\ 0 & \text{if } T_i + d < T_i^* < T_i - d \end{cases}$$

(where d represents some maximally permissible deviation). Refering back to the example problem, a goal of obtaining maximum water quality at site two may be written as:

$$\lambda(Q_2) = 1 - \frac{a_{21}w_1x_1 - (T_{11} + q_2)}{d_{11}}.$$

For example, if the target quality level at site 2 (T_{Q2}) is 6 mg/l for dissolved oxygen and the maximum allowable deviation from the target is 3 mg/l, then for a quality level of 4, $\lambda(Q_2) = 0.33$. For a quality level of 2, $\lambda(Q_2) = 0$, and for $T^*_{Q2} = 6$, $\lambda(Q_2) = 1$.

Since the maximal value of $\lambda(T^*_i \approx T_i)$ is obtained when the target is met, then maximizing λ across all goals becomes the objective function of the fuzzy linear program and $\lambda(T^*_i \approx T_i)$ enters the constraint set.

The fuzzy linear programming model for the multiobjective problem may be written as:

$$\text{MAX} \sum_{i=1}^{p} \lambda_i$$

ST

$\lambda_i(G_i)$
(plus other constraints)

where $\lambda_i(G_i)$ is the fuzzy equivalent of the multiple goals (objectives) of the problem. Here, as in the goal programming formulation, the fuzziness in the goal set permits the transformation of the goals into equivalent constraints.

Referring back to the fuzzy characterization of the goals and constraints, the fuzzy decision set will be valued on the degree to which an alternative simultaneously satisfies all the goals and constraints. A support of unity would indicate that an alternative fully satisfies all goals and constraints, and a support of zero indicates that at least one goal or constraint was not satisfied. Given that the decision problems of concern here include conflicting objectives, the support for any decision is likely to be somewhat less than unity.

The selection of the optimal decision D^* is derived from the conclusion that the ideal solution would be that which fully satisfies the goals and constraints (i.e. $\mu_{D^*}(x) = 1$). Therefore, from the set of decisions, the optimal decision is that which is closest to having a support of unity:

$$\mu_{D^*}(x) = MAX(\mu_D(x)).$$

Thus the optimal decision may be obtained as

$$\mu_D*(x) = MAX\ [MIN(\ \mu_G(x),\ \mu_C(x))].$$

A significant assumption of this approach is that while the goals and constraints of the problem may be fuzzy, the problem (or the system being modeled) is not. That is, it is assumed that the optimal solution is deduced from the given set of goals and constraints. This precludes the possibility that some important constraint may not be considered in the model. Methods based on this fundamental approach defined by Bellman and Zadeh have been proposed by many authors (Yager, 1978, 1981; Znotinas and Hipel, 1979a, b).

Yager's Method.

Yager (1981) describes the decision problem as a mapping from the set of alternatives X into the set of objectives or criteria A:

$$D(x) = f\ [A_1(x),\ A_2(x),\ldots\ A_n(x)] \quad x \in X$$

where $A_i(x)$ indicates some degree of satisfaction of criterion A_i by alternative x. The solution proposed by Bellman and Zadeh and adopted by Yager results in a pareto optimum solution for the finite set of alternatives considered.

Yager extends the maximin criterion for decision making to encompass decision problems with unequal objectives. This method is based on a linear ordering of preference. Yager's approach is to define $Y = \{A_1, A_2, \ldots A_n\}$ as the set of criteria to be satisfied, and $\{X\}$ as the set of alternatives. $A_i(x)$ is defined as above. G is defined as a fuzzy subset of Y in which $G(A_i) = b_i$ indicates the ordinally valued importance of the criterion A_i. In this setting

$$D(x) = M(A_1(x), b_1) \text{ and } M(A_2(x), b_2) \text{ and } \ldots M(A_n(x), b_n)$$

where $M(A(x), b)$ indicates the satisfaction of criterion A modified by importance b.

Bellman and Zadeh (1970) and Yager (1978) define the modification of A by b as A^b. The use of A^b simply provides a non-linear transformation since the importance b will be valued on the interval [0, 1]. Thus, if a criterion is unimportant, $b = 0$ and $A^b = 1$ ensuring that the criterion will not constrain the decision. As a criterion becomes more important, the value of A^b decreases, increasing the possibility that the criterion A will constrain the solution. If the criteria are equally important, $b = 1$ and $A^b = A$ reflecting the unweighted criterion. The effect of this scaling for selected values of A and b is given in Figure 6. The decision model for Yager's approach may now be defined as

$$D = (b_1{}^c \cup A_1) \cap (b_2{}^c \cup A_2) \cap \ldots \cap (b_n{}^c \cup A_n)$$

$$D = \bigcap_{i=1}^{n} (b_i{}^c \cup A_i)$$

The optimal decision $D(x^*) = \text{MAX } D(x) \quad x \in X$.

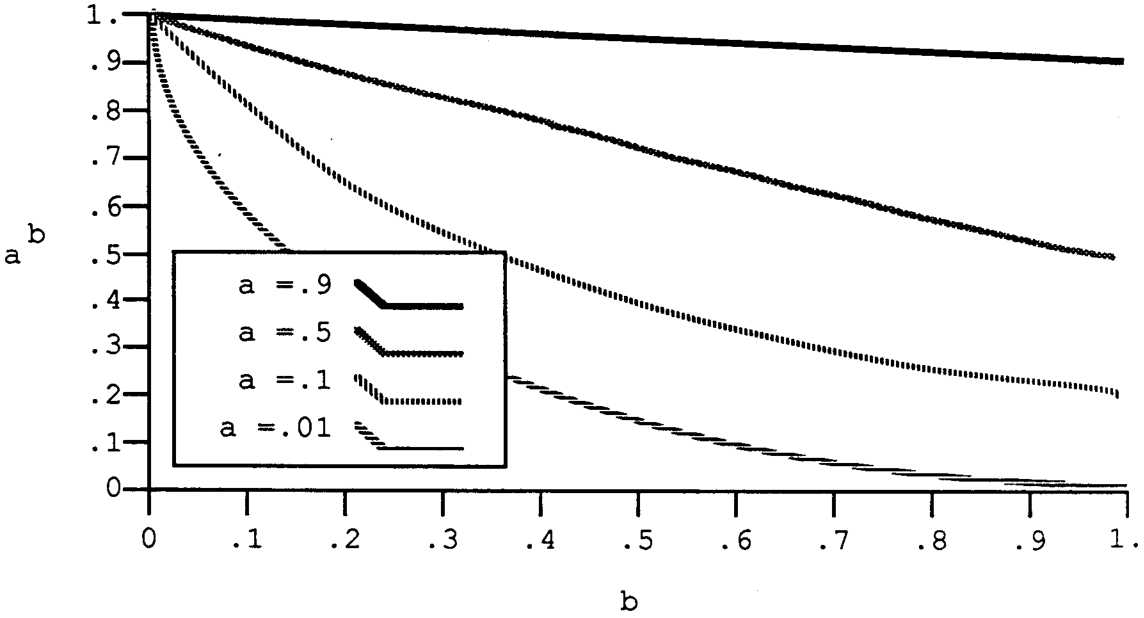

Figure 6. Criterion Weighting Operation for Yager's Method

Yager (1978) suggests that the criterion weights be obtained through the use of Saaty's method. He does not indicate how the remaining weights (i.e. the supports for the alternatives with respect to the criteria) are obtained. In this study, two approaches are used. The first entails pairwise comparisons between alternatives and using the eigenvectors from each level of the hierarchy using Saaty's scaling method. The second approach uses direct assignment of weights for each alternative with respect to each criterion. These approaches are discussed in Chapter Four.

To illustrate the use of these weights in Yager's method, consider the example used in the section describing Saaty's method. From an evaluation of a problem with four criteria and four alternatives, the following sets of supports were obtained.

With respect to criterion 1, the supports for the certainty with which each alternative satisfies the criterion are:

C_1: [.25|A_1, .19|A_2, .06|A_3, .50|A_4].

The remaining supports are:

C_2: [.20|A_1, .10|A_2, .33|A_3, .36|A_4]

C_3: [.26|A_1, .05|A_2, .56|A_3, .12|A_4]

C_4: [.23|A_1, .10|A_2, .30|A_3, .37|A_4].

And the weights for the criteria are:

b_i: [.18|C_1, .18|C_2, .33|C_3, .30|C_4].

The modified supports for the alternatives given the weights for the criteria are:

$C^*_i = A^{b}i,$

$C^*_1 = [.25^{.18}, .19^{.18}, .06^{.18}, .50^{.18}]$

$= [.78, .74, .60, .88]$

$C^*_2 = [.75, .66, .82, .83]$

$C^*_3 = [.64, .37, .83, .50]$

$C^*_2 = [.64, .50, .70, .74].$

The effect of the weighting for the first criterion is to increase the weights for the set of alternatives because with a support of 0.18 the criterion is deemed not very important. Thus any of the weights for alternatives for this criterion are less likely to constrain the solution. If this criterion had been weighted highly (e.g. .9) then the alternatives would have more closely retained their original weights and alternatives with low supports (e.g. A_3) might have constrained the solution.

The solution is:

$\mathrm{Min}(A_j) = [.64, .37, .60, .50]$

$\mathrm{Max}(\mathrm{Min}(A_j)) = .64.$

The first alternative would be the alternative of choice, although all alternatives are closely weighted. For an alternative to be strongly supported it must be weighted highly for all important criteria. To be rejected, it must be weighted low for at least one important criterion. A computer program for calculating the solution with Yager's method, assuming the existence of supports, is given in Appendix 2.

Another characteristic of the maximin criterion for fuzzy decision making requires consideration. The performance of an alternative with respect to a goal, and the goal itself, may be defined as fuzzy sets representing uncertainty in evaluation. It would be expected that this uncertainty would directly affect the choice of alternatives. It is shown here, however, that the maximin criterion does not use fully the uncertainty described in the model.

Assume there exists a fuzzy set defining some measure of, for example, performance (Figure 7a). Assume also an alternative with performance level x. It is seen that the support for the alternative being in the fuzzy set "performance" is unity. Now assume that the set "performance" is redefined to reflect greater uncertainty in possible values (Figure 7b). In this example the support for the alternative (MAX(MIN(C,A))) remains the same. The support does not reflect the greater uncertainty induced in the solution from a change in the structure of the model. In fact, it is possible for the solution to the problem to be invariant even though the model itself varies widely.

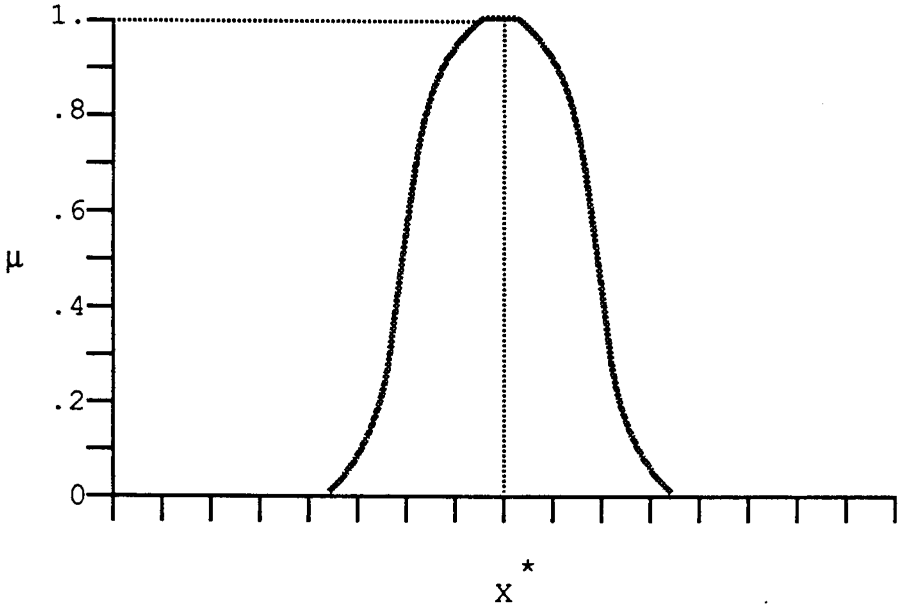

Figure 7a. Support for an Alternative in the Fuzzy Set "Performance"

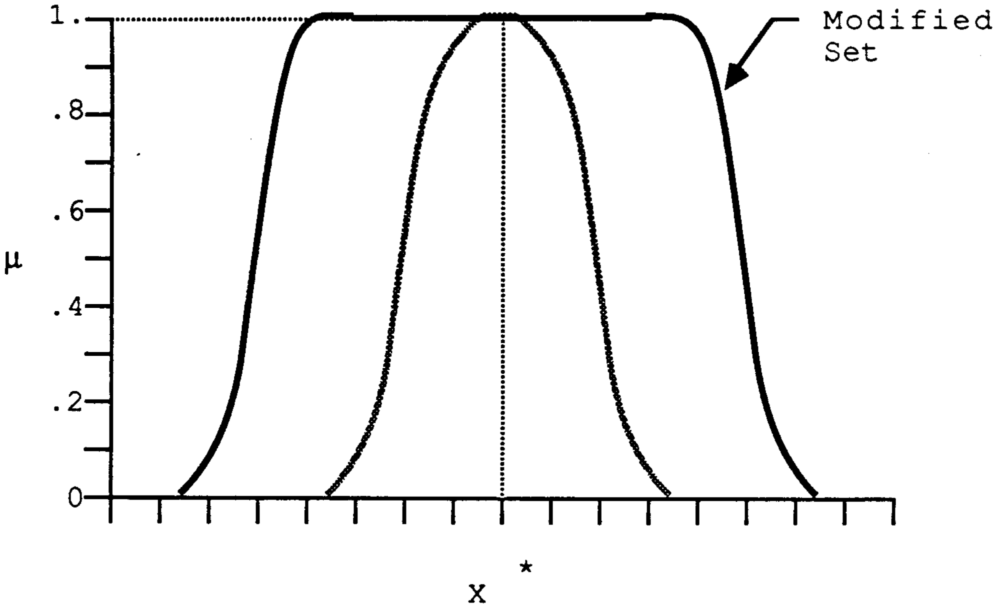

Figure 7b. Support for an Alternative in a Modified Fuzzy Set "Performance"

This suggests several possible sources of difficulty in using the fuzzy formulation of the maximin criterion for decision making. The operations for considering unequal criteria do not appear to be strongly supported in theory and the method results in a certain solution regardless of the uncertainty inherent in the model.

Fuzzy Logic Based Method.

As an alternative to the Bellman-Zadeh model for decision making, a method based on fuzzy logic operations is now described. To support the description, it is necessary to provide further discussion on logic operations.

Within classical single valued logic the propositions of modus ponens and modus tollens are of interest. In the context of decision making modus ponens describes the deduction of a conclusion (the selection of an alternative) from the premises defining the preference for a criterion and a relation between an alternative and a particular criterion. Modus tollens is important in the context of the resolution of inconsistent solutions. This is discussed in more detail in a later section.

As described in Chapter One, implication operations are based on the concept of set inclusion, that is, a set may be a subset of another. If A is a subset of B, then A implies B ($A \rightarrow B$). That is, if A implies B and A exists then it must be concluded that B also exists. Attaching the assumed truth values to these statements the following is obtained

If $p(A \rightarrow B) = 1$, and $p(A) = 1$, then $p(B) = 1$.

The correllary A implies B and B exists, therefore A exists, is not permitted in classical logic. Modus tollens, however permits a conclusion to be drawn about not A given not B and A implies B, that is, if A implies B but B does not exist, then A does not exist either or, using truth values, if $p(A \rightarrow B) = 1$ and $p(B^c) = 1$, then $p(A^c) = 1$.

The truth values of these propositions are restricted to binary (0,1) values. In multivalued or fuzzy logic, continuous truth values on the interval [0, 1] are permitted. To ensure that the fuzzy implication operator supports modus ponens and modus tollens, its properties are examined.

In classical single valued logic, the truth value of the propositions must either be true or false. If the truth values are uncertain, that is, if A may or may not imply B, or A may or may not be known to exist, then the application of fuzzy logic is warranted. Thus there may exist a mapping of truth values for the propositions in the interval between 0 and 1.

The operator for implication used is that by Lukasiewicz (see Chapter One):

$a \rightarrow b = \min(1, 1-a+b)$, or, using fuzzy set notation,

$\mu(a \rightarrow b) = \min(1, 1 - \mu(a) + \mu(b))$.

The characteristics of this operator are such that if $\mu(a) < \mu(b)$ then the support for $\mu(a \rightarrow b)$ is unity. Also, $\mu(a \rightarrow b)$ will equal zero only when $\mu(a)$ equals unity and $\mu(b)$ equals zero, that is, when the support for the proposition A and the disbelief of proposition B are certain. This case implies that A cannot be a subset of B and, as shown before, echos the results of classical logic and implication.

It is of interest to examine the properties of the fuzzy implication operator with respect to its support of modus ponens and modus tollens. Table 7 displays values of the proposition b that may be inferred from the propositions a → b and a. Recall that in classical implication, if a → b is true ($\mu(a \rightarrow b) = 1$) and the proposition a is also true then b is also true ($\mu(b) = 1$). If a → b or a are not true then b is not true. Under fuzzy sets either the proposition a → b or the proposition a may be only partially true. Table 7 (Bandler and Kohout, 1985) illustrates possible values for the proposition b under these conditions.

Table 7. Modus Ponens Values of B For A and A → B ∈ [0,1]

a	0.	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.
a→b											
	b										
0.											0
.1										0	.1
.2									0	.1	.2
.3								0	.1	.2	.3
.4							0	.1	.2	.3	.4
.5						0	.1	.2	.3	.4	.5
.6					0	.1	.2	.3	.4	.5	.6
.7				0	.1	.2	.3	.4	.5	.6	.7
.8			0	.1	.2	.3	.4	.5	.6	.7	.8
.9		0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1.	[0,1]	[.1,1]	[.2,1]	[.3,1]	[.4,1]	[.5,1]	[.6,1]	[.7,1]	[.8,1]	[.9,1]	1

In the extreme cases (indicated in bold type) it is seen that the fuzzy implication operator supports modus ponens in the same manner as classical implication. The table for modus tollens is the same except that table values are for the negation of a (1-a), and the column values are for values of the negation of b (1-b). Note however, that truth values of 1 for a→b and b^c, fuzzy implication allows the possibility that a^c can be inferred; something not possible in classical implication. This result is used to improve inconsistent solutions arising from Tsukamoto and Terano's (1975) method described in a later section.

The selection of an alternative may be perceived as one of testing a series of rules. For example, a rule may state:

> If alternative A is preferred with respect to criterion 1
> and alternative A is preferred with respect to criterion 2
> then select alternative A as the alternative of choice.

The rule may be interpreted as an implication proposition $[C_1(A) \text{ and } C_2(A) \rightarrow C^*(A)]$. If the rule is true and the premises are true then it may be deduced that the conclusion is also true. In the context of fuzzy decision making, however, there may exist uncertainty in the truth values of both the premises and the rules. Thus the decision problem is one of determining the greatest certainty that may be deduced about a conclusion (alternative i is the alternative of choice) and examining the conclusion with the highest degrees of certainty.

In general, a rule of the type above may be expressed as (IF A THEN C), that is, if an alternative is preferred, then some criterion must have been satisfied. This rule may be expressed as a relation on the sets of alternatives and criteria. Mamdami et al., (1975) propose an expression of the form:

$$\text{If A then C} = A*C = R$$

where $A*C$ is the fuzzy cartesian product

$$A*C = \min(a,\ c)\ \forall\, a \in A, c \in C.$$

For example, let

$$A = [.1,\ .3,\ .7,\ 1]$$

$$C = [\ 0,\ .5,\ .1,\ .8].$$

Then

$$A*C = \min(a,c) = R = \begin{bmatrix} 0 & .1 & .1 & .1 \\ 0 & .3 & .1 & .3 \\ 0 & .5 & .1 & .7 \\ 0 & .5 & .1 & .8 \end{bmatrix}.$$

Before proceeding, further interpretation of the relation R is warranted. In the context of decision making, R may be interpreted as an aggregate measure of a decision maker's certainty with which an alternative has satisfied a particular criterion. That is, R_{ij} is a measure of the degree to which alternative i satisfies criterion j. In most problem settings, the relation R would be estimated directly through elicitation of values from the decision maker.

In the context of the decision experiments described in Chapters 4 and 5 the relation R is defined from either direct assessment of each alternative with respect to each criterion or through pairwise comparisons using elements of Saaty's scaling method. Using the example described in the previous sections on Saaty's and Yager's methods the relation R is defined as:

$$R = \begin{bmatrix} .25 & .20 & .26 & .23 \\ .19 & .10 & .05 & .10 \\ .06 & .33 & .56 & .30 \\ .50 & .36 & .12 & .37 \end{bmatrix}.$$

Since the values for A are sought then in addition to the relation R, the set C is also required. The set C may be interpreted as a measure of model completeness, that is, C represents the certainty or importance of the criteria relative to the objective of selecting the best alternative.

