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OPTIMAL OPERATION OF THE SALT RIVER PROJECT, ARIZONA

Eric R. Hooper
Dennis P. Lettenmaier

Water Resources Series
Technical Report No.115
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ABSTRACT

Extended linear quadratic Gaussian (ELQG) control, a nonlinear stochastic control method for the optimization of the operation of multiple reservoirs with multiple objectives, was applied to the Salt River Project (SRP) reservoir system. The optimization problem was posed as the maximization of total hydropower avoided subject to constraints, where the constraints included both physical constraints (maximum and minimum storage) and water supply objectives. ELQG control maximizes the objective function through the use of penalty functions which discourage the algorithm from violating constraints. The physical connectivity of the reservoir system is represented by a set of nonlinear equations that describe the reservoir dynamics in state space form. The SRP reservoir system was conceptualized as 6 equivalent reservoirs: Bartlett Lake and Horseshoe Lake on the Verde River, Roosevelt Lake and an equivalent reservoir representing Apache, Canyon, and Saguaro Lakes on the Salt River, the Granite Reef Diversion Dam downstream of the confluence of the Verde and Salt Rivers, and an equivalent reservoir representing the groundwater system. The total hydropower avoided cost accounted for both hydropower generated by the system, and groundwater pumping cost, both of which varied seasonally. The energy generation avoided cost accounted for the net effect of hydropower generated by the reservoir system on the composite SRP generation system, and effectively represented the cost of power that would have to be generated from other sources absent the net hydropower generated by the reservoir system.

The algorithm was applied to the historic reservoir inflow record for the 51-year period 1931-81 for a 12-month sliding window. At each time period, all past inflows were assumed known, and forecasts of reservoir inflows were assumed to be available for 12 months into the future. Two forecast scenarios were considered: a perfect forecast, for which forecasted inflows were taken from the historical record, and no forecast, for which historical mean reservoir inflow was used as the forecast. In each case, the optimal ELQG control policy was identified for the 12-month future period, and the next month's reservoir releases were taken from the optimal policy. The window was then moved forward, the actual reservoir storages that would have resulted from the optimal policy were determined, and the process was repeated for the new 12-month window. The same process was performed using the current reservoir operating policy as encoded in SRP's monthly simulation model SRPSIM.

Operating policies were developed and compared with the present operating policy for two system configurations: the present configuration, and a reduced storage configuration that mimicked the reduction in usable storage at Roosevelt Lake that will be necessary during the construction period for the planned raising of Roosevelt Dam. The results showed that significant increases in the hydropower avoided cost could be achieved while maintaining water supply reliability at current levels or higher. For the present system configuration, the estimated long-term increase in average hydropower generation was about eight percent for no forecast, and ten percent for the perfect forecast, as compared to the present operating policy. For the reduced storage case, the predicted increases were much more substantial: over 70 percent for no forecast, and over 90 percent for the perfect forecast. For the present storage configuration, most of the increase in the net hydropower avoided cost was attributable to reduction in groundwater pumping under the optimal operation policy. For the reduced storage configuration, the optimal operating policy resulted in greatly reduced spills as compared with the current operating policy; the reduction in spills accounted for most of the increase in hydropower avoided cost.

ACKNOWLEDGEMENTS

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Dr. Aris Georgakakos of the Department of Civil Engineering at the Georgia Institute of Technology provided some of the computer source code that was used in the optimization model. Mr. Thomas Barr, also at the Georgia Institute of Technology, assisted in adapting the code to the SRP system during a visit of the author to the Georgia Institute of Technology in September, 1988. The assistance of Dr. Georgakakos and Mr. Barr, and their hospitality, is greatly appreciated.

Funding for the project was provided by the Salt River Project. System data were supplied by Ms. Nancy Clemm of the Hydrology Department of SRP. Mr. John Keane, also of the Hydrology Department of SRP, provided oversight and helped clarify SRP's present reservoir operating policy. Both Ms. Clemm and Mr. Keane provided useful comments on earlier versions of the report.

Computing resources were provided by the College of Engineering at the University of Washington, and the University of Washington Administrative Data Processing Department, whose IBM 3090 significantly expedited the computer model runs. The efficient and competent organizing abilities of Margo Behler made final production of this manuscript an easier task.

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CHAPTER 1: INTRODUCTION AND OVERVIEW

1.1 Overview

Most large water resource systems are used for multiple purposes. Hydroelectric power production, water supply for municipal, agricultural, and industrial uses, instream flow for fisheries, and flood protection are among the many uses that might be considered in formulating an operating policy for a reservoir system. The purpose of an operating policy is to provide an efficient and consistent mechanism for allocating water to the various uses. The goal of an effective operating policy is to allocate water to the various uses to the fullest extent possible, recognizing the prioritization of the uses. Water allocation, and hence, development of operating policies, is complicated by the inherent uncertainty in natural hydrologic processes and the inability to forecast precisely future hydrologic conditions. These complications introduce the possibility that the system will not be able to meet its objectives. A successful operating policy must account explicitly for the risk of failure as well as for the benefits of system operation.

A number of mathematical models and decision rules have been developed for the operation of multiple purpose water resource systems under uncertainty. Several of these are reviewed by Toebes and Sheppard [1979]. All existing mathematical models of reservoir operation require simplification of the actual reservoir operation being modeled; therefore, they are unable to model precisely all of the complexities of the true system. The shortcomings of most existing models are especially evident for multiple reservoir systems, where attempts to identify optimal operating policies can result in excessively large computational requirements (see, for example, Yakowitz, 1982). Therefore, there is a need to develop improved algorithms for decision making that incorporate more completely the complexities and uncertainties which result from the runoff generating processes and which do not have excessive computational requirements.

1.2 General Reservoir System Conceptualization

A general reservoir system, shown in Figure 1.1, consists of a network of streams containing either a single reservoir or multiple reservoirs in series or in parallel, a set of demands for water, and a set of physical constraints on the stream network. From the standpoint of operations research, the problem is to determine the net benefits of system operation, system reliability, given inflows to and withdrawal from the system over a specified period of time.

For the general reservoir system, releases are used for hydropower production, to meet instream flow targets, and to provide municipal, agricultural, and industrial water supply. Feasible reservoir releases are a function of future inflows, and future demands, and present reservoir storage. Therefore, the operations research problem becomes one of allocating water to various uses given present conditions and forecasted future reservoir inflows in an optimal manner.

The purpose of this report is to describe the application of an optimization algorithm for the operation of a multiple reservoir system with multiple uses under uncertain inflow conditions. The system of interest in this study is the Salt River Project (SRP) reservoir system located in central Arizona. A brief description of the SRP is presented in the following section.

1.3 The Salt River Project

The Salt River Project (SRP) reservoir system (Figure 1.2) provides the water for irrigated agriculture and the municipal and industrial water supply for much of the metropolitan Phoenix

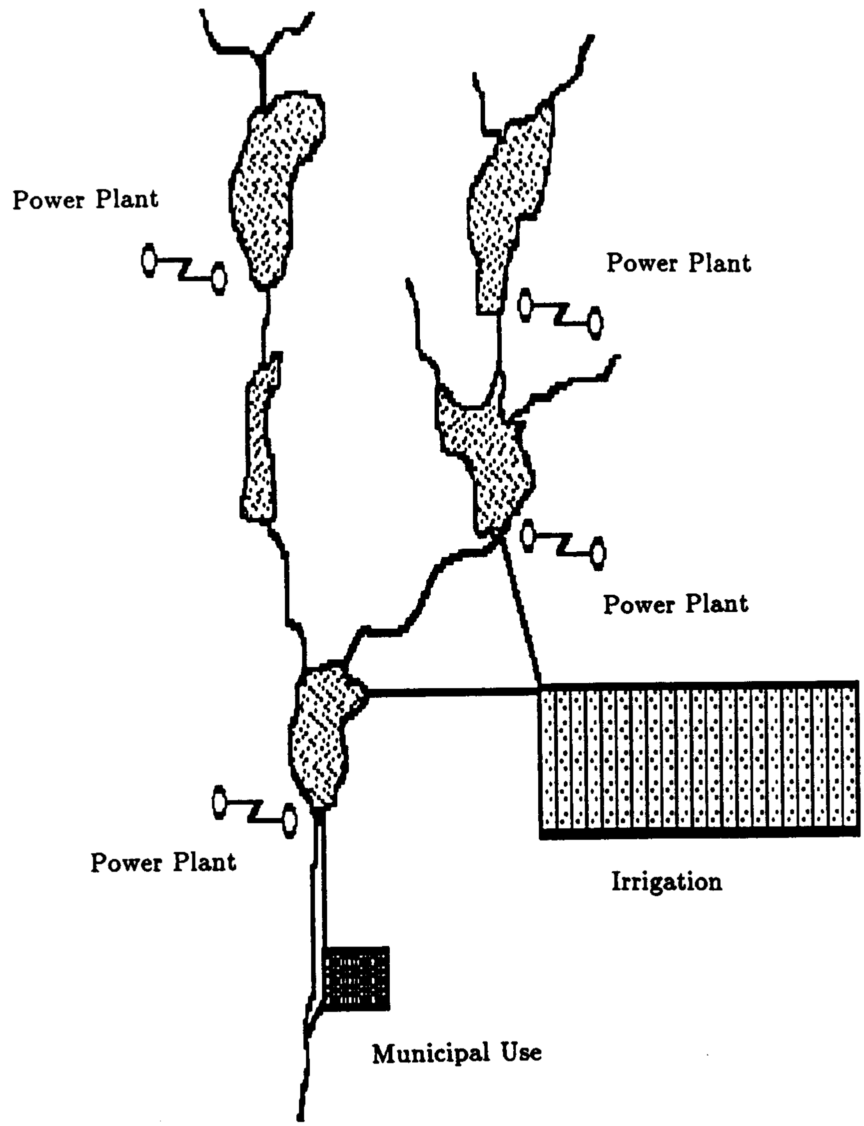


Figure 1.1 General Reservoir System Conceptualization

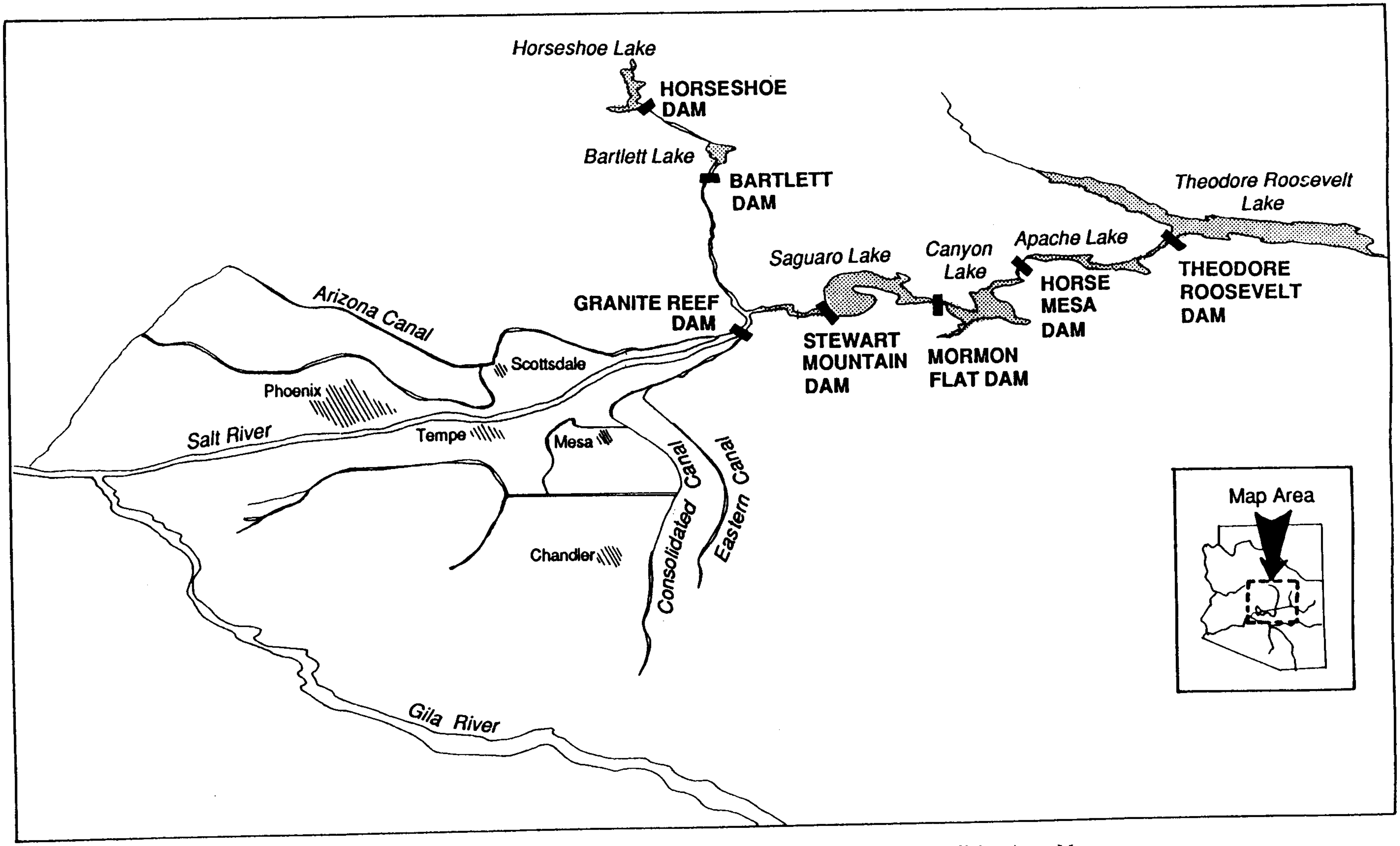


Figure 1.2 Salt River Project Irrigation System and Water Storage Facilities Area Map

area. The SRP has two water supply customers: SRP member lands, originally primarily agricultural, are provided water under SRP's charter; and other water supply organizations, which include the City of Phoenix, are provided water under contract. The primary responsibility of SRP is to providing water to member lands. Hydroelectric power is also generated by the system, primarily for peaking requirements.

SRP is comprised of two organizations. The Salt River Valley Users' Association, the oldest multipurpose project authorized under the Federal Reclamation Act of 1902, is responsible for the storage and delivery of water. The Salt River Project Agricultural and Power District, organized in 1937, is responsible for production and distribution of electricity to more than 350,000 customers. At present, about 15 percent of SRP's annual revenues generated from the sale of power are derived from hydropower. Under Arizona water law, members of the Users' Association (defined by ownership of member lands) exercise complete control over the available supply of water; therefore, the sole statutory requirement affecting current SRP operating policy is to meet the water demands of member lands. Hydropower generation is a benefit realized only through releases to meet a water supply requirement, or releases made to avoid uncontrolled spills. SRP does not have any responsibility for flood control. However, SRP does attempt to mitigate flood damage when possible for specific extreme storm events through modification of its reservoir release policies. Flood mitigation has become more of a concern since floods in 1980 and 1981 destroyed some of the bridges crossing the normally dry Salt River channel in the metropolitan Phoenix area.

The SRP reservoir system is comprised of six reservoirs located on the Salt and Verde Rivers. The reservoirs drain a watershed of over 13,000 square miles. Theodore Roosevelt Dam, completed in 1911, was the first storage dam built in the SRP system. It also forms the largest reservoir in the system, Roosevelt Lake. The present capacity of 1.632 10^9 m^3 (1.337 million Acre-feet) is larger than the other reservoirs on the Salt and Verde Rivers combined. There are three additional dams on the Salt River: Horse Mesa Dam (Apache Lake), completed in 1927, with a present capacity of 0.302 billion cubic meters (245 thousand Acre-feet (KAF)); Mormon Flat Dam (Canyon Lake), completed in 1926, with a capacity of 0.071 billion cubic meters (58 KAF); and, Stewart Mountain Dam (Saguaro Lake), completed in 1930, with a storage capacity of 0.086 billion cubic meters (70 KAF). There are also two dams on the Verde River: Bartlett Dam (Bartlett Lake), completed in 1939, with a storage capacity of 0.220 billion cubic meters (178 KAF); and Horseshoe Dam (Horseshoe Lake), completed in 1946, with a storage capacity of 0.1621 billion cubic meters (131 KAF). Granite Reef Dam, completed in 1908, is located at the confluence of the Salt and Verde Rivers. It diverts water from the storage system into the system of distribution canals. Storage capacity at this reservoir is minimal.