Given R and C the problem is to find the possible values of A such that:

$$C = R \circ A$$

where C is the set of supports for the certainty of the criteria, A is the set of supports for the certainty of satisfaction of all the criteria by the alternatives, and

R ∘ A is the max-min relational composition between R and A:

$$C_j = \bigcup_{i=1}^{m} (r_{ij} \cap a_j) \quad j = 1, 2, \ldots, n.$$

The basis of the problem is the solution of the inverse of the above equation, that is, to solve for the a_j. Solutions are discussed by many authors (Sanchez (1976); Pappis and Sugueno (1985); Tsukamoto and Terano (1975)). There are few differences between the methods. Tsukamoto and Terano begin to address situations in which existence conditions are not met. For this reason the method proposed by Tsukamoto and Terano (1975) is used here.

Tsukamoto and Terano's Method.

In solving for this inverse problem, Tsukamoto and Terano define two propositions regarding the relationship between A and C. The first:

P_j: C_j implies ($\exists i$ (R_{ij} and A_i)), $1 \leq j \leq n$,

may be interpreted as stating that if the support for a criterion C_j exists, then from the relation R and the set of alternatives A, support for at least one alternative related to c_j exists. This proposition in effect states that there may be no irrelevant alternatives. The second proposition:

P_{ij}: A_i implies C_j, $1 \leq i \leq m$, $1 \leq j \leq n$,

simply formalizes the rule "IF A THEN C" discussed in the beginning of this section. This proposition does possess a characteristic of further interest. The proposition in effect states that if an alternative is selected, then it is implied that some criteria have been considered. This proposition, (along with the first proposition) does not preclude the possibility that an important criterion has not been considered in the analysis. The identification of such an occurence is discussed in a later section.

The first proposition is readily translated into a fuzzy set operation. First, the proposition is assumed true which then requires

$$c_j \leq (\exists i \ (r_{ij} \text{ and } a_i))$$

$$c_j \leq \bigcup_{i=1}^{m} (r_{ij} \cap a_i)$$

$$c_j \leq \max_i \ (\min \ (r_{ij}, \ a_i)) \ \ 1 \leq j \leq n.$$

Since there is no restriction on the second proposition, a_i is constrained by the degree to which c_j may be inferred from R_{ij}, that is:

$$a_i \leq (t_{ij} \rightarrow c_j)$$

$$a_i \leq \min_j \ (1, \ 1 - t_{ij} + c_j)$$

where the t_{ij} are the fuzzy valuations of the rule (A implies C).

Given c_j, R_{ij}, and T_{ij}, the a_i that satisfy both equations are sought. The equation associated with proposition P_{ij}

provides an upper bound measure on the certainty with which an alternative satisfies the criteria. The equation associated with the proposition P_i yields the lower bounds on the alternatives, or a measure of the degree to which an alternative remains unconfirmed by the information. This equation, as stated before, requires an inverse solution. The current equation is of the form $C = R \circ A$, but it is desired to solve the equation $A = R^{-1} \circ C$. Tsukamoto and Terano offer a simplification of the solution algorithm proposed by Sanchez. To solve the equation

$$c_j \leq \max_i \, (\min(r_{ij}, a_i))$$

for a,

$$a_i \geq \max_j \, (\min(r_{ij}, c_j)),$$

two compositions on the sets R and C are first defined:

$$r_{ij} \, \omega \, c_j = \begin{cases} c_j & \text{if } r_{ij} > c_j \\ [c_j, 1] & \text{if } r_{ij} = c_j \\ \varnothing & \text{if } r_{ij} < c_j \end{cases}$$

$$r_{ij} \, \varpi \, c_j = \begin{cases} [0, c_j] & \text{if } r_{ij} > c_j \\ \varnothing & \text{if } r_{ij} \leq c_j. \end{cases}$$

The first composition defines the matrix $u_{ij} = r_{ij} \, \omega \, c_j$, and represents the possible outcomes from the implied partial equation $(\min(r_{ij}, c_j))$. Since R defines links between criteria and alternatives, its elements are valued either zero or unity. Thus if $r_{ij} < c_j$, r_{ij} must equal zero and

$\min(r_{ij}, c_j) = 0$. If $r_{ij} > c_j$ then r_{ij} must equal unity and $\min(r_{ij}, c_j) = c_j$. If $r_{ij} = c_j = 0$, u_{ij} remains undefined and is valued [0,1]. Finally, if $r_{ij} = c_j = 1$ then both elements are known with certainty and $u_{ij} = 1$.

The source of the second composition defining the matrix v_{ij} is less clear. If $r_{ij} > c_j$ then $v_{ij} = [0,c_j]$ which appears to be a measure of possible disconfirmation. The same interpretation seems to hold if $r_{ij} < c_j$. It is shown here however that this composition is actually not used in the algorithm and need not be considered further. Tsukamoto and Terano define a third matrix

$$w_{ij}^k = \begin{cases} u_{ij} \text{ for } \exists i \in \{ i \mid u_{ij} \neq 0\} \\ \\ v_{ij} \text{ for other } i\text{'s} \end{cases}$$

There are normally many possible matrices w_{ij} obtained from this definition and k represents an index of these possible solutions. In defining the w_{ij}, only if $u_{ij} = 0$ does $w_{ij} = v_{ij}$. But from the definitions of the matrices U and V, $u_{ij} = 0$ only when $r_{ij} < c_j$, and under the same conditions, v_{ij} would also equal zero. The interval $[0, c_j]$ is never used in the algorithm.

The solutions for a_i^k are given by Tsukamoto and Terano as

$$\max_j(\inf(w_{ij}(k'))) \leq a_i^k \leq \min_j(\sup(\min(1, c_j+1-t_{ij}))),$$

but given the results above, may be rewritten as

$$\max_j(\inf(u_{ij}(k'))) \leq a_i^k \leq \min_j(\sup(\min(1,\ c_j+1-t_{ij}))),$$

where inf is defined as the greatest lower bound and sup is defined as the least upper bound.

Since it is possible to obtain a lower bound solution greater than the upper bound for a given kth solution, the solution is conditioned by

$$k' \in \{\ k \mid \max_j(\inf(u_{ij}(k'))) \leq \min_j(\sup(\min(1,\ c_j+1-t_{ij}))).$$

The final solution is $a_i^{k*} = \max\{\ a_i^k\ \}$, $k \in K$. For the decision experiment Tsukamoto and Terano's method is used as follows.

In addition to the relation R defined earlier, a vector of weights denoting the importance of each evaluation criterion is obtained from the decision maker. For the example the vector is:

$$C_j = [.18,\ .18,\ .33,\ .30]$$

indicating that the third criterion is most important. From the relation R (denoted as T in Tsukamoto and Terano's method to represent fuzzy supports) the upper bound of the solution

$$a_i \leq \min_j(\sup(\min(1,\ c_j+1-t_{ij})))$$

is found:

$$a_i \leq \min_j \begin{bmatrix} .94 & .68 & .85 & 1 \\ .99 & .98 & .97 & .74 \\ 1 & 1 & 1 & 1 \\ .68 & .85 & 1 & 1 \end{bmatrix} = \begin{bmatrix} .68 \\ .74 \\ 1 \\ .68 \end{bmatrix}.$$

The lower bound of the solution is found from the matrix:

$$w_{ij} = \begin{bmatrix} .18 & .18 & .33 & .30 \\ .18 & .18 & .33 & .30 \\ .18 & .18 & .33 & .30 \\ .18 & .18 & .33 & .30 \end{bmatrix}.$$

Four feasible solutions exist, and the lower bound on all alternatives is 0.33. The final solution is therefore:

$$\begin{bmatrix} .33 \le a_1 \le .68 \\ .33 \le a_2 \le .74 \\ .33 \le a_3 \le 1 \\ .33 \le a_4 \le .68 \end{bmatrix}.$$

Identification of Deficient Models.

It is possible that no kth solution is feasible, that is, all possible lower bounds are greater than an upper bound of the solution. Such a situation may occur from inconsistency in the supports provided by the decision maker. Referring to the table of values (Table 7) for fuzzy implication under modus ponens, it is seen that there exists a region in which conclusions may not be drawn from a given range of supports for a premise (criterion) and a rule. Another source of an inconsistent solution may arise from an incorrect or incomplete relation R arising, for example, from a missing criterion. The identification of model sources (criteria) sensitive to this result is desired. Failing to identify such criteria leads to the conclusion that the model itself is deficient, that some important criterion may be missing from the analysis.

To resolve inconsistent solutions, the concept of modus tollens is used. In this setting, being unable to deduce the supports for the satisfaction of the criteria by the alternatives, it is desired to deduce the supports for the degree to which the alternatives do not support the criteria. If an alternative does not support a criterion, then conclusions involving that criterion are unlikely to be a source of the inconsistent solution. As yet, there is no mechanism for identifying which specific criteria may be eliminated from further consideration and which may be contributing to the inconsistent solution. A proposed mechanism is developed here. To identify these criteria, an inverse form of modus tollens is used.

In the original portion of the algorithm, an inverse form of modus ponens is used to derive the possible supports for the alternatives A. In the negation algorithm described by Tsukamoto and Terano, modus tollens (A implies C and C is false therefore A is false) is invoked to identify the alternatives that could not be deduced if the negation of the supports for the criteria were used. Now it is desired to invert modus tollens (A implies C and A is false therefore C is false) to identify specific criteria that may not be eliminated as a source of inconsistency. The negation algorithm is extended to include this operation.

To begin, the propositions P_j and P_{ij} are rewritten to reflect the negation operations of modus tollens and the "inverse" of modus tollens:

P_j': C_j implies ($\exists i$ (R_{ij} and A_i)), $1 \leq j \leq n$

This proposition is interpreted as stating that if a criterion c_j is not eliminated as unimportant then there must exist at least one alternative that may not be eliminated from consideration as the desired alternative. Operationally, this proposition again assumes the form

$c_j \leq \max_i (\min (r_{ij}, a_i))$

which comes directly from the fuzzy operation $C = R \circ A$.

The second proposition is rewritten as

P_{ij}': A_i implies C_j $1 \leq i \leq m$, $1 \leq j \leq n$.

This proposition simply states that if an alternative does not satisfy all criteria then at least one criterion is not satisfied. If it cannot be deduced that an alternative does not satisfy all criteria then some criteria may be satisfied. The operational form of this proposition is

$a_i \leq \min_j(1, 1 - t_{ij} + c_j)$.

The first proposition defines the upper bound on the solution, representing the degree of disconfirmation of the criterion that may be induced from the disconfirmation of the alternative. The second equation, solved for c_j, provides the lower bound.

It remains possible for the negation algorithm to result in an infeasible solution, that is, the lower bounds of c_j are greater than the upper bounds. This condition may be interpreted as indicating that no improvement in the certainty of the alternatives is possible given the existing criteria. It would then be implied that additional criteria might be required to modify the decision problem.

When reviewing the characteristics of the more traditional fuzzy decision model, it was shown that the maximin model may not represent well changes in the uncertainty of the model. A desirable characteristic of an alternative decision model is an explicit representation of uncertainty in the solution set. There may, in fact, be two types of representation. One type arises from uncertainty in elements of a model. If an alternative satisfies a criterion, but it is uncertain that the criterion fully describes the problem, then the support for the alternative satisfying the problem should be less than that arising from the case in which the criterion is known to describe the problem. On the other hand, as uncertainty in the structure of the model itself (in the rule "A implies C") increases, then uncertainty in the solution should be reflected in a broader range of possible solutions.

That is, if the model approaches certainty, then the lower and upper bounds of the derived supports for the alternatives should converge. The following examples will demonstrate that the fuzzy logic method possesses these characteristics.

The first characteristic is examined as follows. Consider a model with two alternatives and two criteria. If the rules are believed to be true ($t_{ij} = 1$), then the certainty of the alternatives are directly related to the certainty of the criteria. Thus if $c_1 = c_2 = 1$, then $a_1 = a_2 = 1$. If $c_1 = c_2 = .5$, then $a_1 = a_2 = .5$. As uncertainty in the model increases, supports for the alternatives are altered. If the supports for the rules are decreased to .5 and the supports for the criteria range from unity to zero then the supports for the alternatives range from [1] to [0, .5]. If the supports for the rules are reduced to zero (maximum uncertainty in the perceived relation) then the values for the alternatives may range on the full interval [0,1] indicating complete uncertainty in the possibility that an alternative satisfies the criteria.

To illustrate the second characteristic, assume a single alternative, and that the supports for the rules are all equal to unity. As the supports for the criteria range from zero to unity, the support for the alternative ranges from zero to unity, but is never valued on an interval because the model itself is assumed known.

Conclusions.

This chapter presented the methodological framework for several decision making methods based on fuzzy sets or related concepts. It was shown that fuzzy set techniques based on the model of Bellman and Zadeh (1970) possessed several undesirable characteristics. A method based on fuzzy logic was presented and proposed as a possible alternative to other fuzzy set based methods. The discussion in Chapter One also resulted in the conclusion that for some types of problems, fuzzy logic based methods may also be more appropriate than probability based methods.

These discussions, however, were unable to prove axiomatically that fuzzy logic based methods are superior to other methods. Fuzzy logic methods may be argued to present yet another artificial means to represent human decisions. To demonstrate whether fuzzy logic methods possess characteristics that better represent human decision behavior, that is, whether it provides a better descriptive model prior to becoming a prescriptive model, requires the weight of empirical evidence. Toward this end, the next chapter discusses an experimental methodology.

CHAPTER THREE

DECISION EXPERIMENT

Fuzzy sets were first proposed as a means of providing a more adequate representation of human decision processes than that provided by other methods (Zimmerman et al., 1984). More specifically, Bellman and Zadeh (1970) have argued that most decision making occurs under conditions in which goals, constraints, and outcomes may not be known precisely. The quantitative tools traditionally used imply that imprecision is an outcome of randomness. It has been suggested (Zimmerman, et al, 1984) that decision analysis techniques employing methods analyzing random effects, specifically the calculus of probability and the principle of maximization of utility, require assumptions about the behavior of human decision makers that are violated in most settings.

The examination of the theoretical characteristics of the decision methods in Chapter One, focusing primarily on probability and fuzzy sets, have shown that neither method may be excluded from application on the grounds of theoretical shortcomings. Probabilistic decision analysis, when there exists no prior information on the probability of an outcome becomes a maximin analysis rather than the averaging analysis of expectation. Fuzzy set analyses are based from the outset on the maximin criterion. A question that remains, however, is if the maximin prescription is more representative of actual decision behaviors, that is, do fuzzy set based methods such as those described in Chapter Two, adequately predict this behavior? This question forms the impetus for this chapter.

There is an abundant supply of papers making the suggestion that fuzzy set based methods are not only viable but more desirable in representing decision problems in which uncertainty and subjective judgment play significant roles. The majority of examples used in these papers are artificially constructed and simple. There is a paucity of examples that do not assume the presence of fuzzy sets or preferences, or use human subjects to obtain these preferences. The work reported in this chapter represents an effort to evaluate the claimed merits of fuzzy set techniques under less artificial conditions. The objective of this chapter is to design an experiment to begin to test the hypothesis that fuzzy set based methods provide an effective mechanism for representing uncertainty and subjective judgment in some types of human decision processes.

An operational definition of effectiveness is required. In experiments evaluating the performance of models and solution methods, various assumptions must be made. Many comparative experiments assume the model completely represents the problem to be solved. In such cases evaluation of different methods may proceed through variation in solution quality or through variation in the operative qualities of the methods, for example, differences in solution time or data requirements. In problems of the nature considered here, relationships between the attributes of the decision alternatives and problem solution with respect to a set of criteria may not be analytically defined and, in fact, are dependent upon the particular decision maker considering the problem. In this case traditional experimental designs measuring the efficacy of a method's solution with respect to an a priori definition of a problem solution are not possible.

When the problem setting is incompletely defined, other approaches to evaluation have been used. One approach (e.g. Wallenius, 1975) is to ask subjects using the method to make judgments regarding their confidence in the efficacy of solutions generated by various methods. Another approach is to use a panel of judges to evaluate the efficacy of solutions generated by different solution methods (e.g. Volkema, 1983). A possible problem with the first approach is that learning effects may affect solution discrimination and in the second, that disagreement among the judges may occur.

The approach to evaluation used here is a blend of these experimental approaches. Because of the nature of the decision problems of interest here, a priori problem solutions are not possible. However, as the stated objective of fuzzy set based methods is to represent decision behavior, the a priori (or baseline) solution is taken here to be a ranking of alternatives obtained directly from the test subjects. The efficacy of solutions generated by the various methods are measured by the degree to which they produce similar rankings of alternatives.

The experiment used here has two major components, the experimental decision setting and the experimental design. The design of an appropriate problem setting is given here with a description of the experimental design in the following section.

Problem Setting.

The problem settings used in the experiment must possess attributes which encourage the application of subjective judgment and yet whose solution must be feasible in the sense that the given alternatives represent relevant solutions to the problem in general and to the evaluation criteria in particular. With this in mind, the decision problems should possess the following characteristics.

1. The problem setting should be familiar to the test subjects so that reasonable consideration may be given to the defined tasks. However, the problem should not be so simple as to preclude the application of subjective judgment.
2. The problem setting should be designed so as to require the application of individual values but must be constrained sufficiently to limit the range of possible solutions.
3. The available alternatives should themselves be unambiguous in nature and reasonably familiar to the student test subjects so as to preclude uncertainty arising from decision options.
4. The problem setting should contain multiple criteria which are non-commensurate in nature so as to require test subjects to consider trade-offs between criteria.
5. The alternatives should be selected so that it may be clearly seen that no single alternative may satisfy each and every criterion to a degree greater than other alternatives (i.e. the alternatives should constitute a non-dominated set).

Several problem settings have been developed to meet these desired characteristics.

The first setting is designed to introduce the nature of problems that are of interest in this study and provide an example with which there is likely to be widespread familiarity. In this context, the problem is one of ranking several brands of ice cream on the basis of flavor, texture, and cost. This problem represents the essence of complex problems in that there exist several criteria, uncertainty (in brand name recognition, for example), and the presence of a non-dominated solution, at least on the basis of objective information. The problem also clearly requires the subjects to use their subjective judgment regarding the evaluation of the criteria.

The second problem setting is the design of a simplified water quality monitoring network. In this setting the criteria are concerned with minimizing cost, protection of fish species, legal considerations, and maintenance of recreational uses. Alternatives emphasize long-term trend monitoring, sampling uniformly in time, sampling uniformly in space, and intensive "crisis" sampling. The alternatives represent a range of accepted practice, but are unable to satisfy all criteria simultaneously.

The third setting is a problem of urban flood management as modified from Novoa and Halff (1977). The proposed criteria include flood protection, neighborhood improvement, project and maintenance costs, and relocation/legal considerations. Alternatives that will be considered include no action, the construction of a park in the floodplain, the construction of a concrete flood channel and the purchase of the floodplain followed by redevelopment. These alternatives were selected on the basis of their ability to best satisfy individual criteria, provide a range of feasible solutions, and in the case of two of the alternatives, were considered very closely ranked in the original analysis by Novoa and Halff. A more detailed description of each problem setting is provided in the survey form (Appendix 1).

The essence of the experimental problem explored here is to analyze the ability of fuzzy set based methods to predict, or at least replicate, the decision behavior of an individual Therefore, a major supposition of the study is that it is not yet possible to consider the generalization of a generic decision problem to a generic decision maker. This microanalytic approach implies that it is not necessary or even desirable to control for variations in judgment or opinion between decision makers. It is desirable to begin to determine if, for an individual decision maker, a particular method performs consistently. It is for this reason that more than one decision problem setting was considered. In general, topics covered by the problem settings are within the scope of topics considered by the engineering student subjects by addressing issues of environmental monitoring and engineering design. The problem descriptions are simple enough that lack of particular expertise should not hinder their analyses of the problems. By identifying discrete solution alternatives and evaluation criteria, the model of the decision problem is defined sufficiently. Finally, the alternatives defined in the problem setting are reasonably non-dominated, thus providing adequate complexity to the decision problem to prevent trivial decision problems and requiring the application of judgment regarding the relative merits of the alternatives.

The general decision problem is defined as a rank ordering of a set of alternatives by evaluating the characteristics of the alternatives with respect to a set of evaluation criteria. The use of ranks in this study is required due to the ordinal nature of the input data (obtained from ordinal scales) and statistical properties of the sample requiring non-parametric analysis. This constraint prevents full consideration of available information; a topic covered in Chapter Four. The problem settings were carefully defined to avoid introducing any implied a priori ranking of the alternatives, or of the relative importance of the evaluation criteria. This is the information that is desired from the test subjects. A secondary class of information to be obtained from the subjects is a measure of the confidence with which the alternatives were rank ordered. How this information was obtained and used is given in the next section.

The objectives of the experiment are to:

1. Formalize the comparison of various treatments of subjective decision information through the examination of decision making by human subjects.
2. Test the ability of several treatments of fuzzy information to emulate human decision behavior in decision problems solved through subjective judgment.

The general hypothesis is that rank orderings of alternatives obtained through the application of the fuzzy logic method will be positively correlated with a baseline ranking of the alternatives obtained directly from each individual test subject. Correlations obtained from the fuzzy logic method are expected to be higher than those obtained from Yager's method.

Development of a Baseline Measure.

The basis for the a priori decision set used as a baseline for comparison with the fuzzy information treatments is the direct assignment of ranks to alternatives given in the problem setting (e.g. Part I, Question 1 of the Questionnaire). To avoid biasing these baseline measures, only the minimal possible instruction is given. To supplement the rank orderings with information regarding the confidence in the ranking on the part of the test subject, the second question requests an estimate, on a scale of zero to one, the confidence with which each alternative might be assigned any of the possible ranks. The identification of confidence provides a means of comparing the strength of the treatment derived rankings with the confidence of the decision maker. These two questions constitute the determination of baseline information and are repeated for each of the three problem settings.

Determination of Treatment Measures.

All three methods for ranking alternatives require that the problem under consideration be decomposed into evaluation of each alternative with respect to each evaluation criterion (Figure 8). The three methods represent these evaluations by way of the assignment of weights indicating, in general, the degree of preference for an alternative, given a particular criterion. The methodology, therefore, provides a means of recomposing these partial evaluations into a composite evaluation of the problem. The issue, beyond the composition of these evaluations, is the means by which they are obtained from the decision maker. Saaty (1977) is rather explicit in defining the basis for evaluation as a pairwise comparison between elements in one level of a hierarchy with respect to an element in the next highest level of the hierarchy. Saaty also goes so far as to suggest a scale by which the pairwise

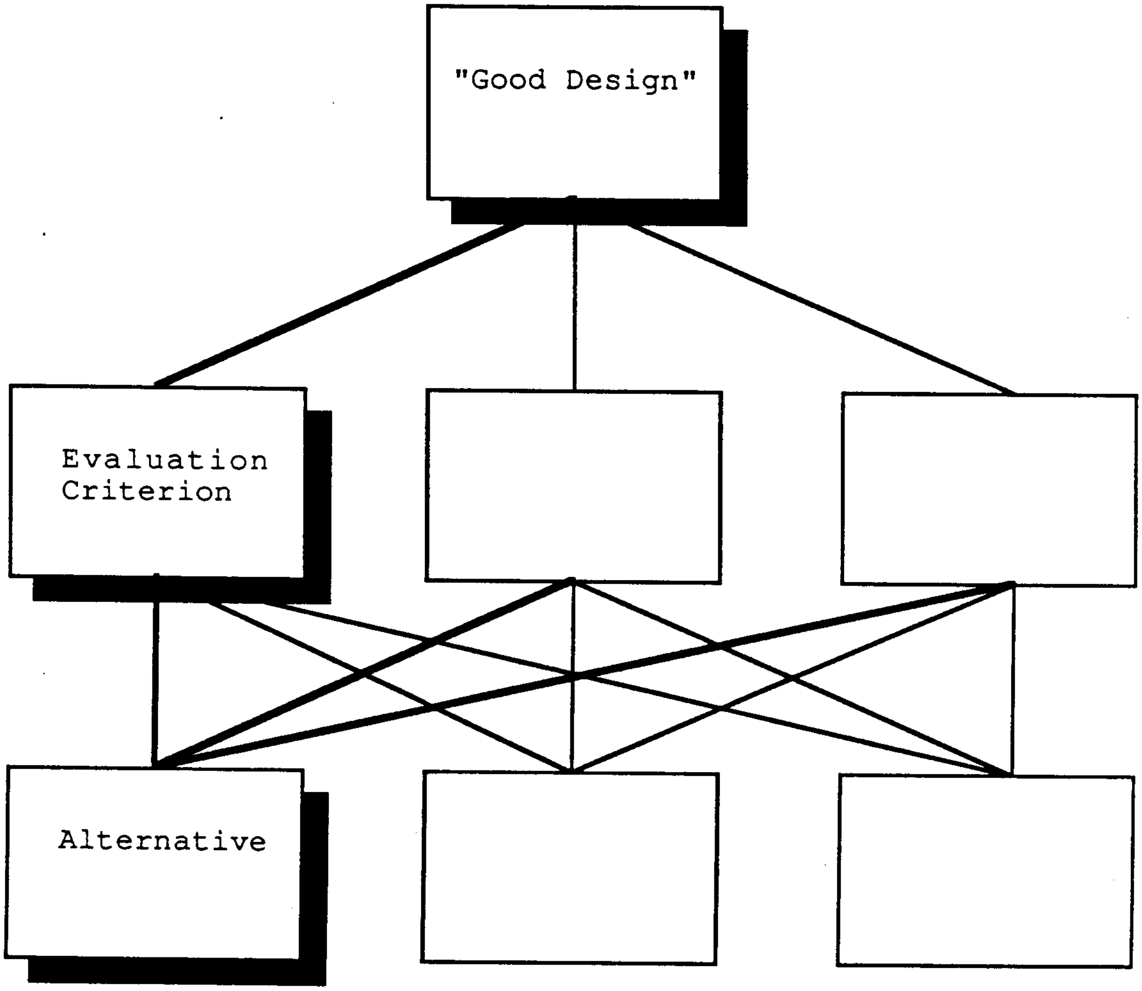

Figure 8. Structure for Evaluation of Alternatives

comparisons might be measured. An intermediate step in the solution of Saaty's method is the determination of a vector of weights relating each alternative (in the context of this study) with each criterion. Yager, in the development of a fuzzy set based method, explicitly uses the vector of weights obtained from pairwise comparisons using the scale suggested by Saaty.

For the first sample group, it was assumed that this approach to obtaining vectors of weights was sufficient. As the fuzzy logic method does not explicitly develop the weights used by the method, the pairwise approach was used for all three methods in this part of the study. There is an implicit assumption that the weights obtained by this approach are valid representations of the subject's judgments. A single weight determination procedure was used for all three methods considered because the first two methods explicitly suggest it be used and because it was desired to control for variation in method results as a function of variability in the input values.

Because the first administration of the questionnaire did not examine the validity of using normalized eigenvectors as the weight vectors for the fuzzy logic approach, a second means of obtaining these weights was developed and incorporated into the analysis. For this approach, no pairwise comparisons are made; the vector of weights for the alternatives with respect to each criterion is obtained directly, that is, for each alternative with respect to each criterion, a fuzzy support for the degree of certainty with which an alternative satisfies each criterion is obtained. This form of weight assessment was introduced to test whether the weights derived from normalized eigenvectors of pairwise comparisons might in general be affecting the results obtained from the fuzzy logic method.

For the baseline ranking the subjects are presented with the problem statement and the descriptions of the evaluation criteria and alternatives. No specific guidance is given as to how this information is to be used in ranking the alternatives. In addition to the ranking of the alternatives, subjects are asked to indicate on a scale of zero to one the certainty of the rank assigned to each alternative.

The remaining two treatments are administered to the subjects in a similar manner. In these cases, the subjects are asked either to provide pairwise judgments (first set of subjects) of the relative certainty that the alternatives satisfy each evaluation criterion, and on the relative satisfaction of the criteria with respect to the solution of the decision problem, or to provide direct judgments (second set of subjects) of the certainty that the alternatives satisfy each evaluation criterion, and that the criteria satisfy the solution of the decision problem. Both the pairwise and the direct comparison treatments are used in each of the three methods under evaluation (Saaty, Yager, and fuzzy logic) to control for variation on input data within each test group (Figure 9).

Each subject's set of pairwise comparisons are used to obtain vectors of weights relating the importance of the alternatives to the each criterion and the importance of the criteria themselves. The computer program CHOICE (Mar et al., 1983) is used to calculate these vectors. For Saaty's method, the vectors were combined using CHOICE to arrive at a composite ranking of the alternatives. For an example, refer to Chapter Two.

For Yager's method, the vectors were used as the base comparison between the alternatives and the criteria and the vector of criteria importance as the set of modifiers B (Chapter Three). Yager (1978) uses non-normalized eigenvectors but normalized eigenvectors do not change the order of the resulting ranks. Normalized eigenvectors are used to maintain consistency between methods. The final rankings were calculated using a computer program (Appendix 2).

Tsukamoto and Terano's method uses data in much the same manner as Yager's method. The relation R, however, is defined by Tsukamoto and Terano (1975) as a matrix identifying the presence or absence of relationships between,

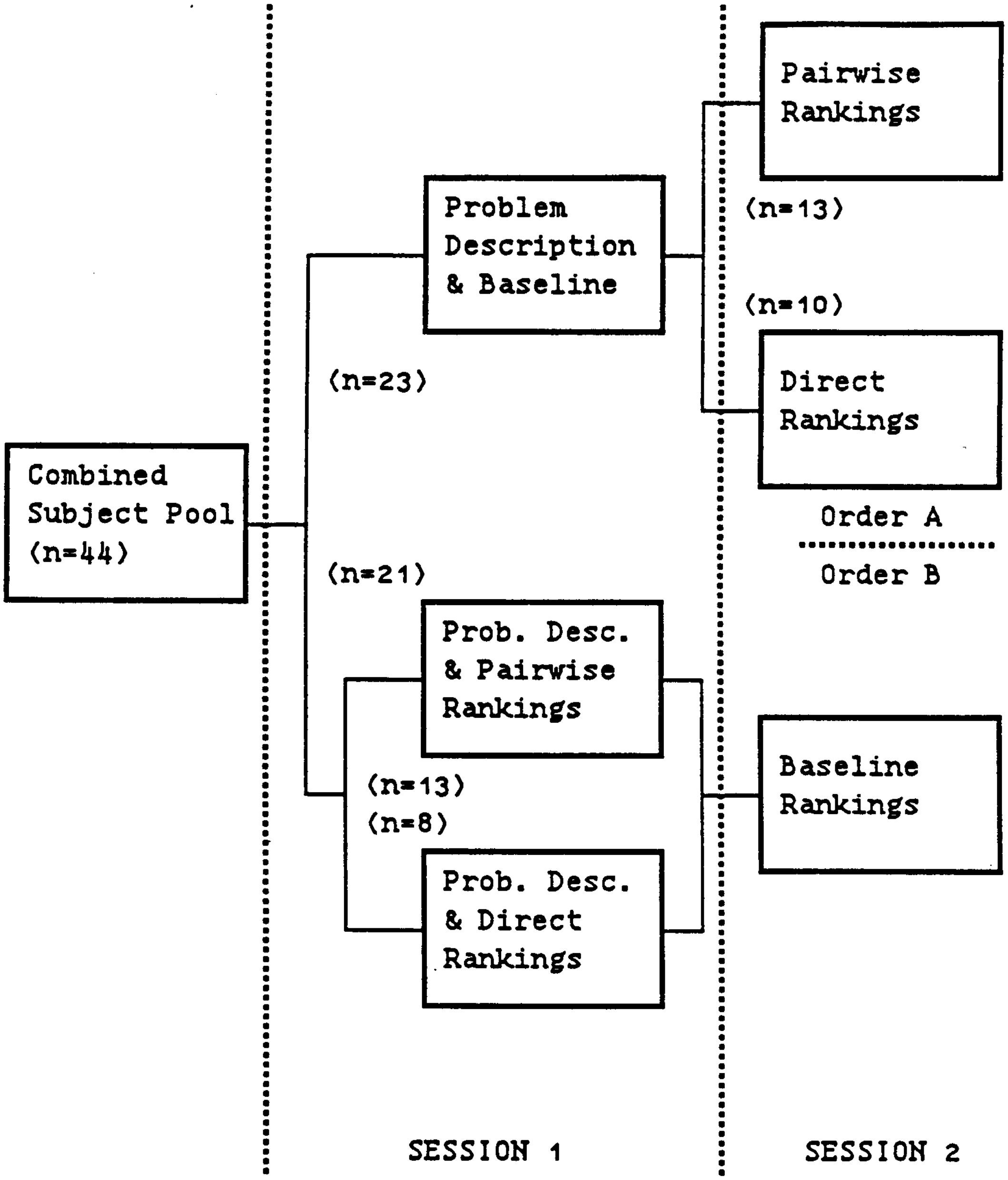

Figure 9 . Questionnaire Administration
Combined Subject Groups

in this study, alternatives and criteria. Each alternative is in this case related to each criterion and the matrix is filled with ones. The eigenvectors obtained from the questionnaire data comprise the matrix T which is the fuzzy support for the satisfaction of the criteria by the alternatives. Finally, the vector of criteria importance is used to form the vector C. From T, R, and C, the possible valuations of the vector A, the support for each alternative, is calculated (Chapter Two). The computer program used to calculate this vector for decision problems with four alternatives and four criteria is given in Appendix 2.

The procedure for calculating the rankings of the alternatives using direct comparisons is similar. In this case, the vectors of weights relating the alternatives to the criteria and the importance of the criteria are obtained directly and used in Yager's method and Tsukamoto and Terano's method in a manner equivalent to the use of the eigenvectors. Because Saaty's method explicitly requires eigenvectors, Saaty's method was not used with direct assessments.

The products of the three methods are vectors of weights indicating the support for each alternative with respect to the goal of finding the best overall alternative. For evaluation with the baseline ranking of alternatives obtained from the test subjects, the vectors of weights are converted to ranks. For example, the vector:

$[\ .6|A_1,\ .2|A_2,\ .5|A_3,\ .4|A_4\ ]$

is assigned the rank order:

$1|A_1,\ 4|A_2,\ 2|A_3,\ 3|A_4.$

In the case of ties, the tied alternatives share the tied rank and the subsequent rank is skipped.

Tsukamoto and Terano's method normally results in a range of possible valuations for the support of each alternative. To obtain rankings, the upper bound of the solution is first considered. If an unambiguous ordering is obtainable only the upper bound, as the highest possible support, is used. If the ranking was ambiguous then the lower bound is also considered. This is best illustrated through an example. Suppose alternative one is valued on the interval [.4, .8] and alternative two is valued on the interval [.6, .8]. The upper bounds are equal but because it is possible for alternative one to assume a value below alternative two, the second alternative is ranked higher. Ties are treated the same as in Yager's and Saaty's methods.

For each test subject and for each decision setting there is now a baseline ranking and a ranking obtained from each method. Correlations between the baseline ranking and each method's ranking can be obtained.

Experimental Design.

The general hypothesis is that the use of the fuzzy logic based method to rank a given set of alternatives with respect to a given set of criteria will achieve a significantly high correlation with the baseline ranking of alternatives, as obtained from a test subject. Further, it is hypothesized that the fuzzy logic based method will yield a mean correlation higher than that obtained from the use of Yager's fuzzy set based method.

The experimental design consists of two sets of subjects and three treatments as diagrammed in Figure 10. Although correlations for each subject will be considered individually, it is assumed that the subjects are sufficiently homogeneous in problem solving ability and other characteristics to permit aggregate measure of each method's results as well. Because two sets of treatments will be administered to the same subjects (the baseline and one of the pairwise and direct comparisons), the two sets of subjects are further divided into two additional groups with the treatments administered in reverse order (orders A and B) to control for the effects of problem familiarity.

Rankings

	Baseline	**Pairwise**	**Direct**
Order A	1 . . 13	1 (n=13) . . 13	27 (n=10) . . 36
Order B	24 . . 44	14 (n=13) . . 26	37 (n=8) . . 44

Figure 10. Experimental Design: Combined Subject Groups

The subjects of the first administration of the questionnaire were senior and first year graduate students in Civil Engineering. For the second administration, the subject group consists of junior and senior Civil Engineering students. The students are being trained in decision analysis techniques in particular and for a profession oriented to the types of problems reflected in the experiment in general, and thus are assumed adequate surrogates for professional analysts.

For the different problem settings described above there are several specific questions that will be addressed and tested:

1) Is there a relationship between ranks obtained from baseline assessments and ranks obtained from fuzzy logic methods?

2) Is there a relationship between ranks obtained from baseline assessments and fuzzy set (Yager) methods?

3) Is there a relationship between ranks obtained from baseline assessments and Saaty's method?

4) Is the correlation of the fuzzy logic method significantly greater than the fuzzy set (Yager) or Saaty methods?

With respect to the evaluation of test results, the following operational measures were selected. First, to measure the similarity of an individual's decision using a treatment of a method to the individual's decision baseline, the rank correlation coefficient Kendall's tau is selected (e.g. Gibbons, 1971). This choice of measure is due to the nature of initial individual (rather than aggregate) evaluations in that it is not possible to determine (and is not reasonable to assume) the distributional nature of variances for each individual. This leads to the experimental hypothesis that a treatment yields the same rank ordering of decision alternatives as the baseline:

H_O: rank correlation between the baseline and treatment decision set for an individual ≤ 0.

H_A: rank correlation > 0.

Examining the total sample group, an aggregate measure of performance may be obtained leading to the experimental hypothesis that the correlations of the sample are significantly greater than zero. In addition, the effects of the order of questionnaire administration and the form of data aquisition (pairwise vs. direct) may also be examined. This may be accomplished through ANOVA tests as diagrammed in Figure 10. The ANOVA test is valid in this particular case because the factors analyzed may be assumed to be independent (they are not matched pairs) and the test is reasonably insensitive to required assumptions of normality even with the sample sizes used here.

Finally, it is desired to compare the relative performance of a method to other methods. In particular, it is hypothesized that the fuzzy logic method will yield higher mean correlations than Yager's fuzzy set based method. Specifically, the hypothesis may be stated as:

H_O: rank correlations for the fuzzy logic method are less than or equal to the rank correlations for Yager's method.

H_A: rank correlations for the fuzzy logic method are greater than the rank correlations for Yager's method.

This hypothesis may be tested using the Wilcoxon signed ranks test. This test was chosen because of the matched pairs of data and lack of distributional information. The results of the experiments are given in the next chapter.

Evaluation of Experimental Assumptions.

As described at the beginning of this chapter the questionnaire was developed to provide a surrogate for actual decision problems by experts in each particular decision setting. The use of actual problems and decision makers was beyond the scope of this study. The validity of the questionnaire based decision setting is thus a major assumption.

Several actions were taken to ensure its validity. First, several different problem settings were developed. The first problem setting was designed particularly to present a problem that the majority of subjects would have had personal experience. In addition, knowledge of the problem context is not unique to engineering and does not require any such skills to provide a solution. The simplicity of this setting provides a benchmark for which high correlations would be expected from all methods. The remaining two problem settings are oriented more specifically to engineering contexts. Because it was assumed that few, if any, of the subjects had direct personal experience with settings of these types, an introductory narrative was provided to supply sufficient background to the settings. The narratives are composed of three parts. The first part describes the general setting of the problem and identifies the nature of the difficulty in providing quantitative solutions (e.g. poorly quantified parameters). The second part identifies the evaluation criteria that are to be used in assessing the alternatives. For each criterion a brief description and rationale is given. The third part of the narrative identifies the set of alternatives to be considered. Factual information about the characteristics of each alternative is given.

To complete the questionnaire, subjects rank the alternatives previously described. To provide an indication of the certainty assigned to each rank a scale is given to be used to indicate levels of certainty. For pairwise comparisons a scale is given corresponding to Saaty's scale and questions directing the pairwise comparisons follow.

Related to the possible difficulties with questionnaire problem settings is the consideration of the questionnaire respondents. The engineering students responding to the questionnaire have been exposed to formal decision analysis techniques but for the most part have not had lengthly experience with actual engineering decision making. To begin to address this potential problem, the first problem setting in the questionnaire was designed to reflect the nature of subjective decision judgments commonly arising, and with which the respondents would have greater experience. Another problem arises from the use of students in that the outcomes of the decisions are not real, that is, there is no incentive for careful consideration of the decision problem. Finally the length of the questionnaire may have caused attrition in responses. These factors could contribute to inconsistent responses by individuals.

A third consideration in the evaluation of the decision methods is that of the definition of membership functions for use in both Yager's and the fuzzy logic method. The explicit assumption used at the outset of this study was that fuzzy membership functions for preferences of alternatives were adequately obtained through the pairwise comparison method of Saaty. This assumption was supported in the literature. To begin to address this situation, an alternative technique, that of direct comparisons of an alternative to an evaluation criterion was used. There exist, however, very few investigations into the merits of these or other assessment techniques. The problem of membership function assessment warrants separate investigation and is beyond the scope of this preliminary investigation. Nonetheless, the quality of the results obtained from the various methods examined here are dependent on the quality of these membership functions.

A final consideration concerns the size of the samples available for each factor. Because of the several control factors and the relatively small pool of subjects available, the power of the results obtained are quite low.

CHAPTER FOUR

DECISION EXPERIMENT RESULTS

This chapter describes the administration of the questionnaire and the results of the analysis of responses. In addition, the results are analyzed and supplemental studies described. In general, all methods in all problem settings performed poorly.

The Kendall's tau scores for each individual for each of the three methods under consideration are given in Tables 8 through 10. The scores represent the correlation between the ranks of preference for the alternatives obtained directly from the respondents and the ranks of the alternatives obtained from the three methods. The respondents from the first sample group are numbered 1 to 26, respondents from the second sample group are numbered 27 to 44. There are no scores for Saaty's method in the second sample group as pairwise comparisons are required.