In addition to the storage reservoirs, SRP can augment its water deliveries with over 250 groundwater pumps located throughout the Salt River valley. These wells have a seasonally varying maximum capacity which averages 0.04 billion cubic meters (30 KAF) per month. Users within the SRP service area currently require more water than can be supplied from the surface reservoirs alone. The difference, roughly 20 percent of total demand (approximately 0.024 billion cubic meters per month), is made up by relatively costly groundwater pumping.

Hydroelectric power is produced at six sites in the SRP reservoir and distribution system: four dam sites, all on the Salt River, and two smaller low head facilities located in the distribution canal network. The Salt River dams have a combined generation capacity of 229 MW, of which 89 MW is conventional generation, and 140 MW is in the form of pumped storage. Hydroelectric power currently accounts for about 11 percent of the total electrical energy provided to SRP customers. However, due to the low costs associated with producing hydroelectric power, hydropower generation accounts for a larger portion of the revenues generated from electric power sales (about 15%).

The majority of inflow to the SRP reservoir system enters the upstream reservoirs on both the Salt and Verde Rivers. Most of the inflow to the system occurs during February, March, and April, and month-to-month correlations are high during these months. However, month-to-month correlations are much lower during the drier summer months, and annual serial correlations at both sites are low. There is a strong cross-correlation between the two sites, especially at the annual level (Lettenmaier et al. 1987).

The Salt River Valley is currently undergoing rapid urbanization. By the year 2000, it is estimated that up to 90 percent of SRP member lands and areas currently under water supply contract will be urbanized (SRP, 1981; 1982), as shown in Figure 1.3. As the SRP service area has become urbanized, agricultural water demand has decreased, while municipal and industrial water use has increased. The net effect, on a per acre basis, is projected to be a short term decrease in water demand.

1.3.1 Current SRP Storage Planning and Operation Decision Policy

The Verde River reservoir system is relatively small compared to the Salt River system in terms of both storage capacity and average inflows, and neither dam on the Verde River has hydroelectric power generation capability. Current SRP operating policy during the summer months is to use releases from the Salt River reservoirs up to the turbine capacity at the Stewart Mountain hydroelectric facility (1300 CFS), and then to augment this if necessary with releases

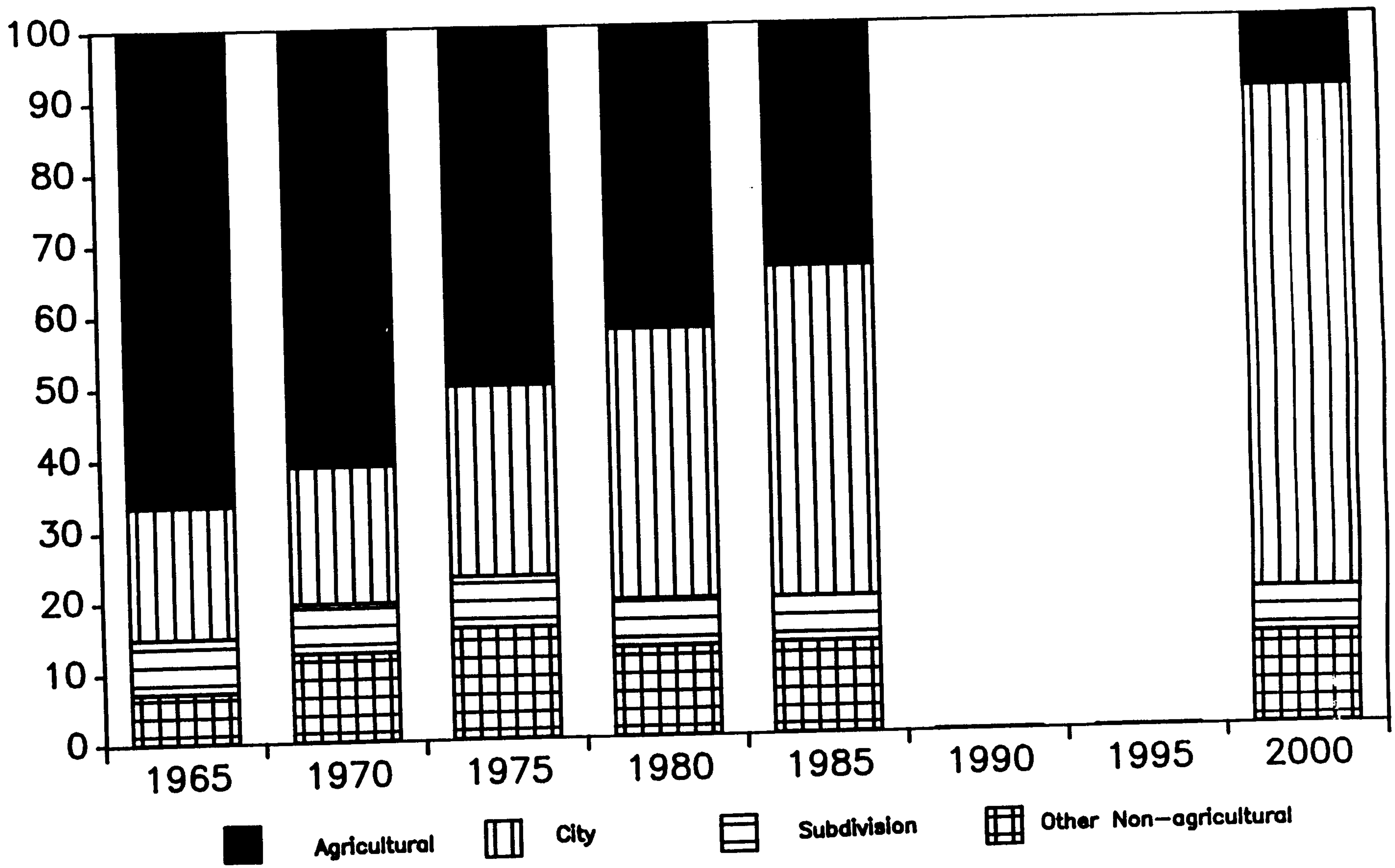


Figure 1.3 Trends in Lands Use in the SRP Service Area, 1965 - 2000 (taken from SRP, 1981)

from the Verde River reservoirs. Current SRP operating policy during the period beginning with October and ending with April takes water from the Verde River reservoirs to allow for distribution canal maintenance and maintenance of turbines at the Salt River dams. The Verde River reservoirs are targeted to be at maximum contents on or about April 1 of each year. Horseshoe Reservoir is targeted to be nearly empty at the beginning of the runoff season (January 1). Based on normal inflows (historical average), SRP has found it desirable to begin the runoff season with a minimum combined storage of the Verde River reservoirs of 0.062 billion cubic meters (50 KAF), which is about 15 percent of capacity. When possible, SRP maintains at least a 50 KAF pool of water in Bartlett Reservoir for the benefit of fisheries habitat, and during the spring spawning season, SRP attempts to maintain constant water elevation. However, water supply operations have priority over these accommodations.

Roosevelt Reservoir is targeted to be at capacity on or about May 1 of each year, that is, near the end of the snowmelt season. Because the reservoirs located on the lower part of the Salt River (Horse Mesa, Mormon Flat, and Stewart Mountain) are operated for pumped storage, storage at these reservoirs is maintained between 90 and 100 percent of capacity at all times.

Release decisions, when not dictated by contractual requirements, are based on historically determined rule curves. SRP has developed system simulation models to assist in evaluating long term operating policy, and also to allocate water to various uses, determine possible shortages, and project upcoming reservoir storage levels based on the rule curve operation of the system.

Storage Planning Diagrams I and III (Figures 1.4a and 1.5a) illustrate storage objectives during the winter runoff season. The maximum historic storage curves, represent the maximum historic storage that has been experienced in Roosevelt reservoir and in the Verde reservoir system respectively. These curves represent the upper limits of storage that might be expected on any given day from May through November. The historic inflow curves are fill curves, any point on which represents the storage level required to achieve the conservation objective if the corresponding runoff occurs. Historic runoff for the Verde River above Horseshoe Dam is defined to be the period of record since 1913. For the Salt River, it is defined as the combined flow of Tonto Creek and the Salt River above Roosevelt, also for the period of record since 1913.

Storage Planning Diagrams II and IV (Figures 1.4b and 1.5b) depict the family of rule curves used for the reservoir operations decision process. Each discrete curve represents a forecasted runoff volume. The point of intersection of any forecasted volume curve with a vertical dateline determines, by following a horizontal line to the unfilled volume axis, the required unfilled storage volume needed to contain that forecasted runoff, taking into account normal surface water demand, from the determination date to May 15. The use of Figure 1.4b can be illustrated as follows. If the runoff forecast on March 1 is 600,000 Acre-feet, the March 1 vertical projection intersects the forecast runoff curve at the horizontal projection representing an unfilled volume at Roosevelt reservoir of 384,000 Acre-feet. This means that 384,000 Acre-feet of reservoir space is needed, which with the normal demand, will allow the reservoir to reach the conservation goal on May 15 if the 600,000 Acre-feet of runoff is realized. As updated forecasts are made at daily or multi-day intervals, the rule curves are applied in the same manner to assist in arriving at a decision on reservoir operation.

1.3.2 SRP Reservoir Operation Simulation Model - SRPSIM

SRPSIM is a monthly reservoir operation simulation model. It models the SRP system as four

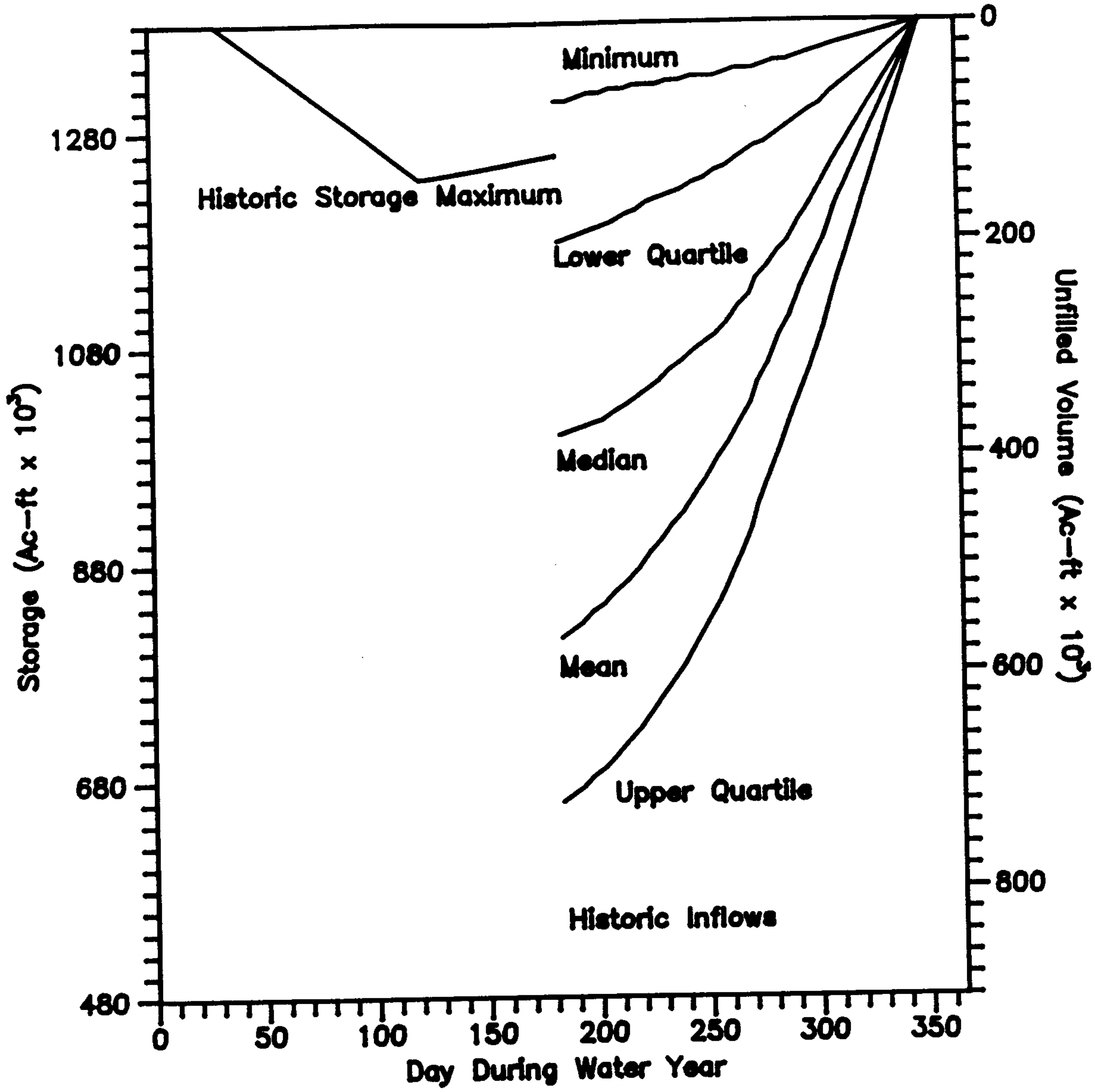


Figure 1.4a Current SRP Operating Policy: Storage Planning Diagram I Roosevelt Reservoir

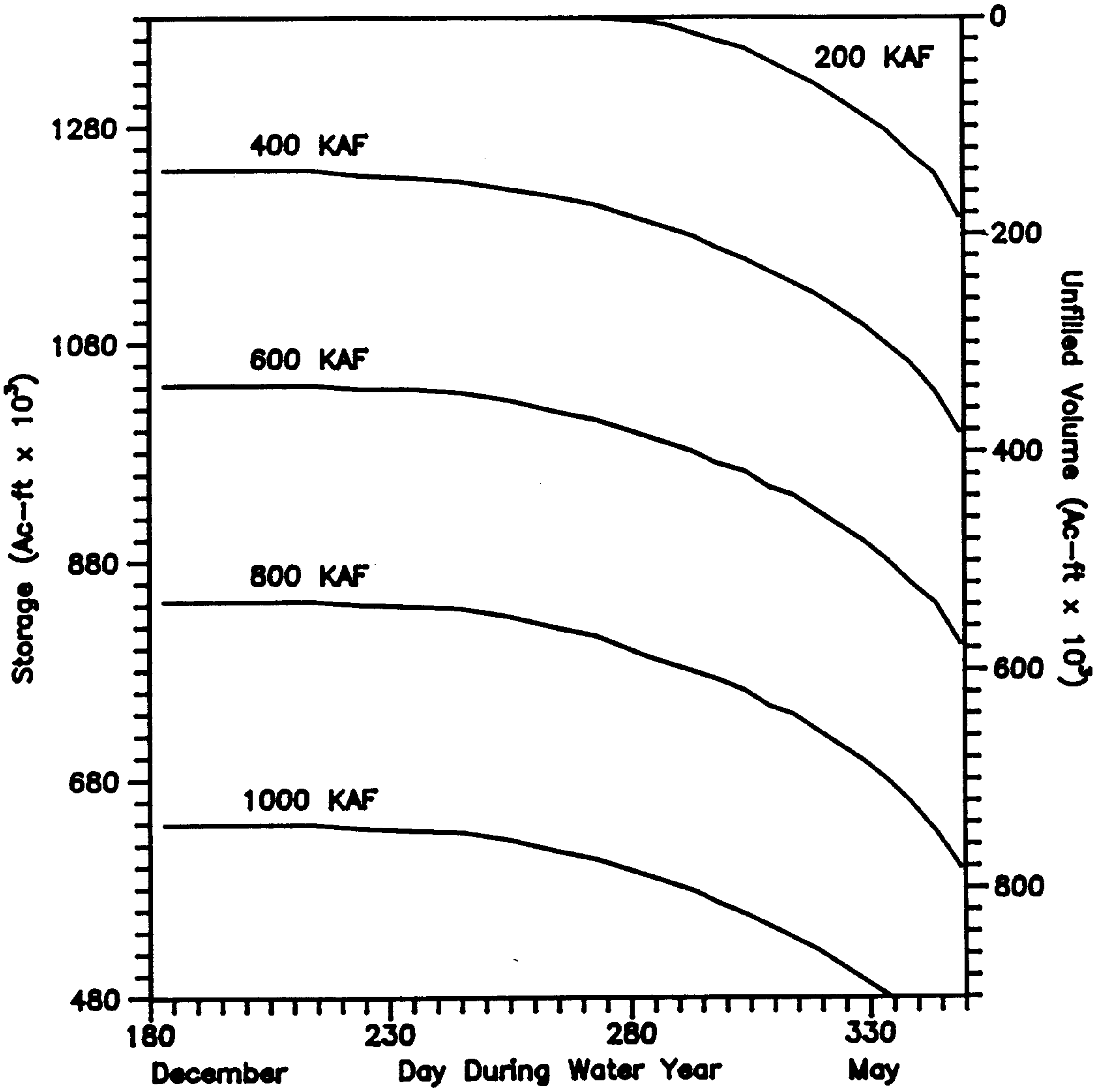


Figure 1.4b Current SRP Operating Policy: Storage Planning Diagram II
Roosevelt Reservoir