With four alternatives to be ranked in each problem setting, there are 4! or 24 possible sets of orderings that may be obtained (not counting ties between alternatives). To reject the null hypothesis of no correlation between sets of ranks,

Table 8. Kendall's Tau Scores For Individual Respondents: Problem A - Selection of Ice Cream Brands

ID	Saaty	Yager	Fuzzy Logic
1	0.333	0.667	0.000
2	-0.667	0.333	0.548
3	0.000	0.333	0.333
4	-0.333	-0.667	-0.548
5	0.000	0.333	0.866
6	0.000	0.000	-0.548
7	0.333	0.667	-0.183
8	1.000	1.000	-0.866
9	0.667	0.667	0.000
10	0.333	0.548	0.548
11	0.667	0.667	1.000
12	-0.667	-0.667	0.000
13	0.667	0.667	-1.000
14	1.000	0.667	-0.183
15	0.667	0.667	0.333
16	-0.333	0.333	0.667
17	0.333	0.333	0.183
18	0.667	0.667	-0.333
19	1.000	0.333	0.289
20	0.667	1.000	0.667
21	0.333	-0.333	0.866
22	0.000	-0.333	0.913
23	-0.333	0.333	0.183
24	0.667	0.866	0.548
25	-0.333	-0.333	0.333
26	0.000	1.000	-0.333
27	n.a.	0.333	0.000
28	n.a.	0.000	0.866
29	n.a.	-0.333	-0.183
30	n.a.	0.000	0.000
31	n.a.	0.913	0.000
32	n.a.	0.000	-0.866
33	n.a.	-1.000	0.000
34	n.a.	0.333	-0.408
35	n.a.	-0.289	0.913
36	n.a.	0.289	0.913
37	n.a.	-0.548	0.000
38	n.a.	-0.548	-0.183
39	n.a.	0.183	0.289
40	n.a.	0.000	0.183
41	n.a.	0.667	0.000
42	n.a.	0.289	0.183
43	n.a.	0.183	-1.000
44	n.a.	0.548	-0.183

Table 9. Kendall's Tau Scores For Individual Respondents: Problem B - Water Quality Monitoring

ID	Saaty	Yager	Fuzzy Logic
1	0.667	1.000	0.000
2	-0.667	0.667	0.548
3	0.667	0.333	0.333
4	0.667	0.333	-0.548
5	0.333	0.333	0.866
6	0.667	0.000	-0.548
7	-0.667	-1.000	0.333
8	-0.333	0.000	0.913
9	0.667	-0.333	0.000
10	0.000	1.000	0.183
11	0.000	1.000	0.183
12	0.000	-0.667	0.333
13	1.000	0.667	-0.333
14	0.000	0.000	-0.183
15	0.000	0.333	0.333
16	0.000	1.000	0.183
17	0.333	0.333	0.183
18	0.000	1.000	-0.333
19	0.333	0.333	0.289
20	0.333	0.333	0.667
21	0.333	0.333	0.866
22	-0.667	0.000	0.183
23	1.000	1.000	0.183
24	0.333	-0.333	0.548
25	-0.333	0.333	1.000
26	-0.333	1.000	-0.333
27	n.a.	0.333	-0.183
28	n.a.	-0.333	0.913
29	n.a.	0.548	0.000
30	n.a.	-0.667	0.000
31	n.a.	-0.667	0.000
32	n.a.	0.000	0.000
33	n.a.	0.289	0.000
34	n.a.	0.667	0.913
35	n.a.	0.548	0.333
36	n.a.	-0.866	0.000
37	n.a.	0.548	0.000
38	n.a.	-0.548	0.000
39	n.a.	-0.667	-0.548
40	n.a.	0.000	-0.667
41	n.a.	-0.183	0.913
42	n.a.	0.913	0.000
43	n.a.	-0.333	0.548
44	n.a.	-0.333	-0.333

Table 10. Kendall's Tau Scores For Individual Respondents: Problem Three - Urban Flood Plain Management

ID	Saaty	Yager	Fuzzy Logic
1	0.000	0.000	0.667
2	0.667	0.667	0.000
3	-0.333	-0.667	-0.183
4	-0.667	-1.000	0.333
5	0.333	0.333	0.289
6	0.667	1.000	-0.667
7	0.667	0.000	0.333
8	0.000	-0.333	0.000
9	1.000	1.000	-0.183
10	0.333	-0.548	0.000
11	0.000	0.333	0.667
12	0.866	0.667	0.183
13	-0.548	0.548	0.000
14	0.333	0.000	-0.333
15	0.000	0.000	-0.183
16	0.183	-0.333	0.549
17	1.000	-1.000	0.000
18	0.000	0.000	0.333
19	0.333	0.333	1.000
20	0.333	0.000	-0.183
21	0.667	0.667	0.000
22	-0.333	-0.333	0.667
23	0.667	0.333	-0.333
24	-0.667	-0.333	0.333
25	0.548	0.548	-0.333
26	0.183	0.183	0.000
27	n.a.	0.667	-0.289
28	n.a.	0.667	-0.548
29	n.a.	0.183	0.000
30	n.a.	0.667	-0.913
31	n.a.	0.913	0.000
32	n.a.	0.000	0.000
33	n.a.	0.000	0.000
34	n.a.	0.667	0.289
35	n.a.	-0.913	-0.289
36	n.a.	0.333	0.000
37	n.a.	-0.548	-0.866
38	n.a.	-0.183	0.000
39	n.a.	0.000	-0.866
40	n.a.	0.000	-0.548
41	n.a.	-0.913	0.183
42	n.a.	-0.816	-0.183
43	n.a.	0.333	0.183
44	n.a.	1.000	-0.548

there must be perfect correlation (tau equal to one). Figure 11 displays the distribution of tau scores for each method.It is readily seen that very few statistically significant correlations are obtained.

Analysis of Sample Group Differences.

Each sample group was divided into subgroups to test the effects of questionnaire ordering, pairwise comparisons, and direct comparisons on correlations. A diagram of the combined administration is given in Figure 10. Four subgroups are indicated. These are:

1) Baseline Ranking followed by Pairwise Comparison (Order A)
2) Baseline Ranking followed by Direct Comparison (Order A)
3) Pairwise Comparison followed by Baseline Ranking (Order B)
4) Direct Comparison followed by Baseline Ranking (Order B).

Scores obtained from these subgroups were analyzed through ANOVA techniques as described in the previous chapter. Results of these analyses are given in Table 11. These analyses only apply to the results obtained from Yager's and

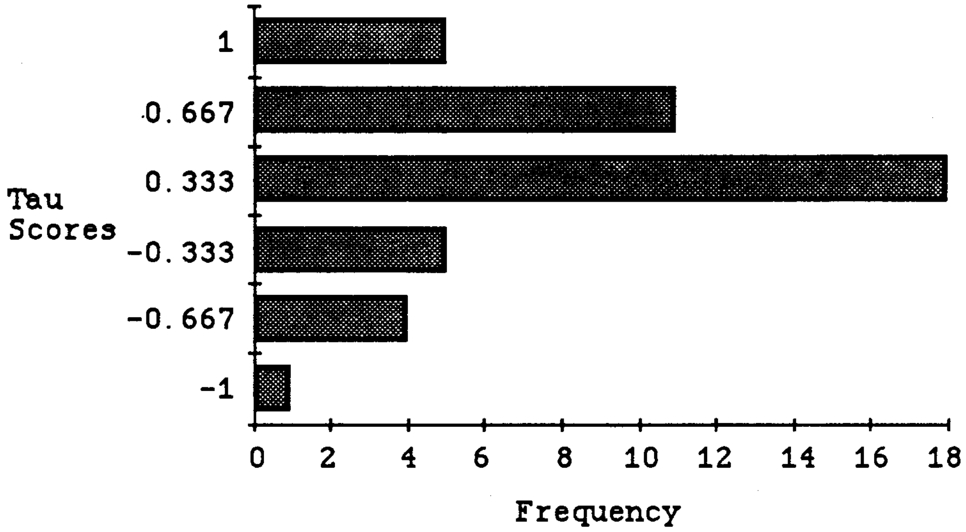

Figure 11a. Kendall's Tau Scores from
Yager's Method
Problem A

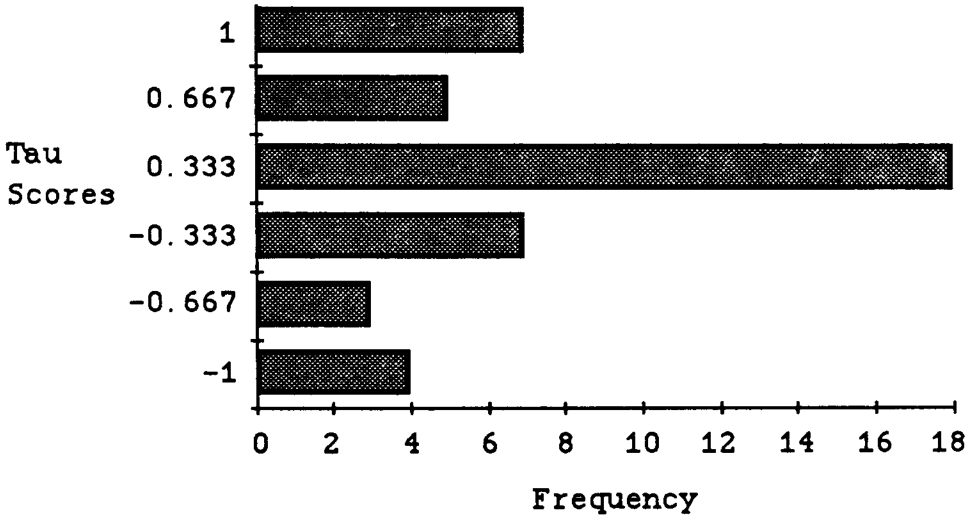

Figure 11b. Kendall's Tau Scores from
Fuzzy Logic Method
Problem A

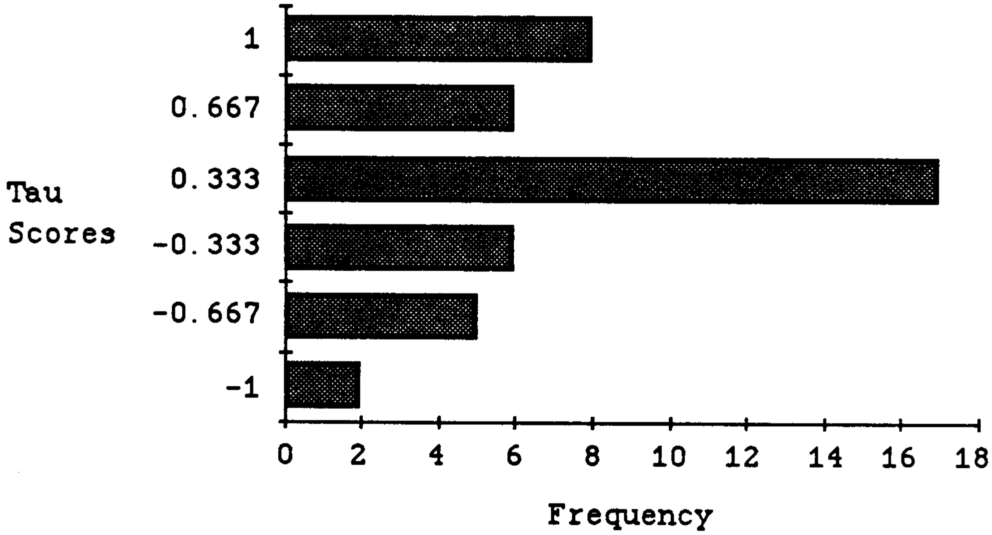

Figure 11c. Kendall's Tau Scores from
Yager's Method
Problem B

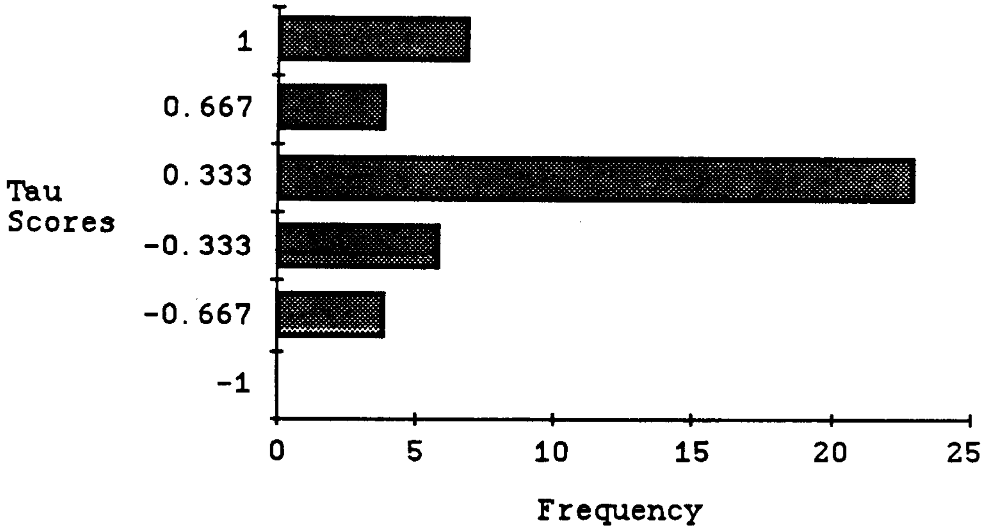

Figure 11d. Kendall's Tau Scores from
Fuzzy Logic Method
Problem B

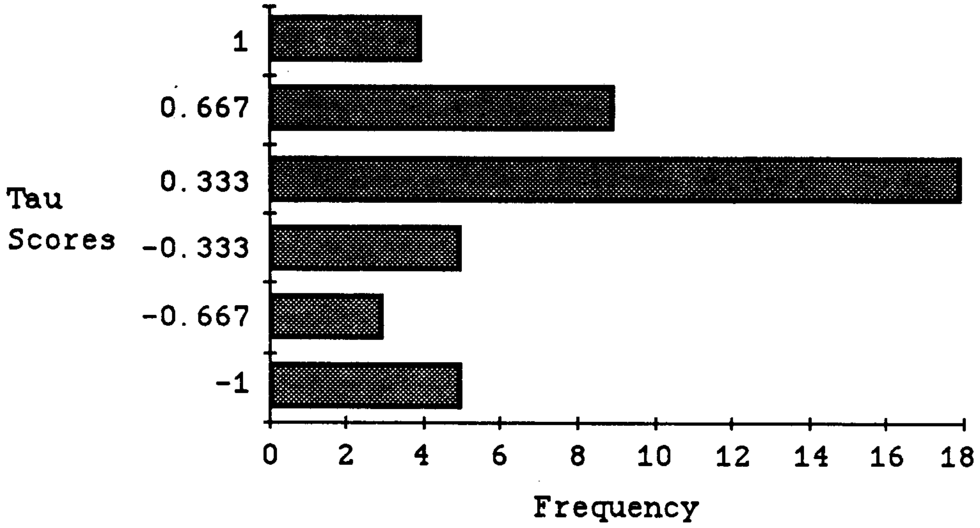

Figure 11e. Kendall's Tau Scores from
Yager's Method
Problem C

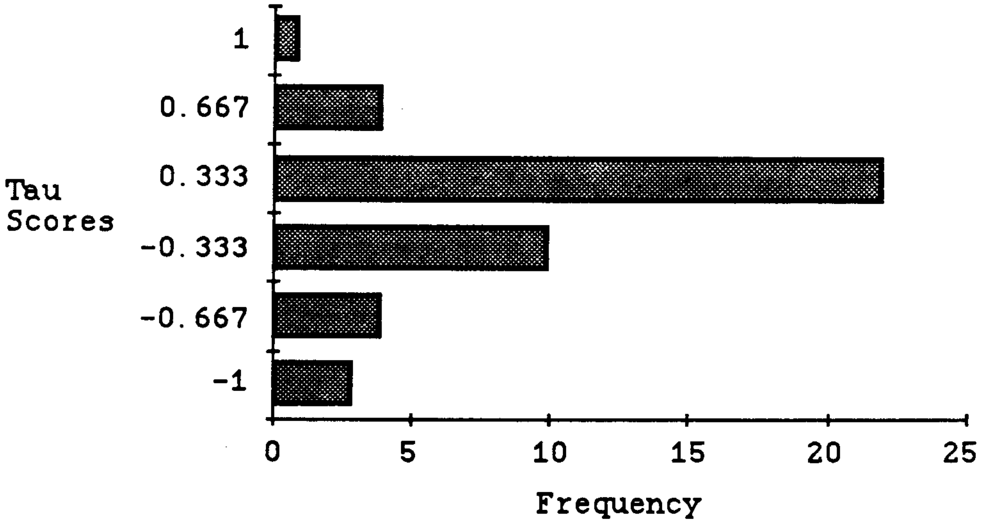

Figure 11f. Kendall's Tau Scores from
Fuzzy Logic Method
Problem C

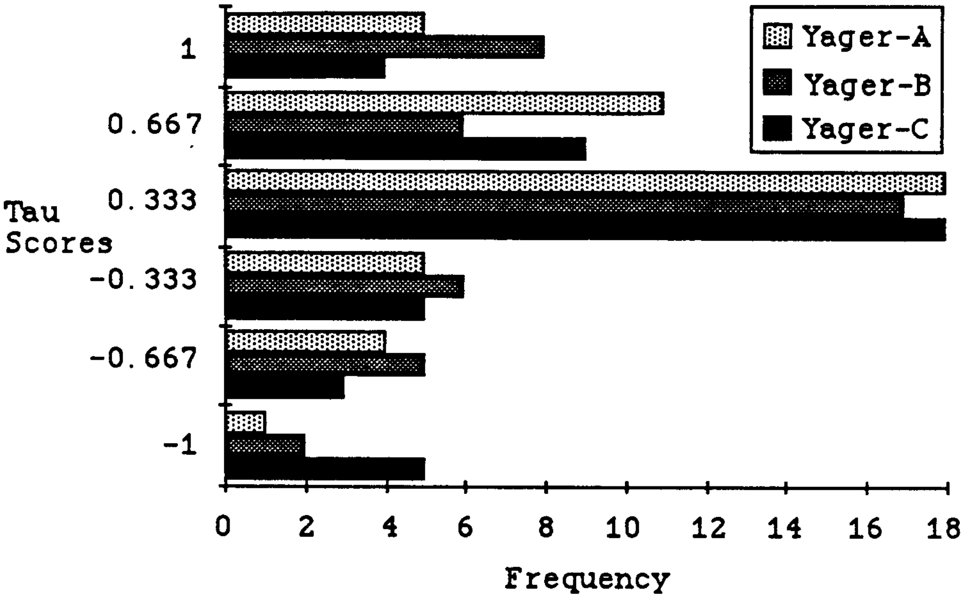

Figure 11g. Kendall's Tau Scores
from Yager's Method
All Problems

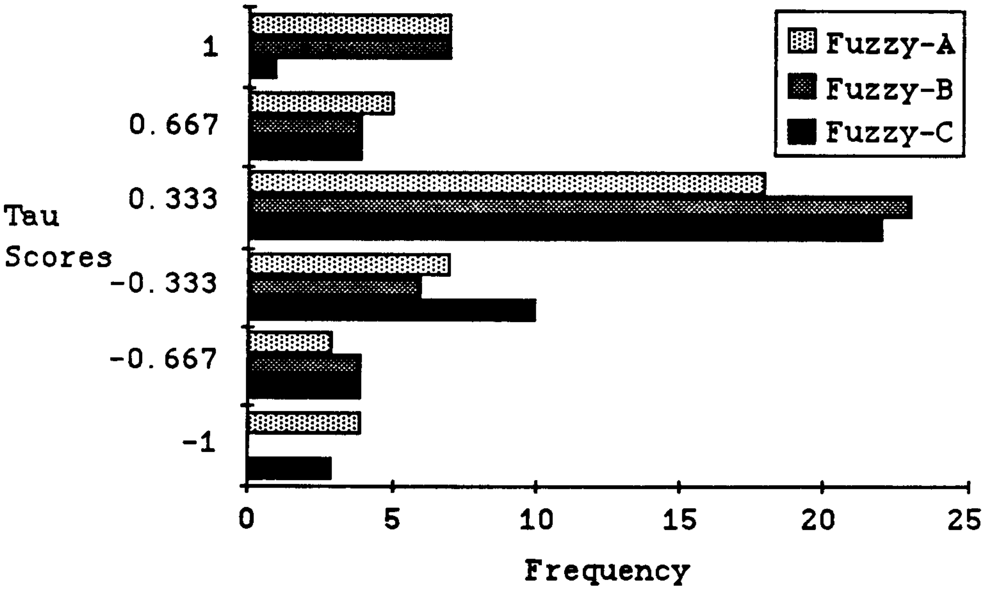

Figure 11h. Kendall's Tau Scores
from Fuzzy Logic Method
All Problems

the fuzzy logic method as Saaty's method is based upon pairwise comparisons. Only in one of six cases (fuzzy logic method, problem C) is there any significant difference between sub-groups (Table 11b). Comparisons of means of the subgroups in this case (Table 11c) indicate significant differences between two subgroups - Order A: Pairwise with Order A: Direct, and Order B: Pairwise with Order B: Direct. In all cases, there is no effect on the scores by the ordering of the questionnaire. The significant difference between the two subgroups might be attibuted to the use of pairwise or direct ranking techniques or to the two sample groups. The first sample group used pairwise rankings while the second group used direct rankings. It was not possible to test this further. The low power available for this group indicates this result might be an aberration.

Comparison of Methods.

The next group of hypotheses to be tested are concerned with the relative performance of each method. The results of the comparison of the methods, that is, that the fuzzy logic method is hypothesized to yield higher correlations than other methods, is given in Table 12. In Table 12a, only in one subgroup of one problem setting can it be said that there is a significant difference between scores obtained from Yager's method and from the fuzzy logic method.

Table 11a. ANOVA Results For Yager's Method

Yager's Method: Problem A
$F_{3,40}$ = 1.537 SS Between Groups: 1.1168
Within Groups: 9.6856

Problem A (Ice Cream)	N	Mean	Std. Dev.
Order A - Pairwise	13	.3498	.5128
Order A - Direct	10	.0246	.5085
Order B - Pairwise	13	.4000	.4814
Order B - Direct	8	.0968	.4506

Yager's Method: Problem B

$F_{3,40}$ = 1.977 SS Between Groups: 1.8251
Within Groups: 12.3085

Problem B (Water Quality)	N	Mean	Std. Dev.
Order A - Pairwise	13	.2564	.6406
Order A - Direct	10	-.0148	.5762
Order B - Pairwise	13	.4358	.4385
Order B - Direct	8	-.0754	.5463

Yager's Method: Problem C

$F_{3,40}$ = 0.627 SS Between Groups: 0.6012
Within Groups: 12.7826

Problem C (Stormwater)	N	Mean	Std. Dev.
Order A - Pairwise	13	.1538	.6429
Order A - Direct	10	.3814	.5351
Order B - Pairwise	13	.0050	.4409
Order B - Direct	8	.0631	.6450

Table 11b. ANOVA Results For Fuzzy Logic Method

Fuzzy Logic Method: Problem A

$F_{3,40}$ = 1.187 SS Between Groups: 1.0063
Within Groups: 11.2997

Problem A (Ice Cream)	N	Mean	Std. Dev.
Order A - Pairwise	13	.0115	.6353
Order A - Direct	10	.1235	.5976
Order B - Pairwise	13	.3179	.4171
Order B - Direct	8	-.0889	.4061

Fuzzy Logic Method: Problem B

$F_{3,40}$ = 0.681 SS Between Groups: 0.4137
Within Groups: 8.1007

Problem B (Water Quality)	N	Mean	Std. Dev.
Order A - Pairwise	13	.1741	.4656
Order A - Direct	10	.1976	.3974
Order B - Pairwise	13	.2758	.4173
Order B - Direct	8	-.0109	.5331

Fuzzy Logic Method: Problem C

$F_{3,40}$ = 3.173 SS Between Groups: 1.4700
Within Groups: 6.1765

Problem C (Stormwater)	N	Mean	Std. Dev.
Order A - Pairwise	13	.1107	.3630
Order A - Direct	10	-.1750	.3461
Order B - Pairwise	13	.1167	.4273
Order B - Direct	8	-.3306	.4353

$F_{3,40}$ crit ($\alpha = .1$) = 2.23

Table 11c. T-Test Results for Cells of ANOVA for Fuzzy Logic Method, Problem C

Problem C (Water Quality)	N	Mean	Std. Dev.
Order A - Pairwise	13	.1107	.3630
Order A - Direct	10	-.1750	.3461
Order B - Pairwise	13	.1167	.4273
Order B - Direct	8	-.3306	.4353

Order A - Pairwise with Order A - Direct

t(pooled variance estimate) = 1.728* df = 40

t(separate variance estimate) = 1.921* df = 21

Order A - Pairwise with Order B - Pairwise

t(pooled variance estimate) = -.039 df = 40

t(separate variance estimate) = -0.39 df = 24

Order B - Pairwise with Order B - Direct

t(pooled variance estimate) = 2.533* df = 40

t(separate variance estimate) = 2.303* df = 19

Order A - Direct with Order B - Direct

t(pooled variance estimate) = 0.835 df = 40

t(separate variance estimate) = 0.824 df = 16

* Significant at $\alpha = .1$

Further, the direction of difference is in favor of Yager's method, that is, the scores obtained from Yager's method are significantly higher than the fuzzy logic method. In examining the difference between Saaty's method and the other two methods (Table 12b), there are significant differences with Yager's method in two problem settings. The direction of the difference is in favor of Yager's method in the first case (.333 versus .440), and in favor of Saaty's method in the second case (.333 versus .000). This of course prevents any statement about consistent performance (at least on the part of Yager's method).

Summary of Findings.

First, the results of the questionnaire responses indicate that there is no effect of the order in which the parts of the questionnaire are administered. This suggests that the timing of the different sessions was sufficient to prevent problem familiarity from biasing the correlations between baseline rankings and the rankings obtained from the various methods.

Second, there is very little evidence to suggest that either differences in the two sample groups or differences between the pairwise comparisons used by the first group and the direct comparisons used by the second group had any effect on the correlations. The experiment, as it was administered,

Table 12b. Results of Wilcoxon Test for Matched Pairs: Comparison of Methods

Yager's Fuzzy Set Method - Fuzzy Logic Method Comparison

		Median Score		
Problem A (Ice Cream)	N	Yager	Fuzzy	Z score
Order A - Pairwise	13	.333	.000	1.293
Order A - Direct	10	.000	.000	.510
Order B - Pairwise	13	.333	.289	.559
Order B - Direct	8	.183	.000	.630
Problem B (Water Quality Sampling)				
Order A - Pairwise	13	.333	.183	.384
Order A - Direct	10	.144	.000	1.070
Order B - Pairwise	13	.333	.183	.629
Order B - Direct	8	-.258	.000	.350
Problem C (Storm Water Management)				
Order A - Pairwise	13	.000	.000	.140
Order A - Direct	10	.500	.000	2.191*
Order B - Pairwise	13	.000	.000	.734
Order B - Direct	8	-.091	-.365	.560

Method Comparison for First Sample Group

		Median Score		
Problem A (Ice Cream)	N	Saaty	Yager/Fuzzy	Z score
Saaty - Yager	26	.333	.440	2.032*
Saaty - Fuzzy	26	.333	.236	.368
Problem B (Water Quality Sampling)				
Saaty - Yager	26	.166	.333	1.346
Saaty - Fuzzy	26	.166	.183	.559
Problem C (Storm Water Management)				
Saaty - Yager	26	.333	.000	2.502*
Saaty - Fuzzy	26	.333	.000	.864

* Significant at $\alpha = .1$

prevents differentiation between the effects of the two groups and the comparison techniques in the one case where there was a significant difference.

Third, there is no evidence to suggest that the fuzzy logic method performs better than the other methods with respect to its ability to generate higher correlations with baseline rankings. There is very little evidence to suggest that Yager's method or Saaty's method perform consistently better as well.

On the basis of the individual correlations given in Tables 8 through 10, there is little evidence to support the hypothesis that any of the fuzzy set methods predict well decisions of individual decision makers for the problems used in the questionnaire and for the decision makers used in the sample groups.

There is no difference between the performance of methods across the three problem settings. All three problem settings were relatively simple with a very small number of discrete alternatives and criteria to be considered. There is no indication that differences in the problem settings were reflected in the poor results. It is reasonable then, to evaluate the results from the point of view of the capabilities of the methods themselves and of the quality of responses obtained from the test subjects.

CHAPTER FIVE

EVALUATION

The inability of the results from the experiments described in the previous chapter to support the study hypotheses may arise from several factors. While it is desirable to simply conclude that the decision methods examined in this study cannot be said to reflect human decision making behavior, there is a reasonable risk of falsely accepting the null hypotheses given the very preliminary nature of this research and the low power available from the experiment. Instead, it becomes necessary to investigate the possible sources of problems that might have contributed to the poor performance reported here and to identify either further tests to contribute to the preliminary conclusions or to identify remedial actions to the experimental design to remove complicating factors.

This chapter examines in detail the behavior of the decision methods under more controlled conditions than those offered by the questionnaire responses. It will introduce the application of the methods in the context of group decision making as application provides further illumination of the operational characteristics of each method. It will also examine elements of the design of the experiment and of the assumptions used in the application of the questionnaire.

Relaxation of Correlation Requirement.

Although there were very few cases in which the ranks generated by the three decision methods are significantly greater than zero, it is of interest to examine the scores for any trends that indicate if any particular method generates correlations significantly greater than the others. This is important for several reasons. First, the source of low correlations may not be due to the methods themselves and better performance from one method would suggest that some evidence exists that a method would yield more significant results under different experimental conditions. Consistent performance also facilitates the examination of the characteristics of the methods to indicate why one method might perform better.

The use of Kendall's tau (or other rank correlation tests) constitutes a very restrictive test of a decision method's capability. While the exact matching of a rank ordering by a method is a desirable characteristic, it may be argued that in practical cases it is more important that the first or second choice alternatives match, rather than perhaps the third or fourth. The ranks obtained from the decision methods were therefore examined in this context. Given the alternative ranked first in the individual baseline rankings, the frequency with which the alternative ranked first by each decision method was the same was first calculated. Relaxing the restriction further, the frequency with which the individual's first choice alternative was the same as the first or second ranked alternative generated by the decision method was computed. Finally, the frequency with which the first or second or third ranked alternative obtained from the decision methods was the same as the baseline first choice was evaluated. The results of these evaluations are summarized in Figure 12 and Table 13. In all three problem settings, the fuzzy logic method matched first ranked alternatives more frequently. The significance of these differences was not tested. One possible source of the apparent performance was that the weights generated by the fuzzy logic method were sometimes equal, in effect causing ties in the ranking.

Concern was noted in Chapter Three regarding the translation of weights into ranks for the purpose of constructing an appropriate test. Closely weighted alternatives cannot be fully considered when they are ranked. To determine if the range of weights were related to resulting rank correlations, scores resulting in strongly positive, strongly negative, and no correlations were identified. There is some indication

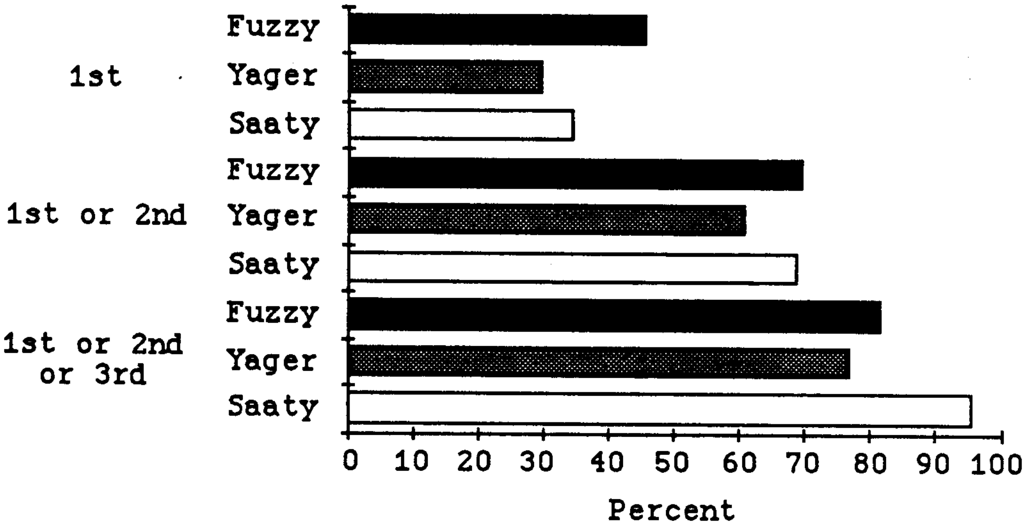

Figure 12a. Percent of Sample Matching Rank: Problem A

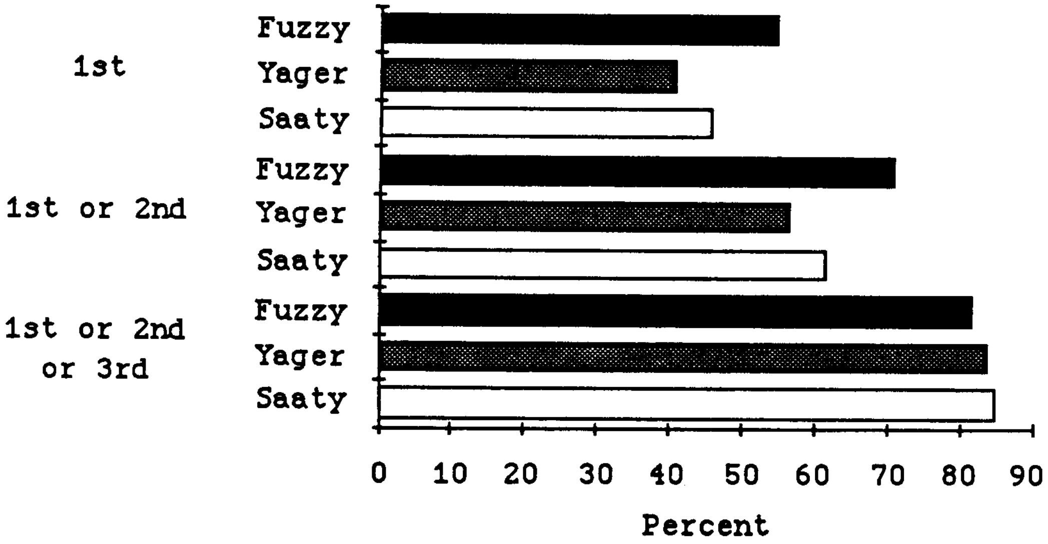

Figure 12b. Percent of Sample Matching Rank: Problem B

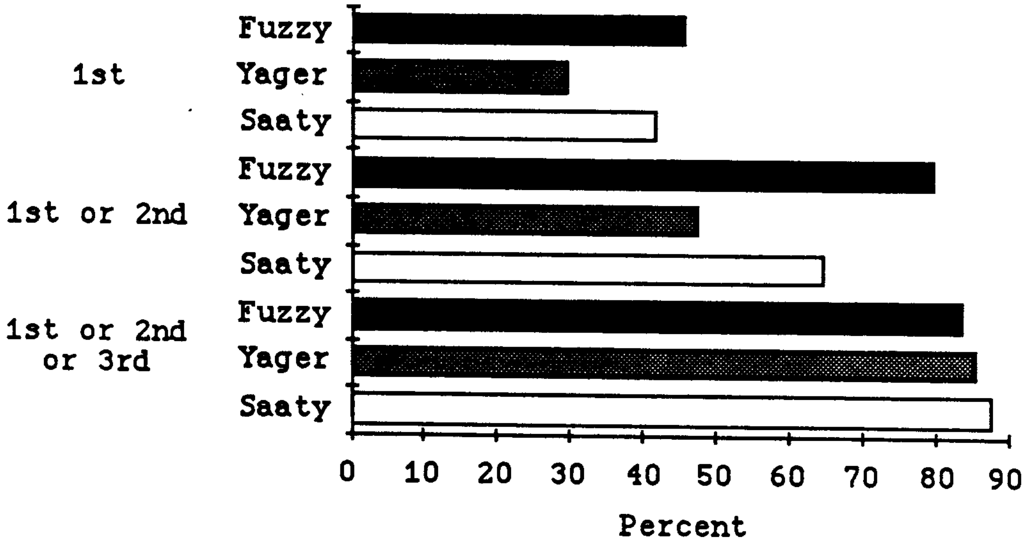

Figure 12c. Percent of Sample Matching Rank: Problem C

Table 13. Frequency of Matched Ranks Between Alternative Ranked First in Baseline and Rank by Method

Problem	Rank by Decision Method	Saaty (n=26)	Yager (n=44)	Fuzzy Logic (n=44)
	1st	**9 (.35)**	**13 (.30)**	**20 (.46)**
A	1st or 2nd	18 (.69)	27 (.61)	31 (.70)
	1st or 2nd or 3rd	25 (.96)	34 (.77)	36 (.82)
	1st	**12 (.46)**	**18 (.41)**	**24 (.55)**
B	1st or 2nd	16 (.62)	25 (.57)	31 (.71)
	1st or 2nd or 3rd	22 (.85)	37 (.84)	36 (.82)
	1st	**11 (.42)**	**13 (.30)**	**20 (.46)**
C	1st or 2nd	17 (.65)	21 (.48)	35 (.80)
	1st or 2nd or 3rd	23 (.88)	38 (.86)	37 (.84)

that the range of weights for scores of no correlation were smaller than the others. However, for large ranges ofweights, the scores were as likely to be highly negatively correlated as positively.

Test of Uncertainty Propagation.

In working with problems in which the inputs to the solution method are well defined and certain, it is expected that the results from the solution method will also be well defined and certain. For example, using the operation of multiplication, 6 * 8 = 48. The inputs to the model, 6 and 8, are unambiguous. Similarly, the conclusion of 48 is equally unambiguous. Now suppose that the inputs are made more ambiguous, say the intervals [5,7], and [7,9] are used rather than 6 and 8. Using the operation of multiplication it might reasonably be expected that the solution will range on the interval [35, 63]. The magnitude of the interval of the input elements are two units each, while the magnitude of the interval of the solution is 28 units. To what may the apparent increase in uncertainty be attributed? Surely not the operation of multiplication - if it were, then the product of unambiguous inputs might be expected to be an ambiguous element. The uncertainty in the solution must therefore be attributed to the the uncertainty in the input elements. The same conclusion may easily be reached for two of the decision methods examined in this study.

Given the trivial example of two alternatives (A, B) and one criterion, assume that alternative A fully satisfies the criterion with complete certainty and alternative B does not satisfy the criterion. Using Yager's method, it is easy to see that the support for alternative A being the alternative that most satisfies the criterion is unambiguous, being equal to unity. Similarly, the support for alternative B being the alternative of choice is equal to zero. Using the same example for the fuzzy logic method yields the same result.

This exercise is made more difficult in Saaty's method. Using the suggested scale for pairwise comparisons yields the pairwise matrix

$$\begin{array}{ccc} & A & B \\ \hline A & 1 & 9 \\ B & 1/9 & 1 \end{array}$$

which results in the (normalized) weights of .9 for alternative A and .1 for alternative B. Thus the ordering is preserved but the use of the scale introduces some ambiguity into the solution. In fact, only the pairwise comparison matrix

$$\begin{array}{ccc} & A & B \\ \hline A & 1 & \infty \\ B & 1/\infty & 1 \end{array}$$

eliminates the ambiguity resulting from the use of a scale. Therefore, it is concluded that the scale used in Saaty's method introduces some ambiguity into the solution. This result has some implications for evaluating the methodology used in this study and will be examined in a later section.

The above example illustrates the behavior of the techniques when the support for one alterative over another is certain. To demonstrate the effects of increasing uncertainty between preferences for alternatives, ten cases are examined. In each case the decision problem is composed of four alternatives (A1, A2, A3, A4) and four criteria (C1, C2, C3, C4). Pairwise comparison matrices for the 10 cases are given in Table 14. The matrices are constructed to range from highest certainty in preference to highest uncertainty (all alternatives are equally preferred). Further, the matrices are constructed to maintain consistency in the ordering of alternatives across cases. Preferences generated by the three decision methods are given in Table 15. The results are reassuring in that decreasing certainty in the input preferences results in decreasing certainty in the aggregate preferences. The ordering of the alternatives (with the exception of the fuzzy logic method) remains consistent. In the extreme, if the ordering of alternatives with respect to each criterion were completely ambiguous (i.e. each alternative is equally preferred) then each decision method would return equally ambiguous results.