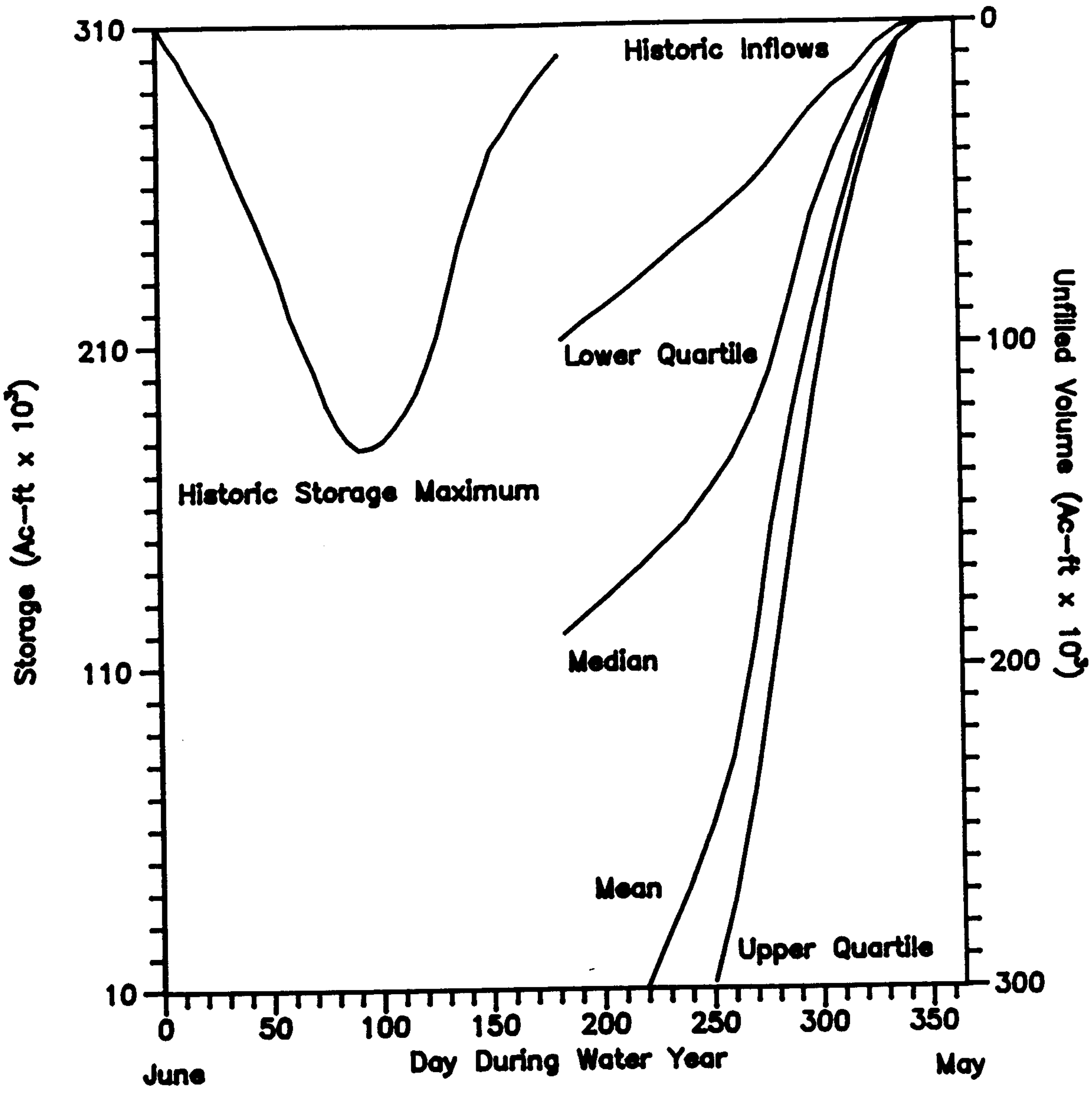


Figure 1.5a Current SRP Operating Policy: Storage Planning Diagram III
Verde Reservoir System

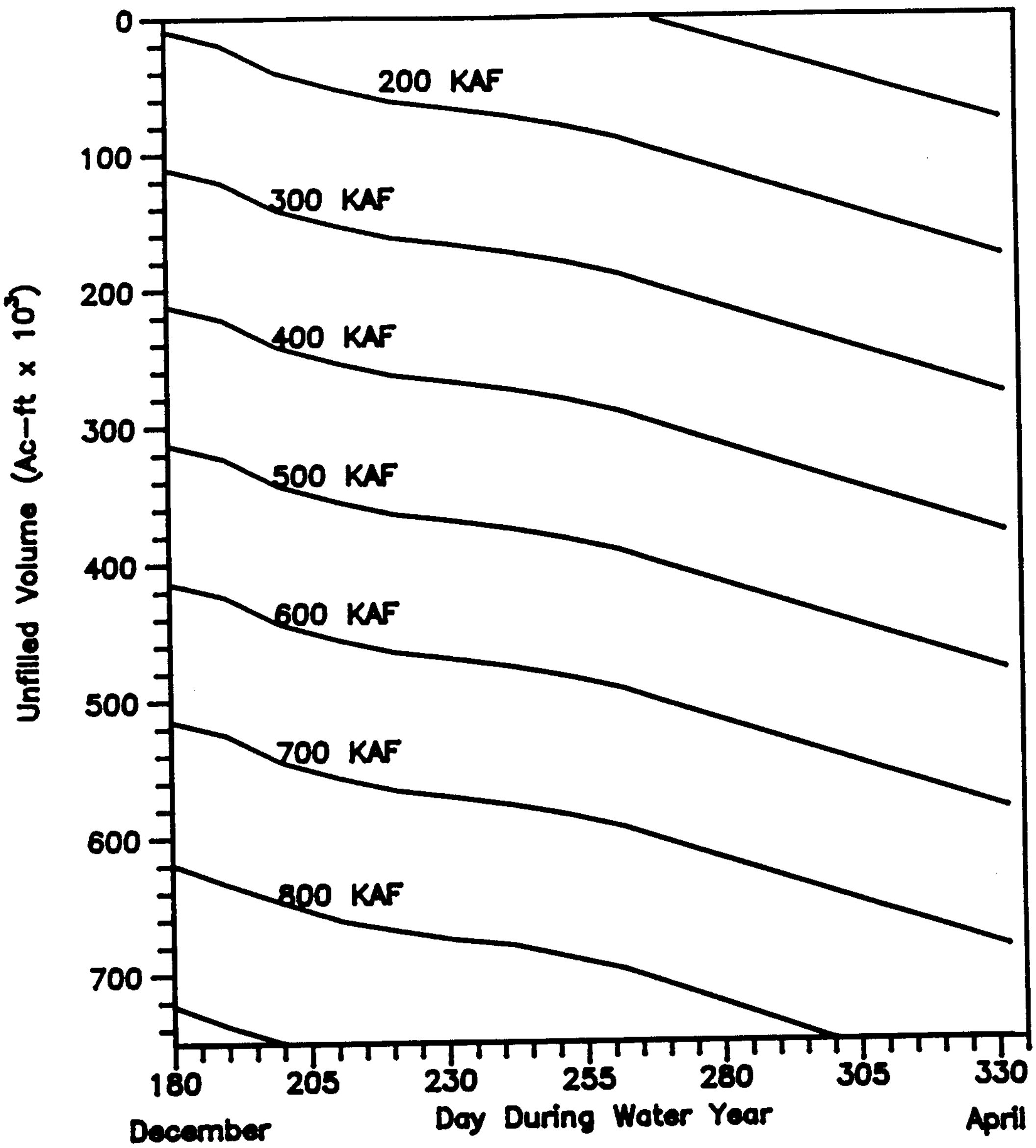


Figure 1.5b Current SRP Operating Policy: Storage Planning Diagram IV
Verde Reservoir System

equivalent reservoirs: Roosevelt, Horseshoe, and Bartlett are each modeled explicitly, while the three reservoirs on the lower Salt River (Horse Mesa, Mormon Flat, and Stewart Mountain), are treated as a single reservoir. The Granite Reef diversion dam is not modeled, although losses occurring in the river channels from the lower reservoirs to the diversion at Granite Reef, and in the distribution canals, are estimated by SRPSIM.

For a given water demand, SRPSIM determines the amount of groundwater and surface water required to satisfy that demand. The amount of groundwater pumping is determined according to the month of the year, reservoir storage at the time, and maximum pumping capacity. The model then operates the reservoirs according to the fixed operating criteria described above, and simulates the distribution of water according to SRP water contracts.

Input to SRPSIM includes initial reservoir storage, annual water demands, and historical monthly inflows. Output includes monthly reservoir releases, reservoir losses, spills, end of month reservoir contents, groundwater pumping volumes, hydrogeneration, and water deliveries for SRP member lands and other users.

1.4 Summary

The heuristic rule curve decision process currently employed by SRP, and most other large multipurpose multi-reservoir systems (Toebes and Sheppard, 1979), is the result of operating experience gained over time. In the case of SRP, 70 years of experience operating the system has resulted in a fairly effective operating policy. The success of the heuristic approach is tied, however, to the physical configuration of the reservoirs during the period of time when the operating experience was gained. If a drastic change in available storage is planned, or new demand levels are specified, the current set of rule curves will not apply. Application of operations research techniques to the management of reservoir systems provides an opportunity to improve upon the heuristic rule curve decision process and to identify operating policies that are not dependent on a specific set of physical parameters. Several optimization algorithms have been proposed in the literature which specify the desired reservoir release as a function of the initial storage volume and inflow during each time period to determine optimal reservoir operating and planning policies.

In the following chapter some representative reservoir control studies and optimization algorithms will be reviewed. A more extensive introduction to optimization, and operations research methods in general, can be found in Hillier and Liebermann [1986], Bradley et al. [1977], and Ossenbruggen [1984]. A detailed introduction to operations research with specific applications to water resource systems can be found in Loucks et al. [1981].

CHAPTER 2: APPROACHES TO RESERVOIR OPERATION MODELING

2.1 Introduction and Overview

Reservoir system operation models fall into one of two categories. Deterministic models assume perfect information concerning future inflows to the reservoir system and the system response to those inflows. Stochastic models assume that only the probability distributions of future reservoir inflows is known.

Deterministic models are generally simpler and easier to apply than stochastic models. Because deterministic models assume perfect information about future reservoir inflows, they do not explicitly account for uncertainty inherent in natural hydrologic processes and therefore are of limited value for operational applications. However, for planning applications, or for screening prior to applications of operation models, deterministic models can be useful. In addition, deterministic models can serve to define a starting point (feasible operating policy) for future decisions concerning system operation.

Stochastic models differ from deterministic models in that the uncertainty inherent in hydrologic processes such as reservoir inflows, and possibly other processes such as system losses due to evaporation and seepage, is accounted for by assigning probability distributions to those variables not known with certainty. In practice, the probability distributions are assumed to be known, and are usually estimated from historical records. Because stochastic models explicitly account for uncertainty, they are more difficult and cumbersome to apply to water resource systems problems than deterministic models. However, because of the inherently stochastic nature of natural hydrologic processes, these models are more useful than deterministic models, especially for operation optimization.

A general multi-objective optimization model of a reservoir system can be expressed in terms of an objective function, which is a mathematical statement of the objectives of system operation, and constraint functions, which express in mathematical terms the physical and other limitations of the system. The general form of an optimization model is:

$$\text{Maximize (Minimize) } [Z_1(\underline{x}),\ldots, Z_n(\underline{x})] \qquad 2.1$$

$$\text{subject to: } g_i(\underline{x}) = b_i;\ i = 1, 2,\ldots, m$$

where: Z_i represents system objectives such as hydropower production,
$\underline{x}$ represents decision variables, usually reservoir releases,
$g_i(\cdot)$ are constraint functions, and
b_i quantifies technical feasibility.

The choice of the algorithm used to solve a water resource system problem depends on the characteristics of the problem being considered, the availability of data, the desired information or result, computational and other available resources, and the decision as to whether to include uncertainty in the analysis. Yeh [1985] groups reservoir operation models into the following general categories:

a) Linear Programming Models (LP),

b) Dynamic Programming Models (DP),

c) Nonlinear Programing Models, and

d) Simulation Models.

The algorithms included in these categories, as well as a further category, control theory, are described briefly in the remainder of this chapter.

2.2 Linear Programming (LP)

As the name implies, linear programming refers to an optimization method where all mathematical functions in the model, that is, the functions describing the system objectives and constraints, are linear. If the optimization problem can be expressed as a linear programming problem, a very efficient algorithm, called the simplex method, can be used to determine the optimal solution. The simplex method was first developed in 1947 by George B. Dantzig, then part of a research group of the U. S. Air Force known as SCOOP (Scientific Computation of Optimum Programs). Dantzig [1963], Press et al. [1986], and Hillier and Liebermann [1980], provide a detailed description of the simplex LP algorithm.

The LP algorithm can be used to optimize system operation for both deterministic and stochastic models. A form of LP known as Chance Constrained Linear Programming, allows inclusion of some of the stochastic aspects of hydrologic variables through the use of probability constraints of the form:

$$\Pr\left[g_i(\underline{x}) \leq b_i\right] \geq P_i \tag{2.2}$$

where the function $g_i(\underline{x})$ contains the non-random decision vector $\underline{x}$ (controlled reservoir release) and b_i is a random variable (hydrologic inflows) whose distribution function can be determined. Chance constraints of the form shown above require that the function $g_i(\underline{x})$ be no greater than the random variable b_i with the probability of P_i. The general form of a chance constrained linear program is therefore:

$$\text{Maximize } Z = \sum_{i=1}^{m} c_i x_i \qquad 2.3$$

$$\text{subject to: } \sum_{i=1}^{m} a_{i,j} x_i \leq b_j, \ j=1, n$$

$$\Pr\,[g_i(\underline{x}) \leq b_i] \geq P_i, \ i=1, m$$

$$x_i \geq 0, \ i=1, m$$

where the probability P_i is a predetermined maximum (minimum) acceptable risk. The solutions to chance constrained LP problems tend to be conservative because only extreme events are included in the model during each time period t. The joint probability of such a sequence is extremely small, but is not accounted for in the model (Loucks et al., 1981). Furthermore, chance constrained LP does not penalize explicitly violations of state constraints, nor does it provide for recourse should constraint violations occur.

LP is of somewhat limited use for reservoir operation and planning models when hydropower is considered. Usually, hydropower generation is a function of both available head (a function of reservoir storage volume) and the reservoir releases and is therefore nonlinear. However, if the nonlinearities are mild, a linear approximation to the objective function may be appropriate.

2.3 Discrete Dynamic Programming (DP)

Dynamic programming provides a systematic approach for reducing a sequence of interrelated decisions into single decision steps. The essential feature of the discrete DP approach is the structuring of the optimization problem into multiple stages, with each stage having discrete feasible states. The state variable trajectory is determined by recursively solving the objective function at each stage, sequentially, one stage at a time. The true objective ('cost to go') function of continuous state variables is replaced by sets of values at discrete points (feasible states). Because of this discretization, the resultant approximate solution may be different from the true solution to the original problem. This problem disappears given a sufficiently fine discretization; however, the computational effort depends on the discretization scheme used - for simple discretization schemes, computational effort increases geometrically with the number of discretizations used.

Dynamic Programming has been used extensively for solving water resource system problems. Stages are typically represented by specific time periods during the planning horizon of the problem, and states are usually represented by discrete reservoir storage levels. Unlike LP, the nonlinear aspects of water resource system problems can be incorporated into the problem formulation easily. The general form of the deterministic DP recursion equation, where the next state is completely determined by the current state and the policy decision at the current stage, can be expressed as follows:

$$v_j(s_j) = \mathrm{Max(Min)}\ [f_j(s_j, x_j) + f_{j-1}(s_{j-1}, x_{j-1}) + \ldots + f_0(s_0, x_0)] \qquad 2.4$$
$$\text{subject to: } s_{i-1} = t_i(s_i, x_i),\ i=1,j$$

where: $f_j(\cdot)$ = the 'cost to go' function which represents the expected value of the discounted objective function from stage j onward given that the state and policy decision at stage j are s_j and x_j respectively,

$v_j(s_j)$ = the optimal value of all previous decisions given state s_j, where state s_j is a predetermined final state and state s_0 is the known initial state,

x_i = the decision variable at stage i (reservoir release),

s_i = physical state at stage i (reservoir storage volume),

i = stage (month during the control horizon), and

t_i = transition function.

The optimal solution to a DP problem can be determined only if both the initial storage level and the storage level at the end of the control horizon are known. In the case of deterministic water resource systems problems, the storage at the beginning of the next time period is determined completely by the storage at the beginning of the current time period and the release decision of the current time period, because hydrologic inputs during the upcoming time period are assumed known.

Discrete dynamic programming solution algorithms require that both the stage and the state during each stage be discretized. While this does not usually create problems for specifying stage (usually a time increment such as a month), the state (usually reservoir storage volume) is, in reality, generally continuous, not discrete. This shortcoming leads to the major limitation of discrete DP, the so-called curse of dimensionality, and results in the major source of error. Computational efficiency requires that the number of states at each stage be limited. However, the error caused by representing a continuous state by discrete values is reduced by increasing the number of discrete state values. Also, DP must be solved recursively, and is therefore of limited use in future predictions where the future values of the state variables are not known. Despite these limitations, early applications of DP to reservoir control optimization were reported by Young [1967], Meier and Beightler [1967], Hall et al. [1968], and Schweig and Cole [1968].

The Discrete Differential DP (DDDP) procedure (Heidari et al., 1971), of which Incremental DP (IDP) (Hall et al., 1969) is a specific application, is an attempt to alleviate the dimensionality problems associated with the DP method. The procedure starts with an assumed sequence of feasible state vectors (trial state trajectory), then uses the DP recursion equation to check the alternate state trajectories that are just above and below the trial sequence. If an alternative sequence is found to result in a better objective function value, then that sequence becomes the trial state trajectory. The algorithm then proceeds in an iterative manner.

Convergence to a local optimum is assumed when no new sequence of states can be found to yield a better objective function value. IDP provides a means of reducing the dimensionality problems of the DP approach but does not guarantee a global optimal solution (Turgeon, 1982).

Another method of relieving the dimensionality problems of DP is by using successive approximations which break a multi-state variable problem into a series of one-dimensional subproblems in such a way that the sequence of optimizations over the subproblems converges to the solution to the original problem. For example, the releases for the joint operation of a six-reservoir system are solved as a series of single state DP problems. This approach is known as Incremental DP with Successive Approximations (IDPSA). Early applications of this approach to multiple reservoir problems were by Larson [1968] and Trott and Yeh [1971]. The advantage of this method is that the solution of a multi-dimensional DP problem is obtained by solving a series of one-dimensional DP problems, thereby reducing the number of computations considerably. However, convergence to the global optimum can not be proven according to Bellman and Dreyfus [1962].

Stochastic DP (SDP) differs from deterministic DP in that the value of the state variable at the next stage is not determined completely by the state and policy decision at the current stage. Rather, a probability distribution describes the possible values of the state variable at the next stage. The resulting general form of the DP recursion equation is as follows:

$$f_j(s_j, x_j) = \sum_i P_i \, [C_i + f^*_{j+1}(s_{j+1})] \qquad 2.5$$

where: $f_j(s_j,x_j)$ = the minimum expected value of the objective function from stage j onward given that the state and policy decision at stage j are s_j and x_j respectively,

$f^*_{j+1}(s_{j+1})$ = minimum $f_{j+1}(s_{j+1}, x_{j+1})$ taken over all feasible values of x_{j+1} state trajectories,

P_i = probability distribution of what the state will be given the current state s_j and the decision x_j at stage j,

C_i = the value of the objective function prior to stage j given previous state i,

s_j = discrete state values (storage levels), and

x_j = discrete policy decisions (release decisions).

Dimensionality problems and discretization error control problems that are encountered with the solution of deterministic DP problems are compounded in SDP, since a discretized probability distribution function must be associated with the value of each state variable at each stage. Askew [1974] developed a stochastic DP algorithm incorporating reliability constraints in an attempt to reduce the dimensionality problem caused by the introduction of stochasticity. This method attempts to exercise control over the probability of failure through the use of a penalty function in the formulation of the stochastic DP recursion equation. Askew's method was further explored by Rossman [1977].

The penalty function acts much like the linear programming chance constraint discussed earlier in that the probability of failure associated with the penalty function can be determined. This method has been employed successfully for single reservoir problems. However gaps in the formulation of a dual (expressing a maximization problem in terms of minimization) have been identified for single reservoir problems (Rossman, 1977), and due to the joint probabilities that must be determined for multi-reservoir systems, computational difficulties arise quickly.

Gradient dynamic programming (GDP) has been suggested as an attempt to reduce the computational requirements by reducing the discretization error in SDP problems (Kitanidis and Foufoula-Georgiou, 1987). In GDP, the function representing the 'cost to go' from the state at one stage to the state at the next stage (Equation 2.5), and its derivative with respect to the state variables are calculated at each state. The 'cost to go' function is then approximated by piecewise polynomials that preserve the values of the objective function and its derivative at each state. A minimum to the approximated function can then be determined using iterative procedures, such as Newton's method or other nonlinear search algorithms.

Another related method of reducing the error caused by discretization in SDP is currently being explored by Stedinger [in press]. In this method, the function is approximated by a spline fit polynomial that matches the calculated value of the objective ('cost to go') function at each discrete point. Preliminary results suggest that accuracy is comparable to the GDP method discussed above. The advantage to this method over GDP is that the first derivative of the 'cost to go' function is not calculated. In addition to simplifying the solution algorithm, introduction of error in the estimated first derivative is avoided.

2.4 Analytical Dynamic Programming

In some cases, a dynamic programming problem can be solved analytically, that is, the problem does not require discretization of state variables. The DP recursion relationships shown before (equations 2.4 and 2.5) do not require a finite number of states at each stage. Therefore, the DP problem can, in theory, be represented in continuous state space, if an analytical expression exists to describe the system dynamics. Analytical DP is an outgrowth of optimal control theory which states that an optimal control trajectory (sequence of predicted decisions) can be found by maximizing, during each decision period, a function representing the sum of present and future benefits.

The most widely studied case (Wasimi and Kitanidis, 1983; Georgakakos and Marks, 1987) is when the objective value function is quadratic, subject to linear constraints. Stochasticity of future inflows is incorporated into the algorithm by assuming that the probability distributions of the hydrologic inputs are normal (Gaussian). The method is known as Linear Quadratic Gaussian Control. The analytical solution to the problem can be determined using the Newton-Raphson method or other suitable nonlinear solution algorithms. Unfortunately, this method does not consider storage or release (state) constraints; therefore, its practical value for water resource applications is limited. However, it is useful in providing a starting point for more sophisticated approaches based on control theory (cf. § 2.6).

2.5 Nonlinear Programming

Nonlinear methods differ from linear programming in that both the objective function and the constraint functions are allowed to be nonlinear. They differ from discrete dynamic programming in that the state variables do not have to be discretized, that is, a continuous function can be evaluated. However, this method has not been as popular as LP or DP for water resource system applications because it is difficult to incorporate the stochastic nature of future inflows. In addition the computational requirements are relatively large.

The general form of a nonlinear problem is as follows:

$$\begin{aligned} &\text{Minimize (Maximize) } f(\underline{x}) \\ &\qquad \text{subject to: } g_i(\underline{x}) \geq b_i,\ i=1,m \\ &\qquad\qquad\qquad x_i \geq 0 \end{aligned} \tag{2.6}$$

where: $f(\underline{x})$ and $g_i(\underline{x})$ are given functions of the n decision variables.

No single algorithm will solve every specific problem of this form, however, methods have been devised to solve special cases by making certain assumptions concerning the functions $f(\underline{x})$ and $g_i(\underline{x})$.

If the problem is unconstrained, the above function reduces to:

$$\begin{aligned} &\text{Minimize (Maximize) } f(\underline{x}) \\ &\qquad \text{over all } \underline{x} = (x_1, x_2, x_3, \ldots, x_n). \end{aligned} \tag{2.7}$$

An optimal solution is reached when $\frac{\partial f}{\partial x} = 0$ (assuming that $f(\underline{x})$ is differentiable). If the problem is a minimization, and if $f(\underline{x})$ is convex, then the condition $\frac{\partial f}{\partial x} = 0$ is sufficient for determining optimality. This type of problem was analyzed by Wasimi and Kitanidis [1983] and was discussed earlier.

A classical method of dealing with a nonlinear problem is the method of Lagrange multipliers. This procedure begins by formulating the problem as:

$$h(\underline{x},\underline{\lambda}) = f(\underline{x}) - \sum_{i=1}^{m} \lambda_i[g_i(\underline{x}) - b_i] \qquad 2.8$$

where the variables $\underline{\lambda} = (\lambda_1,\lambda_2,\ldots, \lambda_m)$ are called Lagrange multipliers, and where for all feasible values of $\underline{x}$,

$$g_i(\underline{x}) - b_i = 0, \text{ for all i.} \qquad 2.9$$

Therefore $h(\underline{x},\underline{\lambda}) = f(\underline{x})$. As a result, the method reduces to analyzing $h(\underline{x},\underline{\lambda})$ as:

$$\frac{\partial h}{\partial x_j} = \frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} = 0, \text{ for } j=1,2,\ldots,n \qquad 2.10a$$

$$\frac{\partial h}{\partial \lambda_i} = -g_i(x) + b_i = 0, \text{ for } i=1,2,\ldots,m \qquad 2.10b$$

The critical points are obtained by solving these equations for $(\underline{x},\underline{\lambda})$. The m equations obtained by expanding Equation 2.10b are equivalent to the continuity constraints of the original nonlinear problem, and the Lagrange multipliers, $\underline{\lambda}$, associated with continuity constraints represent the marginal future value of the state variable $\underline{x}$, i.e. stored water. An optimal trajectory for $\underline{x}$ is reached when both the multipliers and the predicted decision trajectory converge to a desired precision.

Several algorithms exist that trace the function in the direction of the first partial derivative or gradient of the function. The simplest, and one of the least efficient, is known as the method of 'steepest descent'. Another, more efficient group of methods, collectively known as conjugate gradient methods trace the gradient for each step in the direction conjugate to the gradient of the previous step. If the conditions for the general constrained optimization problem (Kuhn-Tucker conditions) are also satisfied, the solution can be considered optimal. For more theoretical treatment of nonlinear optimization methods, and a presentation of some of the more important algorithms, see Press et al. [1986], and Hillier and Liebermann [1986].

2.6 Control Theory

The concept of control arises when the equations describing the dynamics of system behavior contain a set of deterministic (control) variables $\underline{u}(t)$, whose functional form can alter the system dynamic properties. In the case of reservoir system control, $\underline{u}(t)$ represents releases, which affect storage volume, $\underline{s}(t)$, which in turn affect losses due to seepage and evaporation, and available head for hydropower generation. A linear stochastic system having this capability can be represented by the equation:

$$\frac{\partial s}{\partial t} = F(t)\, \underline{s}(t) + G(t)\, \underline{w}(t) + L(t)\, \underline{u}(t) \qquad 2.11$$

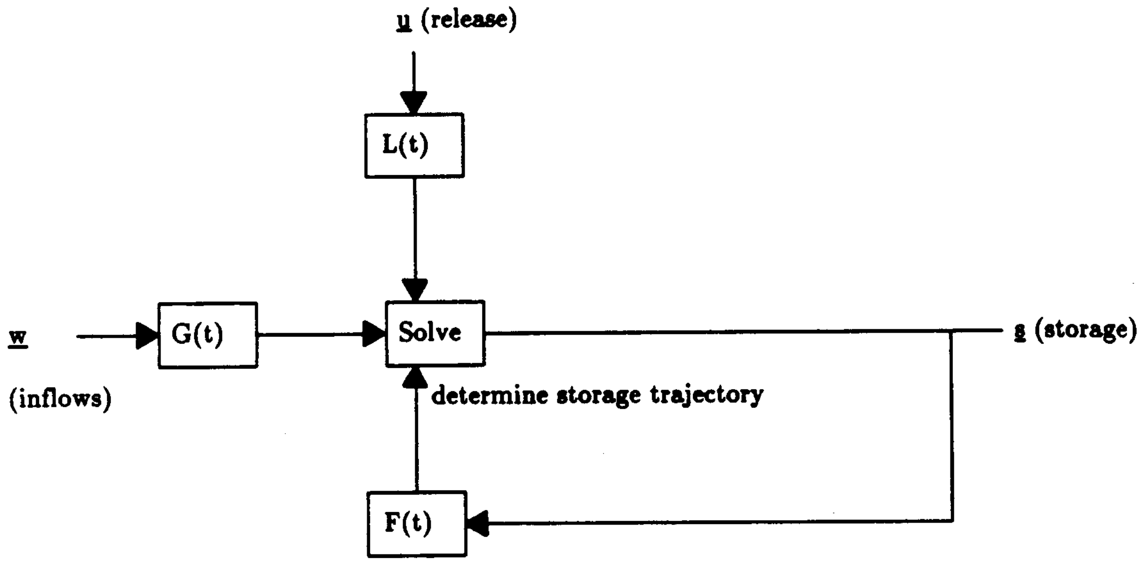
where L(t) is a coefficient matrix describing the influence of the control variable $\underline{u}(t)$ on the state vector, $\underline{s}(t)$, (i.e. the influence of releases on storage volume), and G(t) is a coefficient matrix describing the influence of a random vector $\underline{w}(t)$, (that is, the influence of stochastic inflows on storage volume). If $\underline{u}(t)$ is explicitly a function of time only, the problem is referred to as an open loop control. If $\underline{u}(t)$ is explicitly a function of $\underline{s}(t)$ also, it is called a closed loop or feedback control (Gelb, 1974). Figure 2.1 illustrates the open loop form of Equation 2.11.

To mechanize a feedback control, $\underline{u}[\underline{s}(t), t]$, the state variables $\underline{s}(t)$ must be accessible. In a reservoir system, the system state, represented by reservoir storage, is not directly observable. Therefore, it must be related to other observable quantities such as reservoir pool elevation, and calculable river discharge into and out of the reservoir. Typically, $\underline{s}(t)$ is observed only through available appropriate measurements $\underline{z}(t)$ where:

$$\underline{z}(t) = \mathbf{H}(t)\underline{s}(t) + \underline{v}(t), \qquad 2.12$$

where $\underline{v}(t)$ is Gaussian measurement noise.

To determine a control rule for the system described above in Equations 2.11 and 2.12, it is often desirable to impose a performance criterion that leads to a unique choice, $\underline{u}(t)$. An important class of problems is the so-called 'regulator' problem, where $\underline{s}(t)$ is assumed to have an initial value $\underline{s}_0$ at time t_0, and the control is chosen to drive the state toward zero or toward a target if the desired final state is nonzero. This objective is stated more precisely by requiring that $\underline{u}(t)$ minimize a performance index, J, which provides a measure of the size of $\underline{s}(t)$. A form of J that is found to be convenient and useful for linear systems is the quadratic performance index because the first derivative of a quadratic performance index is linear, while the second derivative is a constant which can be set to zero. In a reservoir system, where the operating objective is to maximize power production, a performance index as shown below will maximize energy generation by quadratically penalizing energy deficits from the maximum possible production.



Mass balance equation:

$$\frac{\partial \underline{s}}{\partial t} = F(t)\, \underline{s}(t) + G(t)\, \underline{w}(t) + L(t)\, \underline{u}(t)$$

where: $\underline{s}$ = reservoir storage,
$\underline{w}$ = inflows, and
$\underline{u}$ = controlled releases.