Table 14. Pairwise Comparisons For Ten Test Cases

ALTERNATIVES

Test Number One

	1	2	3	4
1	1	9	9	9
2	1/9	1	9	9
3	1/9	1/9	1	9
4	1/9	1/9	1/9	1

Test Number Two

	1	2	3	4
1	1	4	9	9
2	1/4	1	8	9
3	1/9	1/8	1	9
4	1/9	1/9	1/9	1

Test Number Three

	1	2	3	4
1	1	3.5	7	9
2	1/3.5	1	7	9
3	1/7	1/7	1	7
4	1/9	1/9	1/7	1

Test Number Four

	1	2	3	4
1	1	3	3	9
2	1/3	1	3	9
3	1/3	1/3	1	3
4	1/9	1/9	1/3	1

Test Number Five

	1	2	3	4
1	1	1.33	2	4
2	3/4	1	2	4
3	1/2	1/2	1	2
4	1/4	1/4	1/2	1

CRITERIA

Tests One Through Five

	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1

ALTERNATIVES

Test Number Six

	1	2	3	4
1	1	9	9	9
2	1/9	1	9	9
3	1/9	1/9	1	9
4	1/9	1/9	1/9	1

Test Number Seven

	1	2	3	4
1	1	4	9	9
2	1/4	1	8	9
3	1/9	1/8	1	9
4	1/9	1/9	1/9	1

Test Number Eight

	1	2	3	4
1	1	3.5	7	9
2	1/3.5	1	7	9
3	1/7	1/7	1	7
4	1/9	1/9	1/7	1

Test Number Nine

	1	2	3	4
1	1	3	3	9
2	1/3	1	3	9
3	1/3	1/3	1	3
4	1/9	1/9	1/3	1

Test Number Ten

	1	2	3	4
1	1	1.33	2	4
2	3/4	1	2	4
3	1/2	1/2	1	2
4	1/4	1/4	1/2	1

CRITERIA

Tests Six Through Ten

	1	2	3	4
1	1	1.33	2	4
2	3/4	1	2	4
3	1/2	1/2	1	2
4	1/4	1/4	1/2	1

Table 15. Ranking of Alternatives for 10 Test Cases

		METHOD					
TEST NUMBER	ALTERNATIVE	SAATY		YAGER		FUZZY LOGIC	
		score	rank	score	rank	score	rank
1	1	.675	1	.906	1	[.25,.575]	1
1	2	.225	2	.689	2	[.25,.575]	1
1	3	.075	3	.523	3	[.25,.575]	1
1	4	.025	4	.398	4	[.25,.575]	1
2	1	.586	1	.875	1	[.25,.664]	1
2	2	.294	2	.737	2	[.25,.664]	1
2	3	.092	3	.551	3	[.25,.664]	1
2	4	.028	4	.409	4	[.25,.664]	1
3	1	.565	1	.867	1	[.25,.685]	1
3	2	.309	2	.745	2	[.25,.685]	1
3	3	.095	3	.555	3	[.25,.685]	1
3	4	.031	4	.419	4	[.25,.685]	1
4	1	.529	1	.852	1	[.25,.721]	1
4	2	.230	2	.740	2	[.25,.721]	1
4	3	.128	3	.598	3	[.25,.721]	1
4	4	.043	4	.454	4	[.25,.721]	1
5	1	.390	1	.790	1	[.25,.860]	1
5	2	.338	2	.762	2	[.25,.860]	1
5	3	.181	3	.652	3	[.25,.860]	1
5	4	.091	4	.549	4	[.25,.860]	1
6	1	.675	1	.858	1	[.39,.715]	1
6	2	.225	2	.559	2	[.39,.663]	2
6	3	.075	3	.364	3	[.39,.506]	3
6	4	.025	4	.237	4	[.39,.416]	4
7	1	.586	1	.812	1	[.39,.805]	1
7	2	.294	2	.620	2	[.39,.752]	2
7	3	.092	3	.394	3	[.39,.595]	3
7	4	.028	4	.248	4	[.39,.505]	4
8	1	.565	1	.800	1	[.39,.825]	1
8	2	.309	2	.632	2	[.39,.773]	2
8	3	.095	3	.399	3	[.39,.616]	3
8	4	.031	4	.257	4	[.39,.525]	4
9	1	.529	1	.780	1	[.39,.861]	1
9	2	.230	2	.625	2	[.39,.809]	2
9	3	.128	3	.449	3	[.39,.652]	3
9	4	.043	4	.292	4	[.39,.561]	4
10	1	.390	1	.693	1	[.39,.843]	2
10	2	.338	2	.655	2	[.39,.909]	1
10	3	.181	3	.513	3	[.39,.791]	3
10	4	.091	4	.392	4	[.39,.700]	4

The results from the fuzzy logic method require further examination. In the first five cases, the supports for the alternatives are equal. This is because the supports for the criteria are equal. It appears then, that the criteria dominate the solution results. The variation in preferences for the alternatives with respect to each criterion is evidenced only in the changing upper bound. This shows that if uncertainty in the inputs increased then the fuzzy logic method returns a solution encompassing a greater range on the unit interval. This property was discussed in Chapter Two. The results from the cases presented here would imply that in the last five cases, the ordering induced by the upper bound values are again a function of the supports for the importance of the criteria.

That the ordering of alternatives by the fuzzy logic method is dependent upon only the ordering of criteria calls the applicability of this technique to the types of problems considered in this study into question. The solution mechanism of this method should be examined in further detail to determine if its application to decision analysis problems is warranted.

The results of the last five cases for Saaty's method illustrate another cause for concern. Note that the weights for the last five cases are identical to the respective weights for the first five cases. The weights for the criteria are different between the two sets of cases however. The reason that the weights are equivalent arises from two factors. First, the pairwise comparison matrices with respect to each criterion were intentionally made the same to ensure that the ordering of the alternatives in the method input would be maintained. This equivalency, in combination with the use by Saaty's method of normalized eigenvectors for the criteria weights ensures that the final weighted vector of the the alternatives remains invariant even as the relative importance of each criterion varies. Perhaps in some settings this property is desirable, but if the vectors of weights reflect uncertainty in preference, then this property would in fact be undesirable. It seems reasonable to require that if the uncertainty in preference between criteria in a problem varies, then that change in certainty should be reflected in the final supports for the preference of the alternatives. It can be seen in Table 15 that this is the behavior exhibited by Yager's method.

Group Decisions.

In addition to the examination of the ability of the three decision methods to replicate human decision behavior of individuals with respect to the ranking of a set of alternatives, it was also desired to examine the ability of the methods to aggregate individual rank orderings to form a group decision. The procedure followed for group decisions was similar to that used for individual decisions. After responding to questions pertaining to the individual assessment of alternatives, the respondents were randomly divided into groups of two or three persons. Each group generated a ranking of the alternatives reflecting the consensus of the individuals. In addition to ranking the alternatives, each individual was requested to evaluate the criteria with respect to their completeness in modeling the decision problem. There were no constraints and/or guidelines as to how groups were to achieve the consensus ranking. The sequence of group decision making in the administration of the questionnaire is shown in Figure 13.

To generate ranks using the decision methods, the pairwise comparisons obtained from each individual were used as input for aggregation as described in Chapter Two. In all methods, the weights assigned to the importance of the individual rankings were equal. The results of the comparison between ranks obtained by each group of respondents and ranks obtained from each of the decision methods are given in Table 16. Again, the performance of the methods is poor with respect to their ability to match the baseline ranks obtained directly from the groups.

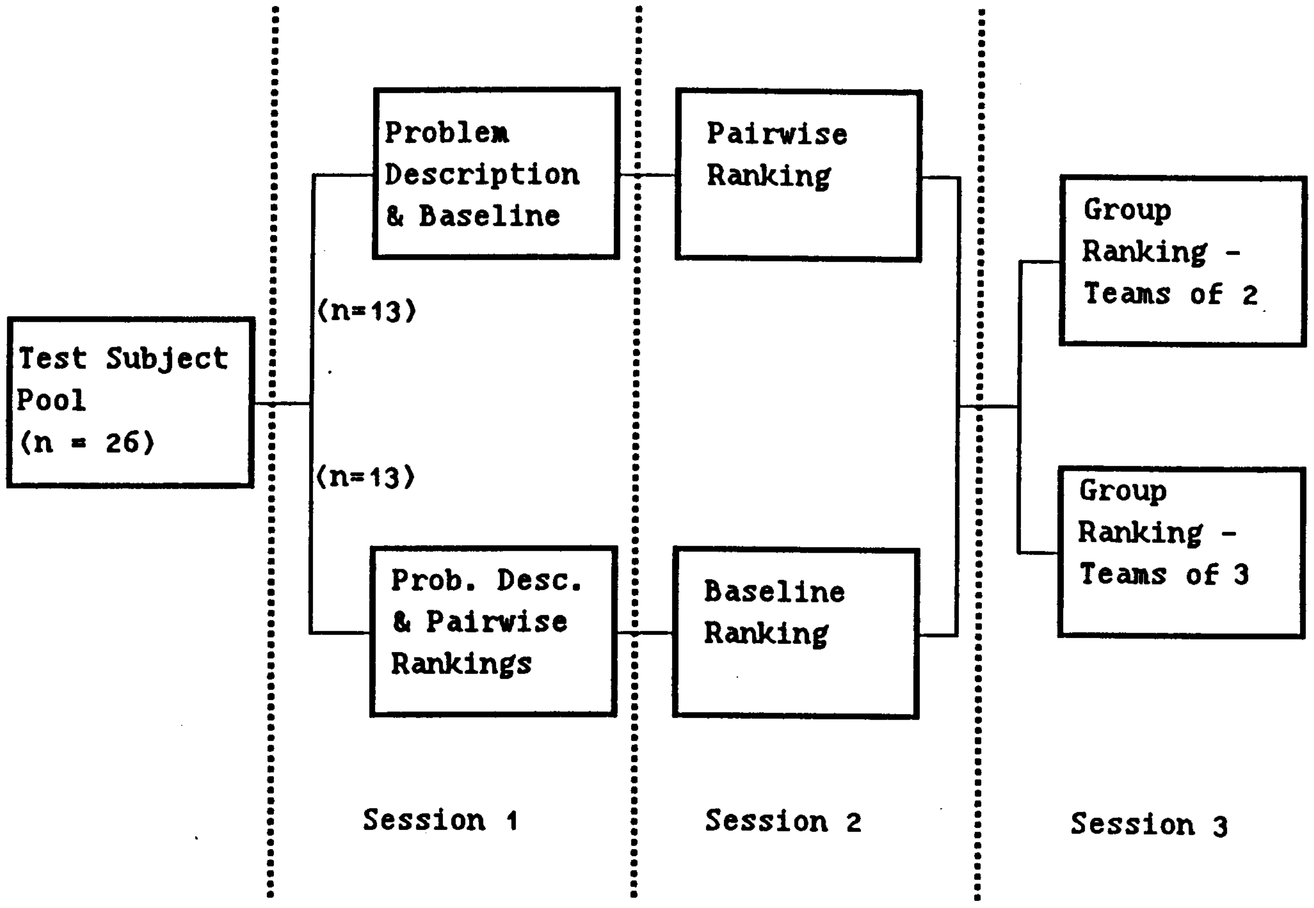

Figure 13. Questionnaire Administration
First Subject Group

Table 16. Kendall's Tau Scores for Group Decisions

Group	Problem	Saaty	Yager	Fuzzy Logic
1	1	0.000	1.000	0.667
2	1	-0.333	-0.667	-0.289
3	1	0.000	1.000	0.866
4	1	-0.333	0.000	-0.333
5	1	0.000	0.333	0.333
6	1	0.333	0.333	-0.408
7	1	-0.333	0.000	0.667
1	2	0.333	-0.333	-.913
2	2	-0.667	0.333	0.000
3	2	0.000	0.333	0.183
4	2	0.183	0.333	-0.548
5	2	-0.548	0.333	-0.913
6	2	0.000	0.913	0.548
7	2	-0.913	0.667	-0.183
1	3	0.333	1.000	-0.913
2	3	0.000	0.000	0.183
3	3	-0.333	0.333	0.289
4	3	0.667	0.667	-0.548
5	3	0.333	0.333	-0.913
6	3	0.000	0.000	0.000
7	3	0.333	-0.667	-0.183

Methods for aggregating the preferences of individual decision makers into a single, group decision are frequently evaluated on their possession of a set of characteristics or principles for rational decision making. While Arrow (1951) shows that no decision method may possess all of these principles, it will be seen that the three decision methods examined in this study do not support one principle that may have been used by the questionnaire respondents. The principles may be given as

1) Neutrality,
2) Independence of Irrelevant Alternatives,
3) Transitive Rationality,
4) Unrestricted Domain,
5) Anonymity.

The reader is referred to other references for discussions of these principles (e.g. Goicochea et al., 1982; Hillier and Lieberman, 1980). Of interest here is the principle of anonymity. This principle in effect states that each of the decision makers has an equal influence on the outcome of the group decision - the "no dominant figure" principle.

At least two of the decision methods considered here do not ascribe to this principle. In the application of Saaty's method, consider the following hypothetical example. Assume there exists four decision makers (D1, D2, D3, D4). Their preferences for a set of four alternatives are given in Table 17. If the decision makers are weighted equally in the aggregation process, then the resulting combined vector of weights for the preference of the alternatives is the average of the preference weights of the individuals. Thus, in this example, one decision maker may influence the outcome of the group's decision by unambiguously supporting one possible

Table 17a. Rankings of Alternatives for Four Hypothetical Decision Makers

	DECISION MAKER							
	D1		D2		D3		D4	
Alternative	weight	rank	weight	rank	weight	rank	weight	rank
A1	.35	(1)	.35	(1)	.35	(1)	.02	(4)
A2	.30	(2)	.30	(2)	.30	(2)	.03	(3)
A3	.25	(3)	.25	(3)	.25	(3)	.05	(2)
A4	.10	(4)	.10	(4)	.10	(4)	.90	(1)

Table 17b. Aggregate Rankings for Four Hypothetical Decision Makers

	METHOD			
	SAATY		YAGER	
Alternative	weight	rank	weight	rank
A1	.268	(2)	.02	(4)
A2	.233	(3)	.03	(3)
A3	.200	(4)	.05	(2)
A4	.300	(1)	.10	(1)

ordering of alternatives. Even though three of the four decision makers are unanimous in their ordering of the alternatives, the extremely strong "opinion" of a single decision maker may change the ordering of the alternatives.

This effect for Yager's method is even more demonstrable. Given that the decision makers weights are themselves weighted equally dictates that the solution to the aggregation problems is the maximin criterion. Thus in the example, the dissenting decision maker determines the order of three of the alternatives, and in effect determines the order of all four.

Strictly speaking perhaps, these characteristics may not violate the principle of anonymity as any of the decision makers in the group are free to express such strong beliefs as the one in the example. Neither is it apparent that such behavior does not, in some settings, actually occur. What is problematic is that there is evidence from the questionnaire results that an aggregation method of majority vote was used quite frequently. This seems to contradict some of the claims of the fuzzy decision analysis literature. This is not an attempt to imply that anonymity is necessary or that the aggregation methods of Saaty or Yager are not valid in the context of the decision problems used in this study. It is possible that the problem settings in the questionnaire, and the use of a questionnaire itself, may not sufficiently represent actual decision making environments. This issue is one of four discussed in the context of future research tasks presented in the final chapter.

CHAPTER 6

SUMMARY AND CONCLUSIONS

Summary.

Fuzzy set analysis has been offered as an alternative approach to decision making for ill-defined problems. More specifically, it has been claimed in the literature that fuzzy set theory may be suitable for problems in which the application of probability techniques are not appropriate due to violations of probabilistic theory. Further, it has been claimed that fuzzy set models more closely reflect actual decision behavior. These general claims form the basis for the study reported here.

In Chapter One, the axiomatic characteristics of both fuzzy set theory (fuzzy logic in particular) and probability theory were investigated. Two objectives were fulfilled. The first objective was to demonstrate that classical set theory and probability theory are subsets of fuzzy set theory. This is accomplished by showing that a valid restriction on fuzzy sets results in the support of the algebraic properties of classical sets and the axioms of probability. It was also shown that probability does not support all the axioms of fuzzy logic, demonstrating that the two theories are not equivalent. The implication that may be drawn is that from a theoretical standpoint, in decision problems where aspects of the problem violate probability axioms, fuzzy set theory, through a relaxation of probabilistic constraints, is an appropriate (although not necessarily unique) alternative.

The second objective was to identify some characteristic decision behaviors that violate probabilistic axioms but do not violate fuzzy set axioms. This represents a preliminary effort to identify more specific decision making environments in which the application of fuzzy sets theory is appropriate. Two environments were identified. In the first, it was shown that fuzzy set theory supports cases in which the preference of one element over another is not unambiguous. It was also shown that this condition violates axioms of probability theory. A related environment is that of transitivity. Here it was shown that fuzzy sets permitted some intransitivity in the ordering of elements. Again, this was shown to violate aspects of probability.

Chapter Two introduces several decision making methods based, to varying degrees, on fuzzy set techniques. These methods were the hierarchical pairwise comparison technique of Saaty (1977), the fuzzy decision making method of Yager (1978), and a fuzzy logic method based on the technique of Tsukamoto and Terano (1975). The derivation of these methods are given and some modifications and extensions to the fuzzy logic method are given.

Because the theoretical investigations of Chapter One and the methodological comparisons of Chapter Two do not in themselves demonstrate the applicability of fuzzy set based methods, Chapter Three describes the development and execution of a decision making experiment. The experiment was designed to test the hypothesis that there is a significant correlation between rankings of alternatives generated by the fuzzy set based techniques described in Chapter Two and direct rankings of alternatives made by test subjects. Correlation was used as a measure of the ability of the methods to replicate the decision behavior of the test subjects.

The questionnaire was administered to undergraduate and graduate students. The analysis of the questionnaire responses indicate that for the sample, the methods generated very few correlations that were significantly greater than zero. Further, no method consistently returned higher scores than another method. These results appear to contradict the claims for the methods made in the literature. There exist, however, several possible sources of poor performance. These include shortcomings in the methods themselves, and the effects of the assumptions made in the design of the experiment.

In the final Chapters, the assumptions used in the experiment were evaluated. It was determined that it would be valuable to expand the types of problem settings and the number and characteristics of the test subjects. Expansion of the scope of the study to include greater realism in the decision making environment is also desirable. This expansion, however, introduces a paradox into the evaluation procedure which is described below.

Future Research Needs.

The assumptions maintained in this study are not unique to the analysis of fuzzy set based decision methods. It is nonetheless desirable to test the validity of the assumptions or to be able to remove them. Future research efforts in this area, in the expansion of the nature of problem settings, and in the consideration of other techniques for fuzzy set based decision making would be valuable. Briefly, it would be fruitful to consider the following topics in future efforts.

First, the assessment of membership functions is a fundamental element of fuzzy set analysis and needs to be given further consideration. A primary characteristic of fuzzy set analysis is the incorporation of linguistic variables into the assessment procedure. Many of the fuzzy set methods for decision making either assume the presence of numeric representations (fuzzy sets) of the linguistic variables or use numerical representations directly. Research in this area should address the questions of whether a fuzzy set represents well the value of a linguistic variable; are any methods for developing fuzzy sets (e.g. direct assessment, pairwise comparisons, graphic representation) more suited than others; and are the fuzzy sets created by a sample general to the population. The investigation of these questions would be most useful in assessing the practical application of fuzzy set techniques.

The second topic for further investigation concerns tests of the appropriateness of the model of the decision problem. Although this study was concerned with investigating the appropriateness of models of the decision maker, as described in the introduction and elsewhere, it is clear that models of the decision problem are an important consideration. This is especially so as the experimental problem settings move toward "real-world" concerns. By appropriate is meant whether or not the model of the decision problem, that is, the alternatives and criteria under consideration, sufficiently (not necessarily completely) represent the actual decision problem. This study used an open question in the questionnaire to obtain an indication of appropriateness, as described in an earlier chapter. The format could be improved upon by direct interviewing with perhaps, repetitive problem solving, or perhaps by placing the subjects into a cooperative or adversarial gaming situation where it would be necessary for the subjects to make explicit all criteria and alternatives being considered.

A third topic for consideration is related to the previous topic. In the course of this study it was assumed that the student subjects were a sufficient model of actual decision makers. It would be more desirable to expand the study and include active representatives of the engineering decision making community. In a similar vein, it would be desirable to use less artificial problem settings, that is, problem settings that have real (or the illusion of real) consequences. This begins to raise something of a paradox, however. A motivating factor for the form of this study was difficulties arising from the use of real decision makers in real problem settings to obtain models of subjective judgment. Beyond problems arising from practicality, difficulties stem from problems of verifying or validating solutions. The more controlled environment of the artificial problem was therefore sought. This leads to the fourth and final topic.

In addition to testing the appropriateness of the decision models used in analysis, the results of this study have so far revealed that additional consideration be given to testing the appropriateness of the methods themselves. Several areas of concern regarding the solutions tendered by the methods examined in this study have been identified. If these areas are truly problematic then modifications to the methods should be made if the modifications are in keeping with the structure or theory of the method. If not, then other methods should perhaps be brought into consideration.

This study represents a preliminary investigation toward the empirical evaluation of several fuzzy set based techniques for multiobjective decision making under conditions of imprecision and uncertainty. To date, researchers have produced very little evidence to either support or refute claims made of the capabilities of fuzzy set techniques. This study is intended to contribute to this effort. Future research may serve to illuminate some of the concerns and issues raised here.

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APPENDIX ONE

QUESTIONNAIRE[1]

Multicriteria decision making is concerned with the ranking of a set of alternatives for a set of criteria or objectives. The ability to determine a ranking is hindered by many problems. First, there is frequently no single alternative that may satisfy all criteria better than other alternatives. Second, the satisfaction of a criterion may not be quantifiable, requiring the application of judgment. Thirdly, many decision makers may be involved in the decision process, each possessing views as to the relative importance of criteria and the desirability of the alternatives. In this questionnaire you will be asked to consider several examples of multicriteria problems and provide the requested information. It is important that you read the examples carefully before answering the questions.

Instructions for Use. This questionnaire will be administered in three parts. In this first part, you will receive a set of three problem descriptions, and a set of questions. For the second part of the questionnaire, you wil receive a different set of questions regarding the three problem descriptions. For these first two parts you are to answer the questions independently from others who are answering the questionnaire. In the third part of the questionnaire, you will be assigned to teams of either two or four persons. Several questions must then be answered by the group.

When the questionnaire has been completed, please return all three parts. The problems are designed to test several decision making methods. They are not designed to test your knowledge of the subject matter. It is expected that you will apply your own judgment, along with the information provided, to answer the questions that follow. There are no right or wrong answers. The first problem will illustrate the role of subjective judgment in decision making. Please answer all questions, giving reasonable time to consider the questions fully.

---

[1] Administered to first sample group.