Figure 2.1 Conceptualization of Continuous Linear Dynamics Equation for Reservoir Operation under Open Loop Control

$$J = \sum_j \int_{t_0}^{t_T} \left[P_j - g_j\left(u_j(t), s_j(t)\right)\right]^2 dt \qquad 2.13$$

where: P_j = the maximum system energy generation, and

$g_j(\cdot)$ = the system energy production function.

The form of Equation 2.13 is preferred because minimization problems are generally easier to solve given Kuhn-Tucker conditions, (cf. §2.5).

For a more detailed treatment of the field of optimal control, see Gelb [1974] and Georgakakos and Marks [1985].

2.7 Simulation Models

Simulation, while not an optimization tool, provides a method of evaluating the performance of a system under various input and operating scenarios. The advantage of simulation is that major simplifying assumptions concerning the system operation can be avoided. Simulation is often used for verification of optimization model results. Simulation alone cannot directly identify optimal operating policies. However, some simulation models have been used in conjunction with optimization schemes in order to evaluate operating decisions (Sigvaldason, 1976).

One complication in simulation is that uncertainty in future inflows is difficult to assess. Repeated runs of a simulation model using different input conditions will produce a probability distribution of possible outcomes (the Monte Carlo technique). However, incorporation of future forecasts is more complicated. Approaches to evaluating the effects of forecast uncertainties have been suggested (Lettenmaier, 1984), but are not widely used.

2.8 Summary

The preceding review of optimization techniques indicates that while a variety of techniques are available, none is able to handle completely and efficiently all of the complexities of a natural reservoir system. The major difficulties faced are system nonlinearities, proper incorporation of hydrologic process stochasticity, dimensionality problems resulting from multiple reservoirs, and incorporation of stochasticity.

In Chapter 3, a model for a general reservoir system developed by Georgakakos and Marks [1985], which is an extension of the Linear Quadratic Gaussian Control method introduced by Wasimi and Kitanidis [1983], is described. This model also accounts for system storage and release constraints through the use of penalty functions in place of constraints on the state variables and can also make use of updated information for future decisions. In Chapter 4 an application of this model to the SRP reservoir system is described.

CHAPTER 3: PHYSICAL RESERVOIR SYSTEM MODEL AND OPTIMAL CONTROL

3.1 Introduction

A reservoir system is a group of reservoirs whose operation is linked physically and/or linked by system operation requirements. The dynamics of a general reservoir system can be described by the following elements:

- A set of hydrologic inputs, which enter the reservoir system at various locations. These can be represented by the historical record or synthetically generated stream flow sequences;
- A set of physical reservoir parameters such as reservoir volumes, and spillway capacities;
- Functional relationships describing net reservoir losses due to evaporation, precipitation on the reservoir surface, and seepage as a function of reservoir volume or surface area; and,
- System operational requirements (constraints) and objectives, such as water supply requirements, freeboard for flood control, recreation, and energy generation revenues.

As shown in Chapter 2, many approaches to determining the optimal control policy for a physical reservoir system have been proposed. None has been entirely successful, either for reasons of inappropriate simplifications or severe computational requirements. The algorithm proposed here makes use of an approach based on control theory first proposed by Wasimi and Kitanidas (1983) and then further advanced by Georgakakos and Marks (1985). The optimization approach is coupled with simulation in order to validate the optimization algorithm.

3.2 General Physical Model for Reservoir Systems

If the index $i = 1,\ldots, N$ refers to a reservoir, and t to a time period within the operating horizon, then according to conservation of mass, the dynamics of the system state (storage volume) at any time can be expressed by the following differential equation:

$$\frac{ds_i(t)}{dt} = I_i(t) - u_i(t) - L_i(t) \tag{3.1}$$

where:

- $s_i(t) =$ the storage at reservoir i,
- $I_i(t) =$ inflow (or infiltration to groundwater) for reservoir i,
- $u_i(t) =$ the controlled discharge (or groundwater pumping) for reservoir i, and
- $L_i(t) =$ uncontrolled losses such as evaporation and seepage from reservoir i.

In vector differential form according to the convention established by control theory as described in §2.6, the mass balance equation above can be expressed as follows:

$$\frac{d\underline{s}(t)}{dt} = F\Big(\underline{s}(t),\, t\Big) + G\, \underline{w}(t) + B\, \underline{u}(t) \tag{3.2}$$

where:

- $\underline{s}(t) =$ the system state vector, that is, reservoir storage volume,
- $\underline{w}(t) =$ a random forcing function vector, that is, hydrologic inputs,
- $\underline{u}(t) =$ a deterministic (control) input vector, that is, controllable releases or pumping,
- $F\Big(\underline{s}(t),\, t\Big) =$ a time varying nonlinear function with system state-dependent terms, that is, evaporation, seepage and direct precipitation, and
- G and B = coefficient matrices that associate each control (release) and input (inflow) with the pertinent differential equations (continuity).

System state (storage volume) dependent terms, specifically reservoir losses through evaporation and seepage, are modeled by a lumped parameter representation as follows:

$$F\left(\underline{s}(t), t\right) = \underline{e}(t) * A\left(\underline{s}(t), t\right) \tag{3.3}$$

where: $\underline{e}(t) =$ a spatially averaged net (evaporation plus seepage minus rainfall) loss rate per unit reservoir area, or a spatially averaged net recharge for the groundwater reservoir, and

$A\left(\underline{s}(t), t\right) =$ the reservoir surface area subject to recharge or loss (a function of reservoir storage volume at time t).

Since the form of Equations 3.2 and 3.3 are in the same form as Equations 2.11 and 2.12, control theory can be applied directly to the reservoir operation problem.

3.3 Modeling of the System Objective

With regard to hydropower, a reservoir system is operated to generate either the maximum possible energy or the maximum possible revenues (alternatively maximizing avoided power production costs). The power production function for reservoir j is given by:

$$P_j = g_j\left(u_j(t), s_j(t)\right), \tag{3.4}$$

where $u_j(t)$ is the downstream release through the turbines and $s_j(t)$ is the reservoir storage at time t. The state variable $s_j(t)$ enters the power production function via a known function relating reservoir storage with hydraulic head. Therefore, over a period $[t_0, t_T]$, the energy generated by reservoir j will be:

$$\int_{t_0}^{t_T} g_j\left(u_j(t), s_j(t)\right) dt. \tag{3.5}$$

Because of the stochastic nature of the storage-related variables, maximization of Equation 3.5 requires that a deterministic scalar quantity be determined to describe the probability density function underlying Equation 3.5. The most common approach is to optimize the expected value:

$$\underset{\underline{u}(t)}{\text{Maximize}} \quad E\left\{ \sum_j \int_{t_0}^{t_T} g_j\big(u_j(t),\, s_j(t)\big)\, dt \right\} \qquad 3.6$$

where j represents the system reservoirs. Alternatively, making use of the performance index approach introduced in §2.6:

$$\underset{\underline{u}(t)}{\text{Minimize}} \quad J = E\left\{ \sum_j \int_{t_0}^{t_T} \Big[M_j(t) - g_j\big(u_j(t),\, s_j(t)\big) \Big]^2 dt \right\} \qquad 3.7$$

where $M_j(t)$ is a known target level.

For optimization purposes, the performance index method is generally easier to solve because $g_j\big(u_j(t),\, s_j(t)\big)$ is usually mildly nonlinear; therefore, $\Big[M_j(t) - g_j\big(u_j(t),\, s_j(t)\big)\Big]^2$ will usually be convex, and the condition $\frac{\partial f}{\partial x} = 0$ will be sufficient for determining optimality.

Because evaluation of the control objective function defined by Equation 3.7 in continuous time is not practical, the discrete form of the objective function becomes:

$$\underset{\underline{u}_k}{\text{Minimize}} \quad J = E\left\{ \sum_j \int_{t_0}^{t_T} \Big(M_j(t) - g_t(\underline{s}_t,\, \underline{u}_t,\, \underline{w}_t) \Big)^2 dt \;+\; g_{T+1}(\underline{s}_{T+1}) \right\} \qquad 3.8$$

where: $\underline{w}(t)$ = a random vector, and

$g_{T+1}(\cdot)$ is determined completely given the conditions during the previous time step, and represents energy generation at the terminal storage volume.

3.4 Optimal Control Approach for the Unconstrained Reservoir System

The control approach (introduced in §2.6) used for optimal control follows ordinary nonlinear programming principles. It is an open loop approach, (because release is not explicitly a function of storage), and is a trajectory-iteration type algorithm. The optimization method proceeds as follows:

A control sequence $\{u_{i(0)}, u_{i(1)}, u_{i(2)},..., u_{i(t-1)}\}$ is assumed (nominal release, where the vector $\underline{u}$ represents controlled release from reservoir i), and propagated forward in time to obtain the corresponding state trajectory $\{s_{i(1)}, s_{i(2)},..., s_{i(t)}\}$, and the associated value of the system objective performance index. Then, feasible control sequences are searched to find the optimal sequence. Because an analytical solution to Equation 3.8 is not likely to exist, some numerical minimization procedure must be employed. The search algorithm used to determine the optimal sequence belongs to a class of optimizing algorithms known as generalized gradient methods. These methods make use of information derived from the gradient ∇ (first partial derivative) of the function being evaluated.

3.4.1 Newton's Method for Optimal Control of Nonlinear Equations

For a typical function $f(\underline{x})$, in the neighborhood of $\underline{x}$, the function can be expanded as a Taylor series,

$$f(\underline{x} + \delta\underline{x}) = f(\underline{x}) + \sum_{j=1}^{J} \frac{\partial f}{\partial x_j} \delta x_j + O(\delta\underline{x}^2) \qquad 3.9$$

by neglecting the terms of order $\delta\underline{x}^2$ and higher, a set of linear equations for corrections $\delta\underline{x}$ that move the function closer to a minimum is obtained. This means that for a real valued function $f(\underline{x})$, the function moves from $\underline{x}_j$ to $\underline{x}_{j+1}$ according to:

$$\underline{x}_{j+1} = \underline{x}_j + \alpha_j \underline{d}_j \qquad 3.10$$

where: $\underline{d}_j$ is the direction of the step, and

α_j is the step size.

In order to determine $\underline{d}_j$, Newton's direction, shown below, is employed:

$$\underline{d}_j = - [\nabla^2_{xx} f(\underline{x}_j)]^{-1} \ \nabla_x f(\underline{x}_j) \qquad 3.11$$

where: $\nabla^2_{xx} f(\cdot)$ is the Hessian of $f(\cdot)$ evaluated at $\underline{x}_j$, and $\nabla_x f(\cdot)$ is the gradient of $f(\cdot)$ evaluated at $\underline{x}_j$, numerical problems can arise if the function being evaluated is too sharply peaked, or if a second derivative of the function does not exist. However, because the quadratic performance index function is used, these problems are not encountered.

This method was chosen because it converges to a minimum efficiently. One drawback to using the Newton method to determine step direction is that the method can fail to converge to a minimum if the function is not smoothly convex, or if initial values are not relatively near optimum conditions if the function is not smoothly convex. However, because of the generally convex form of Equation 3.8, the Newton direction leads to convergence for the optimization problem considered here.

3.4.2 Updating the Control Sequence - The Armijo Stepsize Selection Rule

The nominal control sequence is updated using the Armijo Rule (Bertsekas, 1982). Following Georgakakos (1985), the Armijo stepsize selection method was chosen because it is easily implemented and because it can be conveniently generalized to fit a problem with control sequence constraints. For the general problem of minimizing $f(\underline{x})$ with respect to $\underline{x}$, given an initial point, x_i and direction, d_i, the stepsize, α_i is obtained according to the Armijo stepsize selection rule as:

$$\alpha_i = \beta^{m_i} \qquad 3.12$$

where: $\alpha_i =$ the stepsize

$\beta =$ a scalar satisfying $0 < \beta < 1$, and

$m_i =$ the first non-negative integer for which the following expression holds:

$$f(\underline{x}) - f(\underline{x} + \beta^{m_i}\, \underline{d}) \geq - \sigma\, \beta^{m_i}\, [\nabla_x f(\underline{x})]^T\, \underline{d} \qquad 3.13$$

where: $\sigma =$ a scalar satisfying $0 < \sigma < 0.5$,

$\underline{d} =$ the step direction determined by the gradient,

$f(\underline{x}) =$ the value of the performance index J prior to updating values of $\underline{x}$, and

$f(\underline{x} + \beta^{m_i}\, \underline{d}) =$ the value of the performance index J after updating $\underline{x}$.

The Armijo rule guarantees that each iteration will reduce the performance index value by an amount proportional to the gradient.

The stochastic nature of the mass balance equation (hydrologic inflows) is propagated forward in time through the use of the Ricatti equation, (Gelb, 1974), which gives the covariance of the state function, evaluated at the mean state value, that is, the second derivative of the mass balance equation evaluated at the storage volume at time i, is as follows:

$$\frac{dP_{\underline{s}_i}(t)}{dt} = \nabla_{\underline{s}_i} F\left(\underline{s}_i(t),t\right) P_{\underline{s}_i}(t) + P_{\underline{s}_i}(t)\left(\nabla_{\underline{s}_i} F\left(\underline{s}_i(t),t\right)\right)^T + G\, Q_i(t)\, G^T \tag{3.14}$$

where: $\nabla_{\underline{s}_i} F(\cdot)$ = the gradient of the state vector evaluated at $\underline{s}_i$,

$Q_i(t)$ = the variance of the inflow, and

$P_{\underline{s}_i}(0) = 0$, i.e. the beginning storage is known.

An optimum is reached when the predicted decision trajectory converges to a desired precision, that is, when the gradient $\nabla_{\underline{s}_i}$ becomes zero, resulting in no further change in the objective function value. The equation reduces to a deterministic expression when the future inflow variance is zero. The control approach used follows the Extended Linear Quadratic Gaussian Control (ELQG) approach developed in Georgakakos and Marks [1985].

3.5 Introducing Storage and Release Constraints

Because open loop control is used, only minimum and maximum release constraints can be directly applied, because the choice of the control (release) trajectory is made prior to and independent of the determination of the corresponding state (storage) trajectory. Since the control variable (release) is not a function of storage, storage constraints can not be applied directly. Therefore a quadratic penalty function, Equation 3.15, is included in the objective function, where deviations from target storage volumes are penalized by the square of the deviation.

$$P_{ij} = \left(T_{ij} - s_{ij}\right)^2 \qquad 3.15$$

where: $P =$ the penalty associated with non-optimal storage at reservoir j during period i,

$T_{ij} =$ target storage for reservoir j during period i,

$s_{ij} =$ storage at reservoir j during period i.

Figure 3.1 shows an example of a quadratic penalty function for the storage constraint for the SRP system application described in Chapter 4. Since the value of this function is included in the objective function value, the controller will seek a trajectory of storage volumes that result in a minimum value of the penalty function. Because values of the penalty function are large for storage levels outside the minimum and maximum bounds, storage constraint violations, that is, storage less than zero or greater than reservoir capacity, will not ordinarily occur in optimal solutions. The choice of target storages is relatively unimportant, as the algorithm is largely driven by the power generation portion of the objective function, except in the case where storages, for policy reasons, are constrained to remain within a narrow band, (as is the case at Lower Salt).

An additional means of introducing storage and release constraints is to formulate the energy generation function such that generation decreases significantly if a storage or release constraint is violated (infeasible conditions). Figure 3.2 shows energy generation as function of storage volume at Lower Salt and volume released from Lower Salt. As with the quadratic penalty

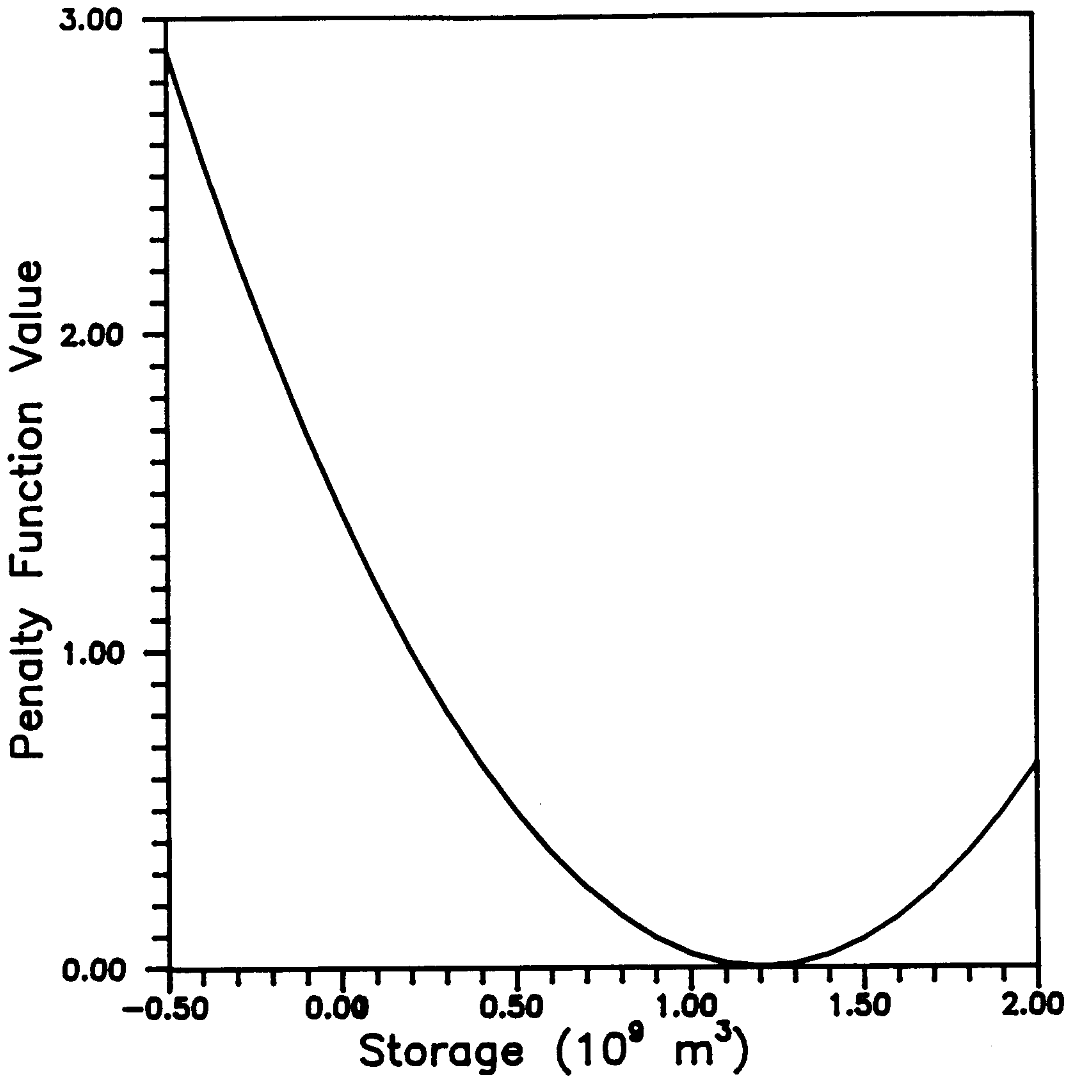


Figure 3.1 **Penalty Function for Introducing Storage Constraints at Roosevelt Reservoir**

Maximum Storage = 1.62 10^9 m^3, Minimum Storage = 0.0 10^9 m^3

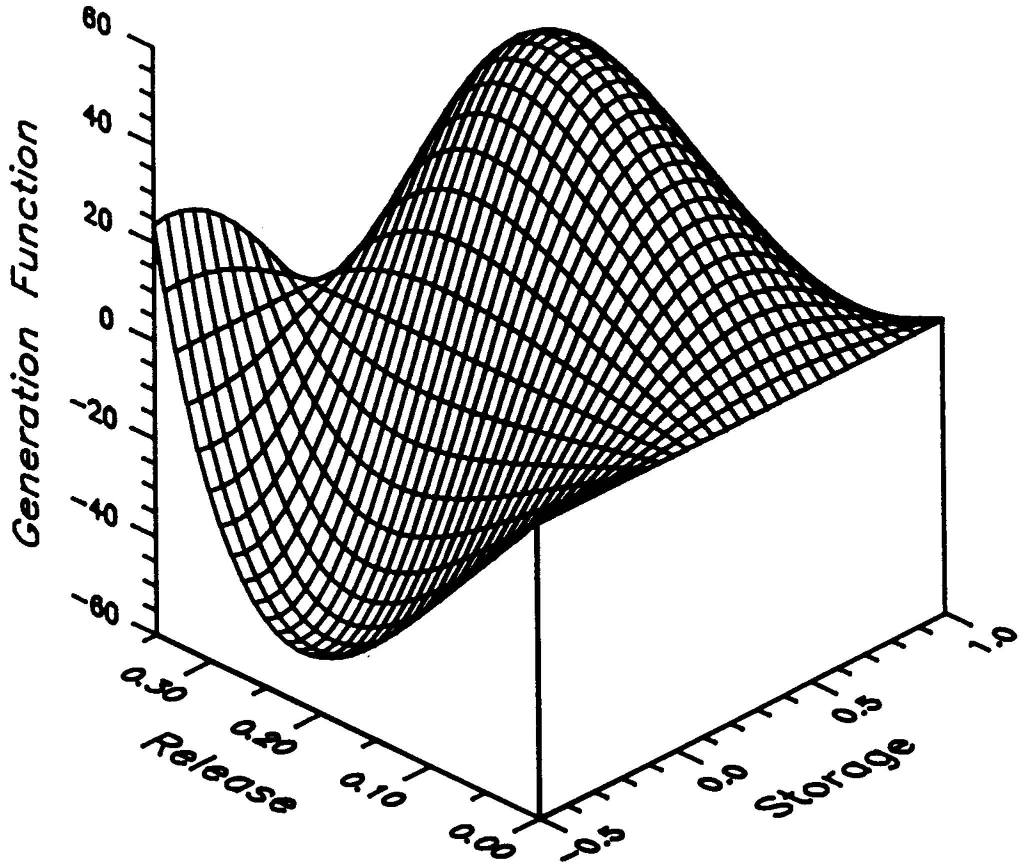


Figure 3.2 Hydroelectric Power Generation Function at Lower Salt Reservoir

function for the storage constraint (Equation 3.15), a quadratic penalty is associated with non-optimal energy generation and is also included in the objective function. Because the controller seeks to minimize the value of the objective function, release and storage trajectories are driven toward target trajectories, specifically, those which yield maximum generation, and away from infeasible storage and release conditions. Figure 3.3 shows the value of the quadratic penalty function for energy generation at Lower Salt reservoir given different releases.

Georgakakos [1989] has presented a refinement to the quadratic penalty function for storage constraint violations via the use of a 'barrier function'. The barrier function becomes large as reservoir storage approaches minimum or maximum levels and is small when storage levels are within preset bounds.

3.6 Salt River Project Reservoir System Model

The SRP reservoir system shown in Figure 1.2 was modeled as a six reservoir system: Roosevelt Lake (Roosevelt Dam), Horseshoe Lake (Horseshoe Dam), and Bartlett Lake (Bartlett Dam) were each modeled explicitly, the lower Salt River reservoirs, Apache Lake (Horse Mesa Dam), Canyon Lake (Mormon Flat Dam), and Saguaro Lake (Stewart Mountain Dam) were modeled as a single equivalent reservoir. The current groundwater pumping well network is treated as a fifth reservoir. The diversion at Granite Reef was also modeled to allow specification of minimum system release requirements, and to allow for the inclusion of a proposed groundwater recharge reservoir, designed to catch overflow spills from the diversion dam at Granite Reef. The recharge basin will be located in the dry riverbed of the Salt River downstream from the diversion at Granite Reef. It will have a capacity of 0.247 x 10^9 m^3, with withdrawals limited to 85 percent of the recharge volume. Operation of the recharge basin was modeled by adding

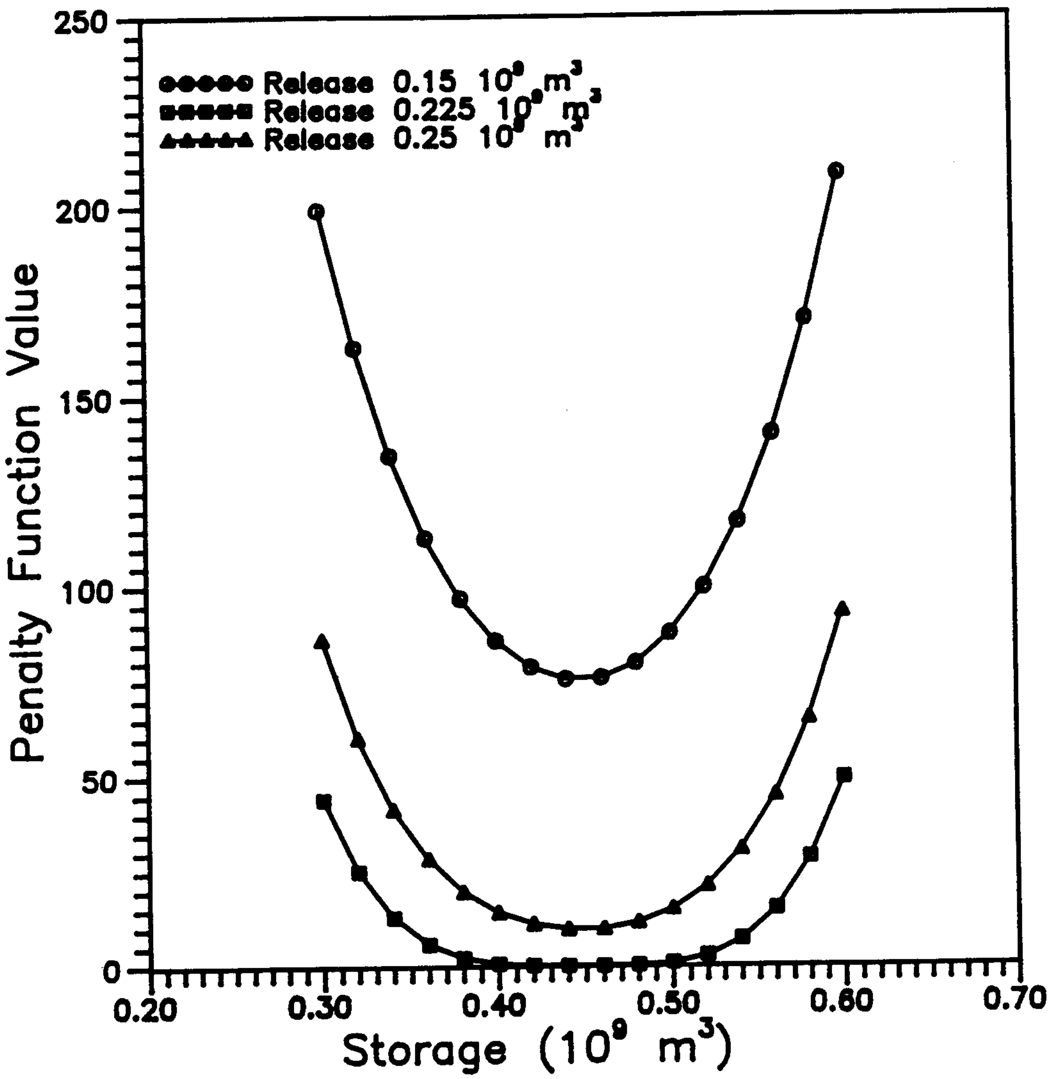


Figure 3.3 Penalty Function for Introducing Hydroelectric Power Generation Constraints at Lower Salt Reservoir

the volume to the conceptualized Granite Reef reservoir and allowing the volume of the combined reservoir to vary. Figure 3.4 is a schematic representation of the SRP system showing the modeled hydrologic variables. Physical data are summarized in Table 3.1.

Table 3.1: Salt River Project Reservoir Data

	Reservoir				
	Horseshoe	Bartlett	Roosevelt	Lower Salt	Groundwater
Storage (10^9 m^3)					
Minimum Allowed	0.0	0.0	0.0	0.414*	0.0
Capacity	0.162	0.220	1.623	0.460	1.5
Flow Rate (10^9 m^3/ month)					
Spillway Capacity	18.61	14.14	8.19	11.16	-
Turbine Capacity	-	-	6.31	4.73	-
Groundwater Pumping (10^9 m^3/ month)					
Minimum Required	-	-	-	-	0.0049
Capacity	-	-	-	-	0.493

* The minimum storage volume of the Lower Salt reservoir is 90 percent of the maximum storage volume of the three reservoirs on the lower section of the Salt River. This restricted volume range assuming that the use of the actual reservoirs for pumped storage, which is not explicitly modeled will not be affected adversely by the operating policy for the remaining reservoirs.

The functions representing the surface area relationship with volume were determined using a best least squares fit of linear equations to each surface reservoir:

$$\text{Area} = c_0 + c_1 * \text{volume} \qquad 3.16$$

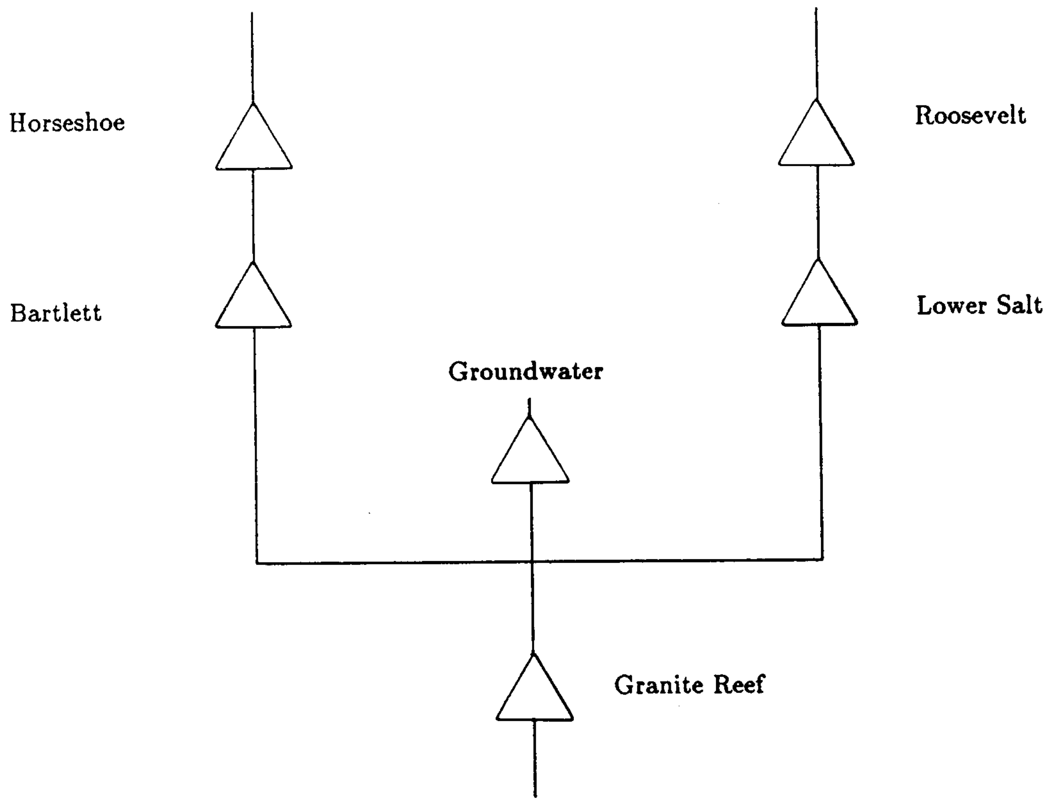


Figure 3.4 Schematic Representation of the Salt River Project Reservoir System

A logarithmic function was initially applied, but was discarded because of its inability to handle the occasional small negative storage volumes that the model can produce. In addition, the effect of evaporation and seepage losses were found to have little effect on the nonlinearity of system dynamics. Therefore, the simple linear form was chosen.