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

In everyday settings multiobjective problems are encountered and even a simple example requires the consideration of conflicting goals and personal judgment.

In this problem you are asked to evaluate and rank several brands of vanilla ice cream. In determining a ranking, several criteria may be considered:

1. COST: The price of ice cream varies greatly from product to product and may reflect the quality of the ingredients.

2. FLAVOR: Since the alternatives are limited to vanilla, flavor refers to the balance between the vanilla flavoring and the cream components (sometimes referred to as "dairy flavor").

3. TEXTURE and BODY: The amount of air contained in ice creams varies greatly. Too little air makes an ice cream heavy, too much air makes an ice cream foamy. Difficulties in manufacture and storage and the absence of additives may affect the texture, making some ice creams icy rather than smooth and creamy.

The "ideal" ice cream would be one that is of low cost, with a distinct vanilla flavor unaffected by "off-flavors." The texture should be smooth and creamy. The ice cream should not be syrupy or bland.

Using the above criteria, the following alternatives are to be evaluated and ranked:

Alternative	Cost per Serving	Calories	% Butterfat
1. Baskin Robbins	24¢	149	11.8
2. Sealtest	15¢	120	9.9
3. Howard Johnsons	27¢	196	16.0
4. Haagen-Daz	44¢	267	15.4

Problem 2: WATER QUALITY MONITORING

Water quality monitoring is important for long-term trend assessment, parameter estimation, and enforcement of water quality standards. In monitoring for trend detection, it is usually recommended that sampling occur at fixed station sites, with sampling distributed uniformly over the monitoring period. With an emphasis on enforcement, it might be desirable to establish a complex network to maximize the chance of recording a violation of water quality standards, should one occur.

There are many factors that hinder the design of an effective monitoring program. First, the very reason for sampling is to estimate the behavior of a very complex stream system. Many of the biological and physical parameters influencing the monitoring process are poorly quantified. Further, it is generally expensive and time consuming to establish and maintain a monitoring network. The problem is to select a monitoring strategy that is best in terms of detecting impacts to the stream environment while minimizing the cost of doing so.

Problem Setting. A small, industrial treatment facility is located on a tributary of a river that supports a large commercial and sport fish industry. The industrial plant produces BOD as its major effluent but is also known to produce relatively small amounts of several toxic substances. The plant has over the last few years expanded its production and it is suspected that the design capacity of the facility is periodically exceeded. In addition to the commercial fisheries, there is a large recreational use of the stream. This use is primarily sport fishing, but also includes some boating, hiking, and camping activities.

In planning a monitoring network, four general criteria are used to assist in the selection of monitoring alternatives.

1. MINIMIZE COST: Regulatory agencies operate under a fixed budget and must allocate this budget among competing programs. It is therefore desirable to minimize the cost of each monitoring activity while maintaining the effective ness of the monitoring program.

2. PROTECT FISH SPECIES: The purpose of monitoring is to protect the health of the stream system, including the commercially valuable fish species. Therefore, any monitoring strategy must give careful consideration to the ability to capture violations that would have a deleterious effect on fish survival, reproduction, and commercial value.

3. PROTECT FROM LIABILITY: While it is important to be able to observe violations of water quality standards, it is also important to be correct when citing dischargers for violations. Possible errors arising from monitoring strategies must be considered. These include sampling error (which suggests the use of replicate observations) and incorrect background level pollution measurements (which may suggest long-term trend monitoring). Incorrect assessment of violations may lead to long-lasting legal action, and damaged credibility.

4. PROTECT RECREATION: For recreational fishing, long-term effects on the consumability of fish may be something of a problem, but maintenance of sport species in the stream is of primary concern. For the other recreational uses, long-term degredation may affect the quality of the stream (hence usage), while toxic doses of pollution may lead to fish kills which would also damage recreational enjoyment.

Four alternatives have been proposed for consideration:

1. TIME SPECIFIC: This alternative recognizes that fish species are particularly vulnerable to pollution effects during specific periods in the species life cycle. There is also a recognition that violations have a higher probability of occurance during low flow seasons due to reduced dilution. However, there is little evidence to suggest that low flow periods and sensitive periods occur simultaneously. Because sampling will occur frequently during the specified period (see table below), there is insufficient funding to provide for many sampling locations.

2. SPACE SPECIFIC: This alternative is composed of sampling stations widely distributed throughout the stream. At least one station will be located above the treatment facility while others will be concentrated in known fish habitat areas and also where it has been estimated that violations are most likely to occur. Because many stations must be located, there is insufficient funding to sample frequently.

3. INTENSIVE: This alternative is designed to maximize the chance of observing a violation of water quality standards by locating several stations within a relatively small area, and sampling as frequently as possible during a specified period. The locations are established from the average downstream mile of minimum dissolved oxygen levels (computed from prior sampling data). The timing is based on the regional low flow period.

4. BASELINE: This alternative is designed to maximize the probability of detecting longer-term changes in water quality. As such, it is composed of a few, widely

distributed stations, with regular sampling throughout the year. Because there are relatively few stations, this alternative is the least costly. It is less likely, however, that violations of stream standards will be recorded.

Alternative	Cost	Station Location	Sampling Frequency
Time Specific	$45,000	1 @ 10 mi. 1 @ 40 mi. 1 @ 70 mi.	once a week from July 1 to December 1
Location Specific	$42,000	1 mi. upstream 10 mi., 20 mi., 30 mi., 40 mi., 60 mi., 80 mi., 100 mi.	once per month
Intensive Alternative	$56,000	same as 2	once a week from July 1 to November 1
Baseline	$37,200	1 mi. upstream 20 mi., 40 mi., 60 mi., 80 mi., 100 mi.,	once per month with replicate sampling

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

Peaks Branch is a stream in the upper Trinity River Basin. The lower portion of Peaks Branch traverses a low-cost housing neighborhood near the downtown of a major city. The present channel is designed to carry a five-year frequency flood. Floods of a greater magnitude would cover a broad, fully developed flood plain.

The stream experiences three types of flooding: backwater flooding from the Trinity River; backwater flooding from White Rock Creek (of which Peaks Branch is a tributary); and headwater flooding caused by runoff on the watershed. A 1908 flood on the Trinity River caused a backwater flood elevation of 409.2 feet above msl in Peaks Branch, but construction of reservoirs on the river have reduced the probability of a similar occurence. Backwater from White Rock Creek has reached elevations of 404.3 feet above msl and the greatest headwater flood on the stream reached an elevation of 411.0 feet above msl. The five year design discharge is 5000 cfs, the 100 year discharge is 11,000 cfs.

Evaluation Criteria:

1. FLOOD PROTECTION. Flood protection is an obvious goal of the project. Open channel floodways provide a greater leeway of safety than do closed conduits. Paved channels are more reliable than grass-covered channels. Concrete channels require little or no periodic maintenance, whereas grass-lined channels must be maintained to support their design conveyance. However, grass-lined floodways can add attractive and useful open space to urban areas.

2. NEIGHBORHOOD ENHANCEMENT. The city is striving to maintain residential neighborhoods near the central business district. The prevention of flooding enhances property values in flood prone areas. Multiple use of flood plains enhances neighborhoods through the provision of open space and recreational areas. Open space may also make an area more desirable, thus increasing property values.

3. PROJECT AND MAINTENANCE COSTS. Funds for this project will come from the sale of bonds and if possible, State and Federal programs. The least expensive design may be attractive if the current financial outlook is uncertain.

4. NEIGHBORHOOD ACCEPTANCE. Local opposition can severely delay or prevent the implementation of any project. Any plan that requires the displacement of residents can expect opposition from at least those who will be displaced. On the other hand, redevelopment provides the opportunity to remove substandard housing. Many families do not believe the threat

of flooding exists, and others would like to see the removal of as many apartments as possible.

Flood Plain Management Alternatives:

1. NO ACTION. Potential losses, computed as a function of depth of flooding and housing value, were estimated from the Residence Flood Damage tables of the United States Department of Agriculture. The five year flood would cause no appreciable damage. The 25 year flood would cause damages of about $19,000. A 50 year flood would result in damages of approximately $150,000, and the 100 year flood would inflict property damage of $1,300,000.

The no action alternative would not aid in the improvement of housing quality in the area but the purchase of flood insurance subsidized by the Federal Flood Insurance Program would permit some losses to be recouped and homeowners would be eligible for lower interest home improvement loans. The project cost of this alternative is zero, and annual maintenance costs are estimated at $12,000.

2. A CONCRETE DRAINAGE CHANNEL. This alternative would widen the Peaks Branch channel. Additional right-of-way would require the purchase of 74 structures, displacing 69 families. The affected area would be used for interior drainage facilities and a sump for the storage of flood water. A levee would separate the sump, with a surface area of 6.4 acres, from the channel. A gravity sluice and a pump station with a capacity of 10,000 gpm would drain the sump when the stream level was low and high, respectively.

The proposed concrete-lined channel has 2:1 side slopes with 10 foot wide berms parallel to, and 8 feet above, the flow line. The channel bottom is 60 feet wide. The project cost of this alternative, including right-of-way and relocations, is $5,600,000. Annual maintenance costs are estimated at $16,000. The concrete channel is believed to be very reliable with respect to it ability to convey design floods over the project life. The benefits associated with this alternative accrue primarily through the elimination of damages by floods up to the 100 year flood.

3. A PARK GREENWAY. This alternative would widen the Peaks Branch channel. Four lakes, hiking and bicycle trails, and parkways would be constructed. Additional right-of-way requirements would entail the purchase of 164 structures (mostly apartments), displacing 419 families. Most of these structures are in a low area that can only be protected from the 100 year flood by levees, gravity sluices, and stormwater pumps. The project cost of $10,900,000 includes the purchase of property, demolition, and relocation expenses. Annual maintenance costs are $41,000.

The detention basins and grass floodways may be less reliable than more structural remedies if maintenance is neglected. Assuming maintenance is adequate, benefits accruing to this alternative include the offset damages from flooding, the provision of open space and recreation opportunities, and a possible resulting increase in the value of adjacent properties.

4. PURCHASE AND REDEVELOPMENT OF THE FLOOD PLAIN. In this plan 524 structures would be purchased and cleared from the floodplain, requiring the displacement of 940 families. A greenbelt, similar to that described in alternative three would be constructed on part of the aquired land. A new development, including a 10 acre shopping center, 336 residential lots and 40 additional acres of parks would be constructed on the remaining land. The land for the shopping center would be sold with the requirement for the provision of neighborhood facilities. Reconstruction would consist of single or duplex residential units with preference given to displaced landowners.

The condemnation of land for resale is prohibited by state statute, except under an urban renewal statute that requires voter approval. The estimated cost of this alternative is $16,300,000, including the cost of aquiring right-of-way, demolition, and relocation. Maintenance costs are estimated at $41,000/year.

The benefits from this alternative are more difficult to assess in that the alternative results in a complete transformation of the neighborhood. In addition to offsetting flood damages, benefits may accrue from the development of park and recreational facilities, profits from the sale of land for housing and the shopping center, and improved housing stock and neighborhood housing values.

Part 1.

Name________________________________

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

1. Please rank the four ice creams ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

Alternative	Rank
1. Baskin Robbins	__________
2. Sealtest	__________
3. Howard Johnsons	__________
4. Haagen-Daz	__________

2. Because there may be some uncertainty about an alternative, an estimate of your certainty regarding your ranking of the alternatives is requested. Since any one alternative may be assigned any of four ranks, utilize the scale given below to indicate the certainty with which an alternative is assigned the given rank.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

EXAMPLE: Alternative 1 may be ranked first with high certainty, second with moderate certainty, third with low certainty and fourth with no certainty. This could be indicated as

1-Baskin Robbins _.7_ 1, _.5_ 2, _.2_ 3, _0_ 4

Indicate the certainty associated with each possible rank assignment:

Alternative	Certainty associated with rank			
1-Baskin Robbins	_____ 1,	_____ 2,	_____ 3,	_____ 4
2-Sealtest	_____ 1,	_____ 2,	_____ 3,	_____ 4
3-Howard Johnsons	_____ 1,	_____ 2,	_____ 3,	_____ 4
4-Haagen Daz	_____ 1,	_____ 2,	_____ 3,	_____ 4

Problem 2: WATER QUALITY MONITORING

1. Please rank the four water quality monitoring alternatives ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

Alternative Rank

1. Time Specific ....__________
2. Space Specific ....__________
3. Intensive ....__________
4. Baseline ....__________

2. Because there may be some uncertainty about the performance of an alternative an estimate of your certainty regarding your ranking of the alternatives is requested.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

Alternative	Certainty associated with rank
1-Time Specific	____ 1, ____ 2, ____ 3, ____ 4
2-Space Specific	____ 1, ____ 2, ____ 3, ____ 4
3-Intensive	____ 1, ____ 2, ____ 3, ____ 4
4-Baseline	____ 1, ____ 2, ____ 3, ____ 4

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

1. Please rank the four flood plain management alternatives ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

Alternative	Rank
1. No Action	________
2. Concrete Channel	________
3. Parkway	________
4. Redevelop Floodplain	________

2. Because there may be some uncertainty about the performance of an alternative, an estimate of your certainty regarding your ranking of the alternatives is requested. Since any one alternative may be assigned any of four ranks, utilize the scale given below to indicate the certainty with which an alternative is assigned the given rank.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

Alternative	Certainty associated with rank
1-No Action	____ 1, ____ 2, ____ 3, ____ 4
2-Concrete Channel	____ 1, ____ 2, ____ 3, ____ 4
3-Parkway	____ 1, ____ 2, ____ 3, ____ 4
4-Redevelop Floodplain	____ 1, ____ 2, ____ 3, ____ 4

Part II.

Name________________________________

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you evaluate pairs of alternatives with respect to each criterion. To assist you in this evaluation a comparison scale is provided:

1 - it is uncertain that alternative A is preferred to alternative B
3 - alternative A may be preferred to alternative B
5 - alternative A is probably preferred to alternative B
7 - alternative A is almost definately preferred to alternative B
9 - alternative A is absolutely preferred to alternative B

Even numbers denote intermediate ranges of certainty. Reciprocals indicate that the order of certainty of preference is reversed.

EXAMPLE: if Alternative 1 may be preferred to Alternative 2, a 3 is indicated. If Alternative 2 may be preferred to Alternative 1, a 1/3 is indicated.

1. With respect to an ice cream's cost, how certain is alternative A preferred to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

2. With respect to an ice cream's flavor, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

3. With respect to the texture and body of an ice-cream, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

4. Finally, it is important to obtain an indication of your belief in the relative significance of the evaluation criteria. With respect to the selection of a best overall ice cream, how certain is the significance of criterion A over criterion B?

A	B	Comparison
1	2	____________
1	3	____________
2	3	____________

Problem 2: WATER QUALITY MONITORING

Part II. You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you evaluate pairs of alternatives with respect to each criterion. To assist you in this evaluation a comparison scale is provided:

1 - it is uncertain that alternative A is preferred to alternative B
3 - alternative A may be preferred to alternative B
5 - alternative A is probably preferred to alternative B
7 - alternative A is almost definately preferred to alternative B
9 - alternative A is absolutely preferred to alternative B

Even numbers denote intermediate ranges of certainty. Reciprocals indicate that the order of certainty of preference is reversed.

1. With respect to an alternative's ability minimize the cost of monitoring, how certain is alternative A preferred to alternative B?

A	B	Comparison
1	2	
1	3	
1	4	
2	3	
2	4	
3	4	

2. With respect to an alternative's ability to protect commercial fish species, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

3. With respect to the ability to protect the regulating agency against false accusation, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

4. With respect to the ability of an alternative to protect recreational uses of the stream, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	
1	3	
1	4	
2	3	
2	4	
3	4	

5. Finally, it is important to obtain an indication of your belief in the relative importance of the evaluation criteria. With respect to the selection of a best overall monitoring policy how certain is the importance of criterion A over criterion B?

A	B	Comparison
1	2	
1	3	
1	4	
2	3	
2	4	
3	4	

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you compare pairs of alternatives with respect to each criterion. To assist you in this evaluation a comparison scale is provided:

1 - it is uncertain that alternative A is preferred to alternative B
3 - alternative A may be preferred to alternative B
5 - alternative A is probably preferred to alternative B
7 - alternative A is almost definately preferred to alternative B
9 - alternative A is absolutely preferred to alternative B

Even numbers denote intermediate ranges of certainty. Reciprocals indicate that the order of certainty of preference is reversed.

1. With respect to an alternative's ability to reliably prevent flood damages for a 100 year frequency flood, how certain is alternative A preferred to alternative B?

A	B	Comparison
1	2	___________
1	3	___________
1	4	___________
2	3	___________
2	4	___________
3	4	___________

2. With respect to an alternative's ability to enhance the desirability and value of the flood plain, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

3. With respect to ability to finance the construction and operation of an alternative, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

4. With respect to the ability of an alternative to maximize neighborhood acceptance of a project, how certain is the preference of alternative A to alternative B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

5. Finally, it is important to obtain an indication of your belief in the relative significance of the evaluation criteria. With respect to the selection of a best overall alternative, how certain is the significance of criterion A over criterion B?

A	B	Comparison
1	2	____________
1	3	____________
1	4	____________
2	3	____________
2	4	____________
3	4	____________

Part III. GROUP ASSESSMENTS

Name________________________________

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

Considering your individual rankings for the four ice cream alternatives, you are now asked to reach consensus on the preferred ranking within the group to which you have been assigned.

(To be answered by each group.) 1. Please rank the four ice cream brands ordinally from 1 to 4, with 1 = most preferred, with respect to the evaluation criteria and the consensus of the group.

Alternative	Rank
1. Baskin Robbins	__________
2. Sealtest	__________
3. Howard Johnsons	__________
4. Haagen Daz	__________

2. For each alternative indicate the group's confidence in the rank that has been assigned.

Alternative	Certainty associated with rank			
1-Baskin Robbins	____ 1,	____ 2,	____ 3,	____ 4
2-Sealtest	____ 1,	____ 2,	____ 3,	____ 4
3-Howard Johnsons	____ 1,	____ 2,	____ 3,	____ 4
4-Haagen Daz	____ 1,	____ 2,	____ 3,	____ 4

(To be answered by each individual.) 3a. Assume that you are able to reconsider this ranking problem and have the opportunity to consider additional criteria that might affect the choice of alternatives. What additional criteria would you consider important in evaluating these alternatives?

3b. If these additional criteria were considered, what would be the certainty associated with each alternative's rank?

Alternative	Certainty associated with rank
1-Baskin Robbins	____ 1, ____ 2, ____ 3, ____ 4
2-Sealtest	____ 1, ____ 2, ____ 3, ____ 4
3-Howard Johnsons	____ 1, ____ 2, ____ 3, ____ 4
4-Haagen Daz	____ 1, ____ 2, ____ 3, ____ 4

4. Please indicate ranks for the alternatives obtained by yourself and by your group:

Yourself

Alternative	Rank
1	_____
2	_____
3	_____
4	_____

Your Group

Alternative	Rank
1	_____
2	_____
3	_____
4	_____

No Consensus Achieved _____

Problem 2: WATER QUALITY MONITORING

Part III. GROUP ASSESSMENTS: Considering your individual rankings for the four stream quality monitoring alternatives, you are now asked to reach consensus on the preferred ranking within the group to which you have been assigned.

(To be answered by each group.) 1. Please rank the four stream quality monitoring alternatives ordinally from 1 to 4, with 1 = most preferred, with respect to the evaluation criteria and the consensus of the group.

Alternative	Rank
1. Time Specific	________
2. Space Specific	________
3. Intensive	________
4. Baseline	________

2. For each alternative indicate the group's confidence in the rank that has been assigned.

Alternative	Certainty associated with rank			
1-Time Specific	____ 1,	____ 2,	____ 3,	____ 4
2-Space Specific	____ 1,	____ 2,	____ 3,	____ 4
3-Intensive	____ 1,	____ 2,	____ 3,	____ 4
4-Baseline	____ 1,	____ 2,	____ 3,	____ 4

(To be answered by each individual.) 3a. Assume that you are able to reconsider this ranking problem and have the opportunity to consider additional factors that might affect the choice of alternatives. What additional criteria would you consider important in considering these alternatives?

3b. If these additional criteria were considered, what would be the certainty associated with each alternative's rank?

Alternative	Certainty associated with rank
1-Time Specific	____ 1, ____ 2, ____ 3, ____ 4
2-Space Specific	____ 1, ____ 2, ____ 3, ____ 4
3-Intensive	____ 1, ____ 2, ____ 3, ____ 4
4-Baseline	____ 1, ____ 2, ____ 3, ____ 4

4. Please indicate ranks for the alternatives obtained by yourself and by your group:

Yourself

Alternative	Rank
1	______
2	______
3	______
4	______

Your Group

Alternative	Rank
1	______
2	______
3	______
4	______

No Consensus Achieved ______

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

Part III. GROUP ASSESSMENTS. Considering your individual rankings for the four flood plain management alternatives, you are now asked to reach consensus on the preferred ranking within the group to which you have been assigned.

(To be answered by each group.) 1. Please rank the four flood plain management alternatives ordinally from 1 to 4, with 1 = most preferred, with respect to the evaluation criteria and the consensus of the group.