The equations describing the dynamic behavior of reservoir storage volume according to conservation of mass (Equation 3.2) for the SRP system reservoirs are as follows:

Horseshoe:

$$\frac{ds_h}{dt} = e_h\,[c_{0_h} + c_{1_h} s_h] + w_h - u_h \qquad 3.17a$$

Bartlett:

$$\frac{ds_b}{dt} = e_b\,[c_{0_b} + c_{1_b} s_b] + w_b + u_h - u_b \qquad 3.17b$$

Roosevelt:

$$\frac{ds_r}{dt} = e_r\,[c_{0_r} + c_{1_r} s_r] + w_r - u_r \qquad 3.17c$$

Lower Salt:

$$\frac{ds_s}{dt} = e_s\,[c_{0_s} + c_{1_s} s_s] + w_s + u_r - u_s \qquad 3.17d$$

The groundwater reservoir (index g) is assumed to be a uniform box with losses occurring on the free surface, therefore, the following equations are applicable:

$$\frac{ds_g}{dt} = (e_g * s_g^{0.667}) + w_g - u_g \qquad 3.17e$$

Because reservoir volume is assumed constant at the diversion reservoir (index d) at Granite Reef, the dynamic behavior of the reservoir can simply be expressed as:

$$\frac{ds_d}{dt} = (e_d * s_d) + u_g + u_s + u_b + w_d - u_d \qquad 3.17g$$

The following notation applies to Equation 3.17:

e_j = Net losses from reservoir j,

s_j = Storage volume of reservoir j,

w_j = Hydrologic input to reservoir j,

u_j = Controllable releases from reservoir j,

sp_d = Spills from the diversion at Granite Reef,

h = Horseshoe Lake,

b = Bartlett Lake,

r = Roosevelt Lake,

s = Lower Salt combined reservoirs,

g = Groundwater reservoir, and

d = Diversion reservoir.

Net losses (e_i) are approximated from pan evaporation rates in conjunction with known pan coefficients, and average precipitation. Seepage losses are assumed negligible compared to evaporation losses.

Values for the parameters c_0 and c_1 for equations 3.17a - 3.17d are given in Table 3.2.

Table 3.2: Mass Balance Parameter Values

	c_0 ($10^9 m^3$)	c_1 (m^2/m^3)
Horseshoe	0.008913	0.028761
Bartlett	0.003140	0.041594
Roosevelt	0.029758	0.023745
Lower Salt	0.008738	0.029355

3.7 Salt River Project Reservoir System Operation Objective

Currently, SRP operates the reservoir system for the single objective of delivering water to SRP member lands and to contract water users. However, by treating the water supply requirements as constraints, the control problem can be formulated to maximize power production or, alternatively, to maximize the avoided cost of generating power at a greater cost facility.

By imposing the performance criterion of a quadratic penalty as described in §2.6 and §3.3, power production maximization can be achieved by applying the following expectation performance index to net power production:

$$\text{Minimize} \quad E\left[\sum_j \int_{t_0}^{t_T} \left[M_j(t) - g_j\left(\underline{u}_j(t), \underline{s}_j(t)\right)\right]^2 dt \right] \qquad 3.18$$

where: $E[\cdot]$ = the expectation operator,

$M_j(t)$ = maximum power production at reservoir j. (In the case of groundwater pumping, $M_j(t)$ is set to 0, which has the effect of penalizing energy use.), and

$g_j(\cdot)$ = power production or pumping energy at reservoir j.

A similar performance index can be used to maximize avoided cost (minimize production costs):

$$\text{Minimize} \quad E\left[\sum_{j}\int_{t_0}^{t_T}\left\{M_j(t) * \underline{c}(t) - \left[g_j\left(\underline{u}_j(t), \underline{s}_j(t)\right) * \underline{c}(t)\right]\right\}^2 dt\right] \qquad 3.19$$

where: $M_j(t)$ = the maximum power production possible at reservoir j,

$\underline{c}(t)$ = the avoided cost of power production at time t, where the avoided cost is the difference in generation costs between a unit of power generated by the most expensive power generation facility operating at time t, and the cost of generating a unit of hydropower at time t, and

$g_j(\cdot)$ = the power production or pumping energy at reservoir j.

The same performance index can be used to maximize profits if $\underline{c}(t)$ is a function of both avoided production costs at time t and the rate charged for electricity at time t.

The hydroelectric power generation relationship with reservoir storage volume for Roosevelt and the Lower Salt reservoirs are given by Table 3.3.

Table 3.3: Hydroelectric Power Generation Relationship with Reservoir Storage Volume for Roosevelt and Lower Salt Reservoirs

Roosevelt	Storage Volume (10^9 m ^{3})	Power Generation (MWH/m^3)
	> 1.6492	0.148
	1.3960 - 1.6492	0.143
	0.8227 - 1.3690	0.130
	0.3976 - 0.8227	0.093
	0.0942 - 0.3976	0.079
	0.0 - 0.0942	0.0

Table 3.3: Hydroelectric Power Generation Relationship with Reservoir Storage Volume (cont.)

Lower Salt	Storage Volume (10^9 m ^{3})	Power Generation (MWH/m^3)
	0.3035 - 0.4478	0.283
	0.1850 - 0.3035	0.251
	0.1456 - 0.1850	0.196
	0.1246 - 0.1456	0.151
	0.0864 - 0.1246	0.112
	0.0321 - 0.0864	0.069
	0.0 - 0.0321	0.0

The power generation-reservoir storage volume relationship functions for the Roosevelt and Lower Salt reservoirs were approximated using a nonlinear equation of the following form:

Roosevelt:

$$g_r(t) = \left(b_1 * \sin(b_2 * \underline{s}_r) + b_3 \sin(b_4 * \underline{s}_r)\right) * \left(b_5 * \underline{u}_r + b_6 * \underline{u}_r{}^2 + b_7 * \underline{u}_r{}^3 + b_8 * \underline{u}_r{}^4 + b_9 * \underline{u}_r{}^5 + b_{10} * \underline{u}_r{}^6\right) * \underline{u}_r \qquad 3.20a$$

Lower Salt:

$$g_l(t) = \left[b_1 * \sin(b_2 * \underline{s}_l)\right] * \left(b_3 * \underline{u}_l + b_4 * \underline{u}_l{}^2 + b_5 * \underline{u}_l{}^3 + b_6 * \underline{u}_l{}^4\right) * \underline{u}_r \qquad 3.20b$$

where: $g_j(t)$ = power generation at reservoir j, MWH/10^9 m^3 discharge,

$\underline{s}_j$ = storage volume for reservoir j, and

$\underline{u}_j$ = release volume for reservoir j.

A Newton approach was used to find least squares parameter estimates. The form of Equation 3.20 was chosen because it yielded lower parameter estimate standard errors compared with standard errors of parameter estimates of other similar functions. Because minimum allowable storage for the Lower Salt reservoir is 90 percent of capacity, power generation can never be less than zero.

The energy required in MWH for groundwater pumping was approximated by:

$$g_g(t) = (b_0 * \exp[b_1 * \underline{u}_g]) * \underline{u}_g \tag{3.21}$$

where $\underline{u}_g$ is the volume of groundwater pumped (billion cubic meters), and the values of parameters b_0 and b_1 are 0.456 MWH/m^3 and -6.00x10^{-9}/m^3 respectively.

Finally, the quadratic penalty function was included in the objective function to introduce storage and turbine capacity constraints, which results in an objective function for maximizing avoided cost during a control horizon of length T given by:

$$\min \sum_j \int_{t_0}^{t_T} \left[M_j(t) * \underline{c}(t) - g_j\left(\underline{s}_j(t), \underline{u}_j(t)\right) * \underline{c}(t)\right]^2 dt +$$

$$\int_{t_0}^{t_T} \left(\underline{S}_j(t) - \underline{s}_j(t)\right)^2 dt + \int_{t_0}^{t_T} \left(\underline{U}_j(t) - \underline{u}_j(t)\right)^2 dt \tag{3.22}$$

where: $M_j(t)$ = the maximum power production at reservoir j during period t, MWH,

$c(t)$ = the avoided cost of power generation during period t, \$/MWH,

$g_j(\cdot)$ = power production at reservoir j (MWH),

$\underline{S}_j(t)$ = target storage volume at reservoir j during period t, m^3,

$\underline{s}_j(t) =$ storage volume at reservoir j during period t, m^3,

$\underline{U}_j(t) =$ target release volume at reservoir j during period t, m^3, and

$\underline{s}_j(t) =$ release volume at reservoir j during period t, m^3.

3.8 Simulation Post-Processing of Optimal Release and Storage Trajectories

Because storage constraint violations are not prohibited by the model (the occurrence is infrequent however, due to the influence of penalty functions for storage and energy generation), the optimal storage trajectory is post processed to determine if any storage constraint violations have occurred. Because of the large penalties associated with constraint violations, corrections are usually small; however, post processing is necessary to assure that the reservoir operation policy is feasible. The procedure to adjust for constraint violations is described below.

If the storage volume determined for the first month of the control horizon is found to exceed capacity levels, the excess is spilled to a lower reservoir, (if excess capacity is available at the lower reservoir), or otherwise is spilled from the system. Spills are assumed not to pass through turbines, so the effect on total power production is small. If reservoir storage during the first month of the control horizon is found to be less than minimum levels, release from upper reservoirs is increased, or release is reduced to the point where the reservoir is at a minimum allowed level. The adjusted storage and release volumes are used as the initial conditions for the next control horizon.

3.9 Summary

In this chapter, the approach that was used to develop an optimal operating policy for the SRP has been described. The approach is based on linear quadratic Gaussian (LQG) control, but also allows for nonlinearities in system dynamics, and incorporates the stochasticity inherent in hydrologic processes (through the Ricatti equation which propagates uncertainty). Because continuous functions are evaluated, the dimensionality problems associated with dynamic programming are not encountered.

In Chapter 4, the result of the application of ELQG control to the SRP reservoir system is presented. Differences between current SRP operating policy and an operating policy determined by ELQG control given the current system configuration are evaluated given historical streamflows. Operating policies are also determined and compared given an altered configuration of the SRP reservoir system, specifically reduced storage capacity at Roosevelt Reservoir during scheduled construction of increased flood and conservation storage.

CHAPTER 4: OPTIMAL CONTROL OF THE SALT RIVER PROJECT RESERVOIR SYSTEM - CASE STUDY

4.1 Introduction

The extended linear quadratic Gaussian (ELQG) control approach described in Chapter 3 is computationally efficient, especially for multiple reservoir systems, and allows explicit incorporation of the effects of stochasticity of future inflows. The conceptualized SRP reservoir system consists of four surface reservoirs, a groundwater reservoir, and a diversion reservoir. In this chapter the current operating policy of SRP is evaluated in comparison with the operating policy determined by applying ELQG control under both existing conditions, and under reduced storage capacity conditions.

4.2 Hydrologic Characteristics - Inflow

Most of the inflow to the SRP reservoir system (over 90 percent on average) enters the upstream reservoirs on both the Salt and Verde Rivers. Most of the inflow is derived from snowmelt during February, March and April, and month-to-month correlations are high during these months. However, month-to-month correlations are much lower during the drier, summer months, and annual serial correlations at both sites are low. There is a strong cross-correlation between the two sites especially at the annual level (Lettenmaier et al., 1987).

A lengthy, relatively good historical record exists for streamflow on the Salt and Verde Rivers. Lettenmaier et al., (1987) describe adjustments to the historical stream gauge record necessary to produce a 52 year (1931 - 1982) record of monthly inflows to the SRP reservoir system.

Summary statistics for the historical record of monthly Salt and Verde River reservoirs inflows are shown in Table 4.1.

Table 4.1 Salt River and Verde River Reservoir Inflow Summary Statistics
Salt River Inflows (1000 Acre-feet)

	Mean	Standard Deviation	Skew	Lag 1 Serial Correlation*
October	32.0	56.7	5.6	0.55
November	23.7	25.5	0.0	0.67
December	62.2	124.0	3.4	0.72
January	68.4	101.2	2.6	0.30
February	98.3	154.5	3.5	0.44
March	150.6	170.7	2.7	0.66
April	121.5	100.3	1.2	0.86
May	67.2	73.1	2.9	0.90
June	31.5	22.0	2.4	0.70
July	31.5	12.6	1.6	0.25
August	48.5	33.8	1.9	0.04
September	36.4	29.6	2.4	-0.01
Annual	771.0	611.0	1.8	0.09

Verde River Inflows (1000 Acre-feet)

	Mean	Standard Deviation	Skew	Lag 1 Serial Correlation*	Cross Correlation**
October	19.6	34.2	6.5	0.38	0.93
November	20.4	15.8	2.9	0.74	0.75
December	40.4	58.5	3.1	0.39	0.92
January	38.0	37.2	2.0	0.44	0.80
February	70.2	106.5	3.3	0.37	0.96
March	100.8	111.1	2.5	0.38	0.92
April	57.0	67.0	2.6	0.80	0.79
May	18.8	10.7	4.8	0.35	0.84
June	13.6	2.4	2.7	0.40	0.33
July	16.8	4.2	1.7	0.08	0.44
August	25.5	12.7	2.1	0.07	0.67
September	21.2	15.2	3.8	-0.10	0.37
Annual	443.0	275.0	1.5	0.01	0.94

* Lag correlation is calculated for a given month using the preceding month; annual lag value is the correlation of total annual flow with previous year's flow.

** Cross correlation is calculated for a given month using inflows to the Salt and Verde River for the same month

4.3 Hydrologic Characteristics - Evaporation

Pan evaporation and temperature data have been collected at Bartlett Dam since 1939 and at Roosevelt Dam since 1915, although there are some missing values in each record. Because it is known that pan coefficients (the ratio of free surface evaporation to pan evaporation) vary seasonally, an estimate of the seasonal variation in the pan coefficient reported by Todd [1970], (adjusted reported values for Yuma, Arizona,) was applied to all pan evaporation records. Mean monthly pan evaporation and the monthly pan coefficients are shown in Table 4.2.

Table 4.2: Mean Pan Evaporation and Pan Coefficients at Bartlett and Roosevelt Reservoirs (inches)

Month	Bartlett	Roosevelt	Pan Coefficient
October	9.84	6.83	0.64
November	6.07	3.47	0.57
December	4.56	2.17	0.40
January	4.16	2.25	0.33
February	4.97	3.47	0.40
March	6.31	7.49	0.51
April	10.55	9.55	0.63
May	14.60	12.89	0.75
June	16.73	15.63	0.88
July	16.66	15.40	0.91
August	14.35	12.69	0.76
September	12.69	10.54	0.68

4.4 Evaluation of Operating Policy Using Performance Indices

In order to evaluate the effectiveness of an operating policy, it is of interest to estimate the cumulative distribution function (cdf) of selected measures of the performance of the system. Consider a performance index r, for instance the number of months in a 50-year sequence in which monthly water supply requirements are not met. Simulation of system operation using a

50-year streamflow sequence will result in 50 values of r (r_i, i=1, 50). The cdf of r can then be estimated from the 50 values r_i by first ordering them from smallest to largest, then assigning the probability of occurrence $P_i = \frac{i}{n+1}$ to the i^{th} largest value of r (Weibull plotting position rule), where n is the number of simulations, in this case 50. The cdf represents the probability that the performance index will be less than any (arbitrary) value r. The effectiveness of alternative operating polices can be evaluated quickly using the cdf of any relevant performance index. In the case of water supply, for instance, it may be of interest to define system operating policies that give a specified reliability (for example, 98 percent) for a forecasted long-term water demand.

4.5 Case Study Results

ELQG control was applied to the SRP reservoir system for a 50-year period, given monthly inflows for the 51-year period, 1931 - 1981. A twelve-month planning horizon was modeled, with the control horizon allowed to 'slide' one month forward at a time. After the optimal release trajectory for a twelve-month control horizon was determined, the storage and release volumes during the first month of the control horizon were post-processed as described in §3.8. The control horizon was then advanced one month. A new optimal release trajectory for the control horizon was determined making use of any new information concerning the inflow during the new twelve-month control horizon, and using as initial trajectories, the optimal release and storage trajectories from the previous control horizon. The algorithm proceeded in this manner for 600 months (50 years). A 51-year record of inflows was required, because only the first month of the twelve-month control horizon is retained to pass through simulation post-processing. Therefore, to determine the optimal storage and release volumes for month 600, inflows for the entire twelve-month control horizon starting in month 600 are required. Optimal release trajectories were determined for two twelve-month forecast scenarios: perfect forecast ability, where inflows were observed values from the historical record; and no forecast ability, where forecasted inflows were historical mean inflows. In the latter case, following determination of an optimal release trajectory for the control horizon is determined, the sliding control horizon was advanced one month, and the assumed mean inflow for the first month of the control horizon was replaced by actual flow from the historical record.

The objective function used was the maximization of net revenues generated from hydropower production and groundwater pumping, subject to minimum and maximum storage and release constraints, and minimum water supply demands as shown in Equations 3.21. The source code, SRPOPT, was written in FORTRAN 77.