Alternative	Rank
1. No Action	__________
2. Concrete Channel	__________
3. Parkway	__________
4. Redevelop Floodplain	__________

2. For each alternative indicate the group's confidence in the rank that has been assigned.

Alternative	Certainty associated with rank			
1-No Action	____ 1,	____ 2,	____ 3,	____ 4
2-Concrete Channel	____ 1,	____ 2,	____ 3,	____ 4
3-Parkway	____ 1,	____ 2,	____ 3,	____ 4
4-Redevelop Floodplain	____ 1,	____ 2,	____ 3,	____ 4

(To be answered by each individual.) 3a. Assume that you are able to reconsider this ranking problem and have the opportunity to consider additional factors that might affect the choice of alternatives. What additional criteria would you consider important in considering these alternatives?

3b. If these additional criteria were considered, what would be the certainty associated with each alternatives rank?

Alternative	Certainty associated with rank			
1-No Action	____ 1,	____ 2,	____ 3,	____ 4
2-Concrete Channel	____ 1,	____ 2,	____ 3,	____ 4
3-Parkway	____ 1,	____ 2,	____ 3,	____ 4
4-Redevelop Floodplain	____ 1,	____ 2,	____ 3,	____ 4

4. Please indicate ranks for the alternatives obtained by yourself and by your group:

Yourself		Your Group	
Alternative	Rank	Alternative	Rank
1	____	1	____
2	____	2	____
3	____	3	____
4	____	4	____

No Consensus Achieved ____

5. Names of other members in your group.

1. ______________________________
2. ______________________________
3. ______________________________

QUESTIONNAIRE[2]

Multicriteria decision making is concerned with the ranking of a set of alternatives for a set of criteria or objectives. The ability to determine a ranking is hindered by many problems. First, there is frequently no single alternative that may satisfy all criteria better than other alternatives. Second, the satisfaction of a criterion may not be quantifiable, requiring the application of judgment. Third, many decision makers may be involved in the decision process, each possessing views as to the relative importance of criteria and the desirability of the alternatives. In this questionnaire you will be asked to consider several examples of multicriteria problems and provide the requested information. It is important that you read the examples carefully before answering the questions.

Instructions for Use. This questionnaire will be administered in two parts. In this first part, you will receive a set of three problem descriptions, and a set of questions. For the second part of the questionnaire, you will receive a different set of questions regarding the three problem descriptions. For these first two parts you are to answer the questions independently from others who are answering the questionnaire.

When the questionnaire has been completed, please return both parts. The problems are designed to test several decision making methods. They are not designed to test your knowledge of the subject matter. It is expected that you will apply your own judgment, along with the information provided, to answer the questions that follow. There are no right or wrong answers. The first problem will illustrate the role of subjective judgment in decision making. Please answer all questions, giving reasonable time to consider the questions fully.

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[2] Administered to second sample group. Only sections different from the first sample group are included.

Part 1.

Name________________________________

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

1. Please rank the four ice creams ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

Alternative Rank

1. Baskin Robbins ....__________
2. Sealtest ....__________
3. Howard Johnsons ....__________
4. Haagen-Daz ....__________

2. Because there may be some uncertainty about an alternative, an estimate of your certainty regarding your ranking of the alternatives is requested. Utilize the scale given below to indicate the certainty with which an alternative is assigned the given rank.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

EXAMPLE: Alternative 1 may be ranked with moderate certainty. This could be indicated as

1-Baskin Robbins _.5_

Indicate the certainty associated with each rank assignment:

Alternative Certainty associated with rank

1-Baskin Robbins ____

2-Sealtest ____

3-Howard Johnsons ____

4-Haagen Daz ____

Problem 2: WATER QUALITY MONITORING

1. Please rank the four water quality monitoring alternatives ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

<u>Alternative Rank</u>

1. Time Specific ....__________
2. Space Specific ....__________
3. Intensive ....__________
4. Baseline ....__________

2. Because there may be some uncertainty about the performance of an alternative an estimate of your certainty regarding your ranking of the alternatives is requested.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

<u>Alternative Certainty associated with rank</u>

1-Time Specific ____

2-Space Specific ____

3-Intensive ____

4-Baseline ____

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

1. Please rank the four flood plain management alternatives ordinally from 1 to 4 with 1 = most preferred, with respect to their satisfaction of the stated criteria.

Alternative	Rank
1. No Action	______
2. Concrete Channel	______
3. Parkway	______
4. Redevelop Floodplain	______

2. Because there may be some uncertainty about the performance of an alternative, an estimate of your certainty regarding your ranking of the alternatives is requested.

Alternative	Certainty associated with rank
1-No Action	____
2-Concrete Channel	____
3-Parkway	____
4-Redevelop Floodplain	____

Part II.

Name________________________________

Problem 1: SELECTION OF A FAVORED BRAND OF ICE CREAM

You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you evaluate each alternative with respect to each criterion. Utilize the scale given below to indicate the certainty with which an alternative satifies the requirements of each criterion.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

EXAMPLE: if it is highly certain that Alternative 1 satisfies the criterion of low cost, assign
Alternative 1 .8

1. With how much certainty does each alternative satisfy the criterion of low cost?

Alternative	Certainty
1-Baskin Robbins	_____
2-Sealtest	_____
3-Howard Johnsons	_____
4-Haagen Daz	_____

2. With how much certainty does each alternative satisfy the criterion of flavor ?

Alternative	Certainty
1-Baskin Robbins	
2-Sealtest	
3-Howard Johnsons	
4-Haagen Daz	

3. With how much certainty does each alternative satisfy the criterion of texture and body of an ice-cream?

Alternative	Certainty
1-Baskin Robbins	
2-Sealtest	
3-Howard Johnsons	
4-Haagen Daz	

4. Finally, it is important to obtain an indication of your belief in the relative significance of the evaluation criteria. With respect to the selection of a best overall ice cream, how certain is the significance of each criterion?

Criterion	Certainty
1. Cost	
2. Flavor	
3. Texture and Body	

Problem 2: WATER QUALITY MONITORING

Part II. You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you evaluate each alternative with respect to each criterion. Utilize the scale given below to indicate the certainty with which an alternative satifies the requirements of each criterion.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

EXAMPLE: if it is highly certain that Alternative 1 satisfies the criterion of low cost, assign
Alternative 1 _.8_

1. With how much certainty does each alternative minimize the cost of monitoring?

Alternative	Certainty
1-Time Specific	____
2-Space Specific	____
3-Intensive	____
4-Baseline	____

2. With how much certainty does each alternative protect commercial fish species?

Alternative	Certainty
1-Time Specific	____
2-Space Specific	____
3-Intensive	____
4-Baseline	____

3. With how much certainty does each alternative protect the regulating agency against false accusation?

Alternative	Certainty
1-Time Specific	____
2-Space Specific	____
3-Intensive	____
4-Baseline	____

4. With how much certainty does each alternative protect recreational uses of the stream?

Alternative	Certainty
1-Time Specific	____
2-Space Specific	____
3-Intensive	____
4-Baseline	____

5. Finally, it is important to obtain an indication of your belief in the relative importance of the evaluation criteria. With respect to the selection of a best overall monitoring policy how certain is the importance of each criterion?

Criterion	Certainty
1. Minimize Cost	__________
2. Protect Fish	__________
3. Protect Agency	__________
4. Protect Recreation	__________

Problem 3: URBAN FLOOD PLAIN MANAGEMENT

You are requested to provide in greater detail your preferences for the listed alternatives. Because the evaluation criteria play a significant role in the final selection of an alternative and because there may be some uncertainty in the performance of an alternative with respect to a criterion, it is requested that you evaluate each alternative with respect to each criterion. Utilize the scale given below to indicate the certainty with which an alternative satifies the requirements of each criterion.

1 -- .9 -- .8 -- .7 -- .6 -- .5 -- .4 -- .3 -- .2 -- .1 -- 0
complete high moderate low no

EXAMPLE: if it is highly certain that Alternative 1 satisfies the criterion of low cost, assign
Alternative 1 .8

1. With how much certainty does each alternative reliably prevent flood damages for a 100 year frequency flood?

Alternative	Certainty
1-No Action	_____
2-Concrete Channel	_____
3-Parkway	_____
4-Redevelop Floodplain	_____

2. With how much certainty does each alternative enhance the desirability and value of the flood plain?

Alternative	Certainty
1-No Action	_____
2-Concrete Channel	_____
3-Parkway	_____
4-Redevelop Floodplain	_____

3. With respect to ability to finance the construction and operation of an alternative, with how much certainty is the criterion satisfied?

Alternative	Certainty
1-No Action	_____
2-Concrete Channel	_____
3-Parkway	_____
4-Redevelop Floodplain	_____

4. With how much certainty does each alternative maximize neighborhood acceptance of a project?

Alternative	Certainty
1-No Action	_____
2-Concrete Channel	_____
3-Parkway	_____
4-Redevelop Floodplain	_____

5. Finally, it is important to obtain an indication of your belief in the relative significance of the evaluation criteria. With respect to the selection of a best overall alternative, how certain is the significance of each criterion?

Criterion	Comparison
1. Flood Prevention	
2. Value Enhancement	
3. Cost	
4. Neighborhood Acceptibility	

APPENDIX TWO

COMPUTER PROGRAMS

```
C*************************************************************
C    MAIN PROGRAM FOR FUZZY LOGIC METHOD
C*************************************************************
     REAL S(4), X(4), R(4,4), T(4,4), SOL(4,2), E(4,4)
     REAL UBS(4), U(4,4), V(4,4), W(4,4,2)
     CHARACTER*12 NAME

C    M = NUMBER OF ALTERNATIVES
C    N = NUMBER OF CRITERIA

     DATA R /1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
    1        1., 1., 1., 1., 1., 1./

     OPEN(2,FILE='FUZ.OUT',STATUS='NEW')

99   WRITE(*,*) 'ENTER FILENAME'
     READ(*,101) NAME
     IF (NAME .EQ. 'STOP') STOP
     OPEN(1,FILE=NAME,STATUS='OLD')
     WRITE(2,*) ' '
     WRITE(2,101) NAME

     M = 4
     N = 4
     DO 1 I = 1,M
     DO 1 J = 1,N
        READ(1,*) T(I,J)
1    CONTINUE

     DO 2 J = 1,N
        READ(1,*) X(J)
2    CONTINUE

     CALL SOLVE (M, N, R, T, X, SOL)
     GOTO 99

101  FORMAT(A12)

     STOP
     END
```

```
C
************************************************************
C    SUBROUTINE FOR SOLUTION OF UPPER AND LOWER BOUNDS
C***********************************************************

     SUBROUTINE SOLVE (M, N, R, T, X, SOL)
C
C    ASSUME PRESENCE OF RELATION MATRICES
C    R(m,n),  T(m,n)
C    where
C       m = # OF ALTERNATIVES
C       n = # OF CRITERIA
C
C
     REAL R(3,4), T(3,4), X(4), E(3,4), UBS(4), U(3,4,2)
     REAL V(4,4,2), W(4,4,2), SOL(4,2), RMAX(4), LBS(4)
     INTEGER JMAX(4), K(4)

C    ***** FIND UPPER BOUND SOLUTION *****

1    DO 10 I = 1, M
     DO 10 J = 1, N
        E(I,J) = AMIN1(1., (X(J) + 1. - T(I,J)))
10   CONTINUE

     DO 20 I = 1, M
        XMIN = 99.
        DO 25 J = 1, N
           IF (E(I,J) .LT. XMIN) XMIN = E(I,J)
25      CONTINUE
        UBS(I) = XMIN
        WRITE(*,*) 'UPPER BOUND SOLUTION =', UBS(I)
20   CONTINUE

C    ***** FIND LOWER BOUND SOLUTIONS *****
C
C    ***** DEFINE U(I,J) AND V(I,J) *****
C              | [X(J)]        IF R(I,J) > X(J)
C    U(I,J) =  | [X(J), 1]     IF R(I,J) = X(J)
C              | [0]           IF R(I,J) < X(J)
C
C    V(I,J) =  | [0, X(J)]     IF R(I,J) > X(J)
C              | [0]           IF R(I,J) <= X(J)
C
     DO 30 I = 1, M
     DO 30 J = 1, N
        IF (R(I,J) .GT. X(J)) THEN
                U(I,J,1) = X(J)
                U(I,J,2) = X(J)
                V(I,J,1) = 0.0
                V(I,J,2) = X(J)
```

```
            END IF
            IF (R(I,J) .EQ. X(J)) THEN
                    U(I,J,1) = X(J)
                    U(I,J,2) = 1.0
                    V(I,J,1) = 0.0
                    V(I,J,2) = 0.0
            END IF
            IF (R(I,J) .LT. X(J)) THEN
                    U(I,J,1) = 0.0
                    U(I,J,2) = 0.0
                    V(I,J,1) = 0.0
                    V(I,J,2) = 0.0
            END IF

 30      CONTINUE

 C       ***** DEFINE W(I,J) *****
 C
 C       **** K = Kth COMBINATION SATISFYING:
 C       **** W(I,J,K) = U(I,J) IF U(I,J,1) .ne. 0
 C                     = V(I,J) ELSE
 C
         DO 40 J = 1,N
         DO 40 I = 1,M
            IF (U(I,J,1) .NE. 0) THEN
               W(I,J,1) = U(I,J,1)
               W(I,J,2) = U(I,J,2)
            ELSE
               W(I,J,1) = V(I,J,1)
               W(I,J,2) = V(I,J,2)
            END IF
 40      CONTINUE

 C
 C       **** FIND 'K' SOLUTIONS
 C       **** K = (# NON-ZERO REPLICATES IN COL. 1)*
 C       ****     (# NON-ZERO REPLICATES IN COL. 2)* ...
 C
         K2 = 0
         K1= 1

         DO 42 J = 1,N
            K(J) = 1
            DO 43 I = 1,M
               IF (W(I,J,1) .EQ. 0) GOTO 43
               DO 44 IA = I+1,M
                  IF (W(I,J,1) .EQ. W(IA,J,1)) K(J) = K(J)+1
 44            CONTINUE
               GOTO 45
 43         CONTINUE
            K1 = K1 * K(J)
```

```
42    CONTINUE

      DO 50 I = 1,M
51       RMAX(I) = 0.
         DO 60 J = 1,N
            IF (W(I,J,1) .GT. RMAX(I)) THEN
               RMAX(I) = W(I,J,1)
               JMAX(I) = J
            END IF
60       CONTINUE
C
         IF (RMAX(I) .EQ. 0.) GOTO 50
         IF (RMAX(I) .GT. UBS(I)) THEN
            K2 = K2 + 1
            W(I,JMAX(I),1) = 0.
            GOTO 51
         END IF
C
         LBS(I) = RMAX(I)
50    CONTINUE

C     **** SOLUTION FOUND
         DO 68 I = 1, M
            SOL(I,1) = LBS(I)
            SOL(I,2) = UBS(I)
68       CONTINUE
C     **** TEST FOR INCONSISTENT SOLUTION
         IF (K2 .GE. K1) THEN
            WRITE(*,*) 'INCONSISTANT SOLUTION'
            RETURN
         ELSE
            DO 69 I = 1,M
               WRITE(2,690) SOL(I,1),SOL(I,2)
69          WRITE(*,*) SOL(I,1), SOL(I,2)
         ENDIF

64    CONTINUE

690   FORMAT(2(1X,F5.3))

      RETURN
      END
```

```
C     *****************************************************
C     MAIN PROGRAM FOR YAGER'S METHOD
C     *****************************************************
C     C = ARRAY OF WEIGHTS (FROM SAATY PROGRAM) FOR EACH
C     ALTERNATIVE RELATIVE TO EACH CRITERION
C     P = VECTOR OF WEIGHTS FOR EACH CRITERION (FROM SAATY)
C
      REAL C(4,4), P(4), D(4,4), DMIN(4)
      CHARACTER*12 NAME
      OPEN (2, FILE='YAGER.OUT',STATUS='NEW')

99    WRITE(*,*) 'ENTER FILE NAME'
      READ(*,1) NAME
      IF (NAME .EQ. 'STOP') STOP
1     FORMAT(A12)
      OPEN (1, FILE=NAME,STATUS='OLD')
      WRITE(2,*) ' '
      WRITE(2,1) NAME
      WRITE(*,*) 'ENTER NUMBER OF CRITERIA  '
      READ(*,*) NCRIT
      NALT = 4

      DO 10 I = 1, NALT
         DMIN(I) = 10.
      DO 10 J = 1, NCRIT
         N = N+1
10    READ(1,*) C(I,J)

      DO 20 J = 1, NCRIT
20    READ(1,*) P(J)

      DO 30 I = 1, NALT
      DO 30 J = 1, NCRIT
         D(I,J) = C(I,J)**P(J)
30    CONTINUE

      DO 40 I = 1, NALT
      DO 40 J = 1, NCRIT
         IF (D(I,J) .LT. DMIN(I)) DMIN(I) = D(I,J)
40    CONTINUE

      DO 50 I = 1, NALT
         WRITE(2,200) DMIN(I)
50    CONTINUE
      WRITE(2,*) ' '
      WRITE(2,1) NAME
      CLOSE(1)
      GOTO 99
200   FORMAT(1X,F8.5)
690   FORMAT(2(1X,F5.3))
      END
```

```
C     PROGRAM TO CALCULATE KENDALL'S TAU FOR RANKED (X,Y)
C     PAIRS INCLUDES SPECIAL FORMULA FOR ADJUSTING FOR TIES
C     (IN Y ELEMENTS ONLY)

      PROGRAM TAUTEST
      REAL S(4,4), T(40)
      CHARACTER*12 NAME

      OPEN(2, FILE='TAU.OUT',STATUS='NEW')

      WRITE(*,*) 'ENTER FILE NAME'
      READ(*,10) NAME
      OPEN(1, FILE=NAME,STATUS='OLD')

      READ(1,*) NG, NA
      NGA = INT(NG*NA)
      CALL CALC(S, T, NG, NA, NGA)

10    FORMAT(A12)
      STOP
      END


      SUBROUTINE CALC(SCORE, TAU, NGROUP, NALT, NGA)
      REAL SCORE(4,4), TAU(NGROUP)
      INTEGER TIE, TY

C     **** NGROUP = NUMBER OF STUDENTS, GROUPS
C     **** NALT   = SAMPLE SIZE, (NUMBER OF ALTERNATIVES)

C     LOOP THROUGH EACH GROUP
      DO 10 I = 1, NGROUP
      N=0
      DO 11 J = 1, NALT
        N = N+1
        READ(1,*)SCORE(N,1),SCORE(N,2),SCORE(N,3),SCORE(N,4)
11      WRITE(*,*)SCORE(N,1),SCORE(N,2),SCORE(N,3),SCORE(N,4)


C     SORT BY SCORE(N,3)
      DO 30 J = 1, NALT-1
         K = NALT - J
         DO 40 L = 1, K
            IF (SCORE(L,3) .LE. SCORE(L+1,3)) GOTO 40
            T1 = SCORE(L,1)
            T2 = SCORE(L,2)
            T3 = SCORE(L,3)
            T4 = SCORE(L,4)
            SCORE(L,1) = SCORE((L+1),1)
            SCORE(L,2) = SCORE((L+1),2)
            SCORE(L,3) = SCORE((L+1),3)
```

```
            SCORE(L,4) = SCORE((L+1),4)
            SCORE((L+1),1) = T1
            SCORE((L+1),2) = T2
            SCORE((L+1),3) = T3
            SCORE((L+1),4) = T4
40        CONTINUE
30     CONTINUE

C      **** WRITE OUT SORTED ARRAY
       DO 41 M = 1, NALT
41     WRITE(*,*)SCORE(M,1),SCORE(M,2),SCORE(M,3),SCORE(M,4)

C      **** CALCULATE P AND Q
C      **** AND SEARCH FOR TIES IN Y COLUMN
       TIE = 0
       TY = 0
       DO 50 K = 1, NALT-1
          N1 = N1 + 1
          KY = 1
          DO 60 K1 = K+1, NALT
             IF (SCORE((K),4) .LT. SCORE((K1),4)) P = P + 1
             IF (SCORE((K),4) .GT. SCORE((K1),4)) Q = Q + 1
             IF (SCORE((K),4) .EQ. SCORE((K1),4)) THEN
                 KY = KY+1
                 TIE = 1
          END IF
60        CONTINUE
          TY = TY + KY*(KY-1)
50     CONTINUE
       TY = TY/2

C      **** CALCULATE TAU FOR EACH GROUP
C      **** CHECK FOR TIES IN THE DATA

       IF (TIE .NE. 0) THEN
          DENOM1 = SQRT(.5*NALT*(NALT-1))
          DENOM2 = SQRT(.5*NALT*(NALT-1) - TY)
          TAU(I) = (P-Q)/(DENOM1*DENOM2)
       ELSE
          TAU(I) = (P-Q)/(NALT*(NALT-1)/2.)
       ENDIF

       WRITE(*,100) SCORE(1,1), TAU(I)
       WRITE(2,100) SCORE(1,1), TAU(I)
       P = 0
       Q = 0
10     CONTINUE
 100    FORMAT(1X,f8.0,5X,F6.3)

       RETURN
       END
```