4.5.1 Optimal Operation under the Current Physical Configuration of the SRP Reservoir System

ELQG control under both forecast ability scenarios was applied to the operation of the SRP system under the present physical configuration (see Table 3.1). Summary results for mean monthly releases, power production, and net power revenues are given in Table 4.3

Table 4.32 shows that roughly a 9.9% increase in net revenues from hydropower generation for a perfect forecast was achieved under ELQG control as compared with the current SRP operation policy. For no forecast, an 8% increase was achieved, where it was assumed that the 'forecast' was always the historic mean for the given month.

Figure 4.1 shows mean reservoir storage levels for the different operating policies. Mean storage levels are significantly higher at Horseshoe under ELQG control (40 - 50 percent of capacity as

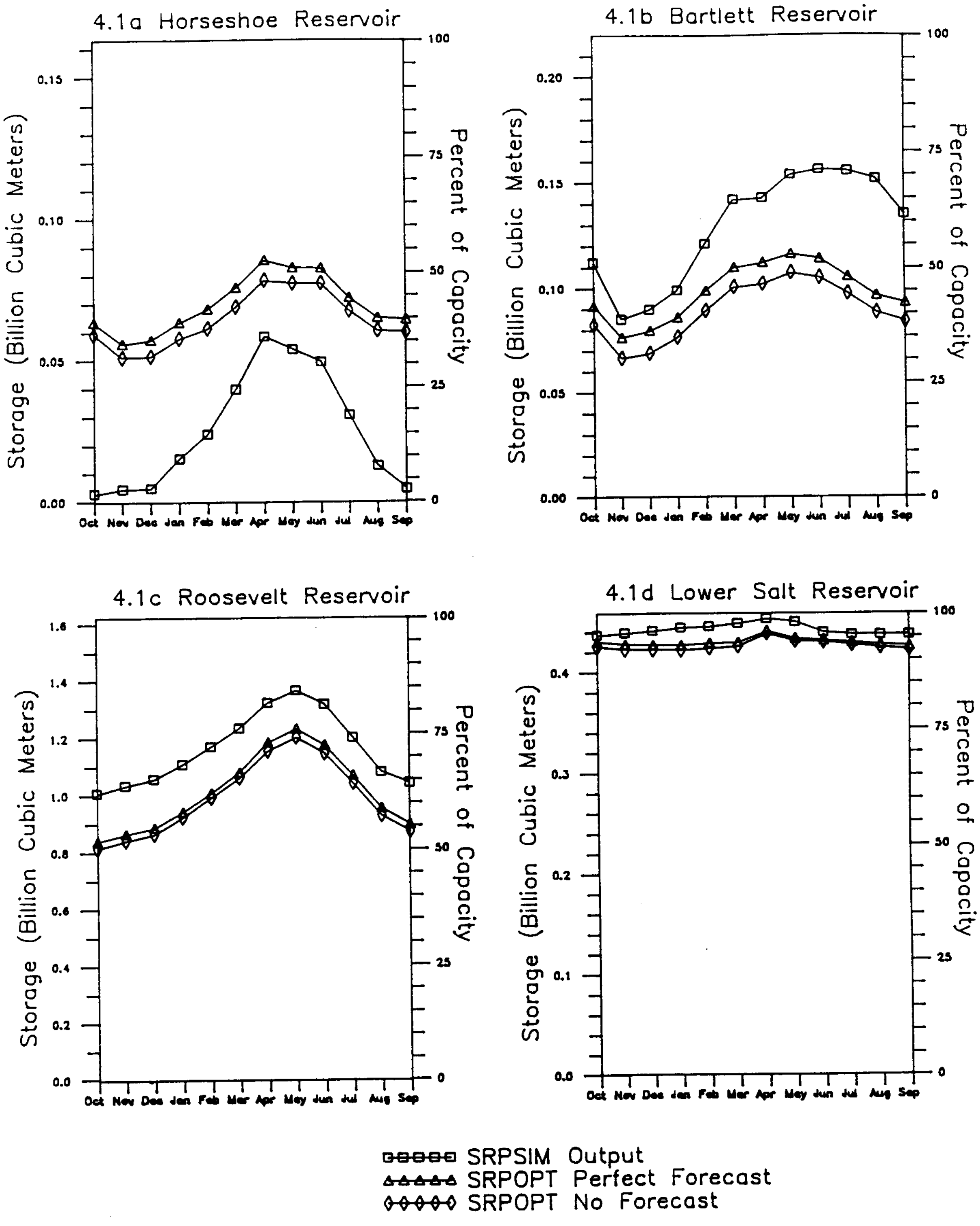


Figure 4.1 Fifty-Year Mean Reservoir Storage at Horseshoe, Bartlett, Roosevelt, and Lower Salt Reservoirs given Present Physical Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

compared to 5 - 30 percent of capacity under current SRP operating policy). Mean storage at the Lower Salt reservoir and Bartlett Reservoir were roughly comparable under the two operating policy algorithms, while mean storage at Roosevelt was slightly lower under ELQG control.

Table 4.3: Mean System Release, Net Energy Production, and Net Avoided Cost for the 50-Year Period 1931 - 1980 Under the Present Physical Configuration

	SRPSIM			SRPOPT Perfect Forecast			SRPOPT No Forecast			Nominal Demand
Month	Release	GWH	Million$	Release	GWH	Million$	Release	GWH	Million$	$10^9 m^3$
Oct	0.078	-10.5	-2.1	0.075	-6.8	-1.4	0.075	-7.5	-1.5	0.075
Nov	0.039	-7.2	-1.4	0.038	-4.2	-0.8	0.038	-4.4	-0.9	0.037
Dec	0.052	-5.4	-1.1	0.050	-4.6	-0.9	0.050	-5.0	-1.0	0.049
Jan	0.041	-1.6	-0.3	0.038	-2.5	-0.5	0.038	-3.3	-0.7	0.037
Feb	0.070	-3.2	-3.2	0.064	-3.4	-3.4	0.063	-4.0	-4.0	0.062
March	0.130	2.3	2.3	0.113	0.3	0.3	0.113	0.1	0.1	0.112
April	0.151	19.0	19.0	0.125	13.9	13.9	0.125	13.3	13.3	0.125
May	0.148	34.5	34.5	0.137	37.8	37.8	0.137	37.7	37.7	0.137
June	0.171	40.7	40.7	0.162	39.9	39.9	0.162	39.6	39.6	0.162
July	0.188	39.9	39.9	0.175	40.3	40.3	0.174	39.8	39.8	0.174
Aug	0.162	17.5	17.5	0.150	27.8	27.8	0.150	26.9	26.9	0.149
Sept	0.133	8.7	8.7	0.125	21.3	21.3	0.125	20.9	20.9	0.125
Annual		134.7	154.5		159.9	174.4		153.9	170.1	

Figure 4.2 shows mean releases made from the Verde River at Bartlett Reservoir, the Salt River at the Lower Salt reservoir, pump discharge from the groundwater reservoir, and total system release at Granite Reef (includes groundwater discharge volume). According to stated SRP operating policy, no releases are to be made from the Salt River reservoirs from October to April. However, as shown in Figure 4.2b, historically, releases would have been made from the Salt River reservoirs under the current operating policy, especially during March, when releases from the Salt River reservoirs accounted for about one-third of total system demand. Under

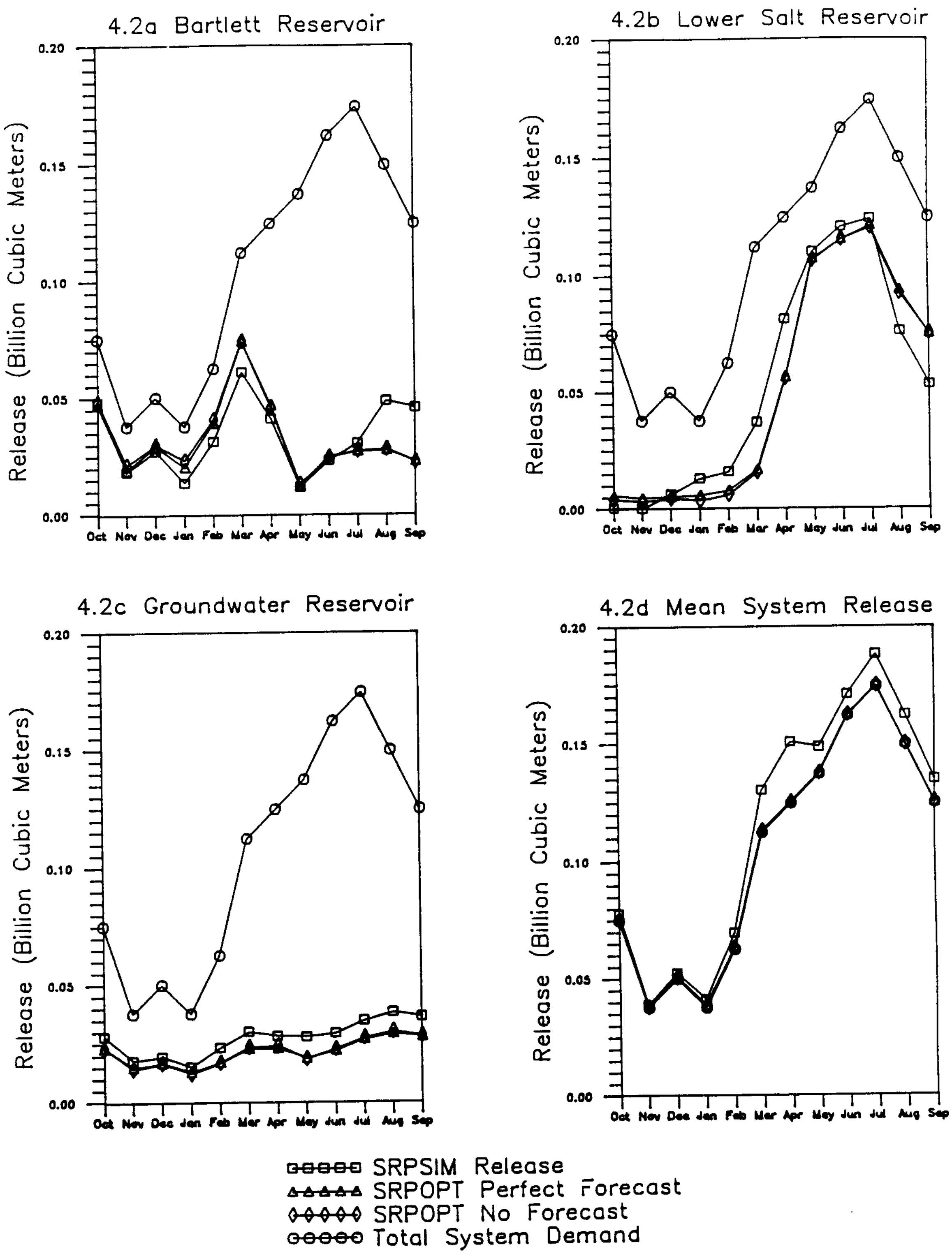


Figure 4.2 Fifty-Year Mean Verde River Release at Bartlett, Salt River Release at Lower Salt, Groundwater Pumping, and System Release at Granite Reef given Present Physical Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

ELQG control, mean Salt River reservoir releases were significantly lower, and as a consequence, closer to stated SRP operating policy.

Under current operating policy, SRP releases excess water during the high demand months (March through September). Under ELQG control, mean system release was very close to system demand during all months. The primary differences were lower groundwater pumping volume under ELQG control, and more release from the Salt River reservoirs later in the water year (August and September) under ELQG control, as opposed to Spring (February through April) under current operating policy.

Figure 4.3 shows the cumulative distribution frequency (CDF) plots for system release under ELQG control and under current SRP operating policy. The figure shows that under ELQG control, for a perfect forecast, there was a smaller range of releases. Most releases cluster around the expected system demand. Under the current policy, there were more releases significantly above the expected system demand levels, especially during the snowmelt months of March, April, and May, and releases were uniformly above system demand during the summer period, June through September.

Figure 4.4 shows monthly energy generation avoided cost for the Salt River reservoirs, mean monthly pumping costs for the groundwater reservoir, and net system energy generation avoided cost. Avoided cost from energy generation were roughly comparable between the current operating policy, and the policy determined by ELQG control. The main difference was a significantly higher pumping cost under the current operating policy: mean annual pumping cost under ELQG control is about 75% of the cost under the current operating policy ($98 million under current operating policy, and $75 million under ELQG control).

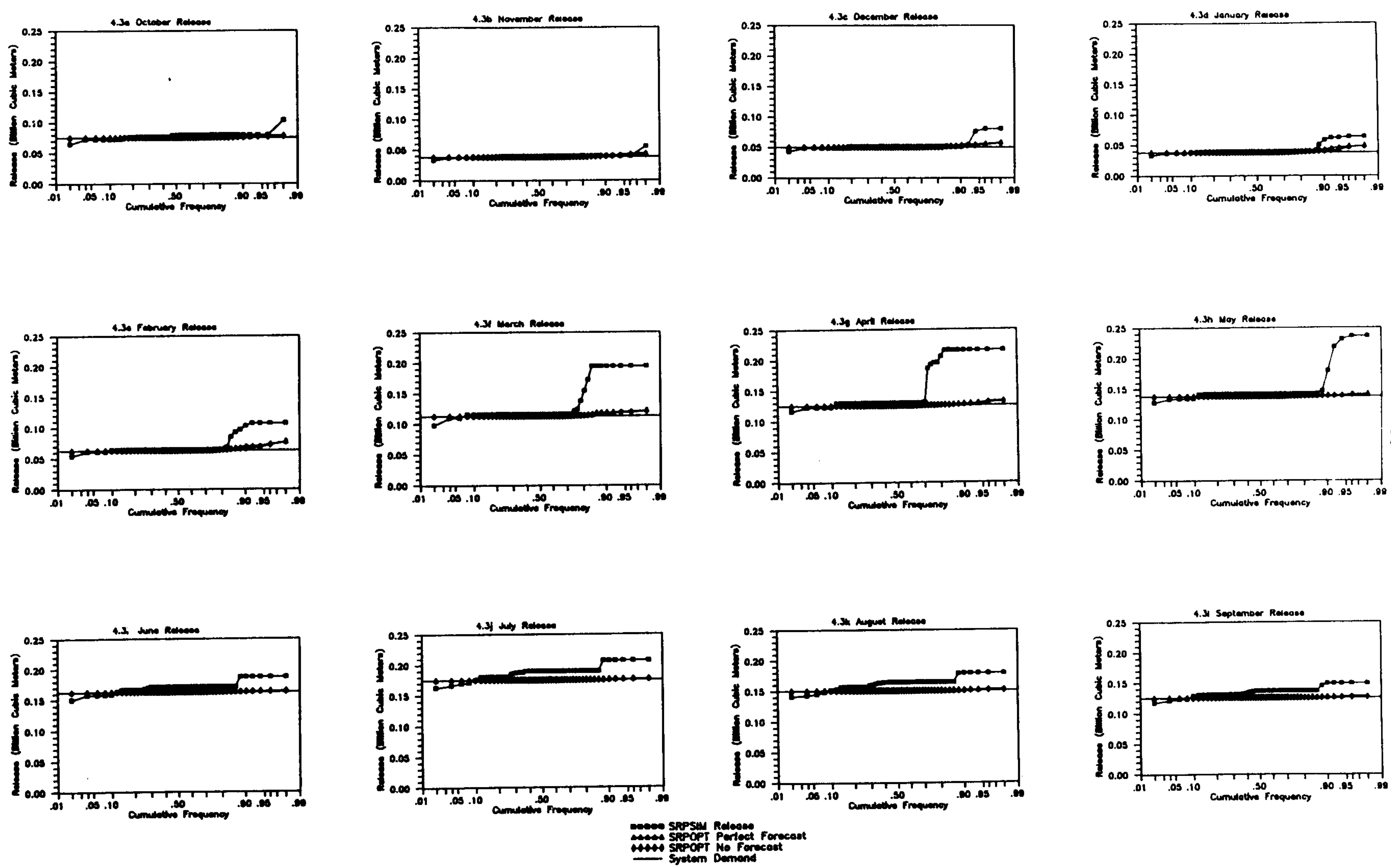


Figure 4.3 Fifty-Year Empirical Cumulative Frequency Distribution Functions for System Release at Granite Reef given Present Physical Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

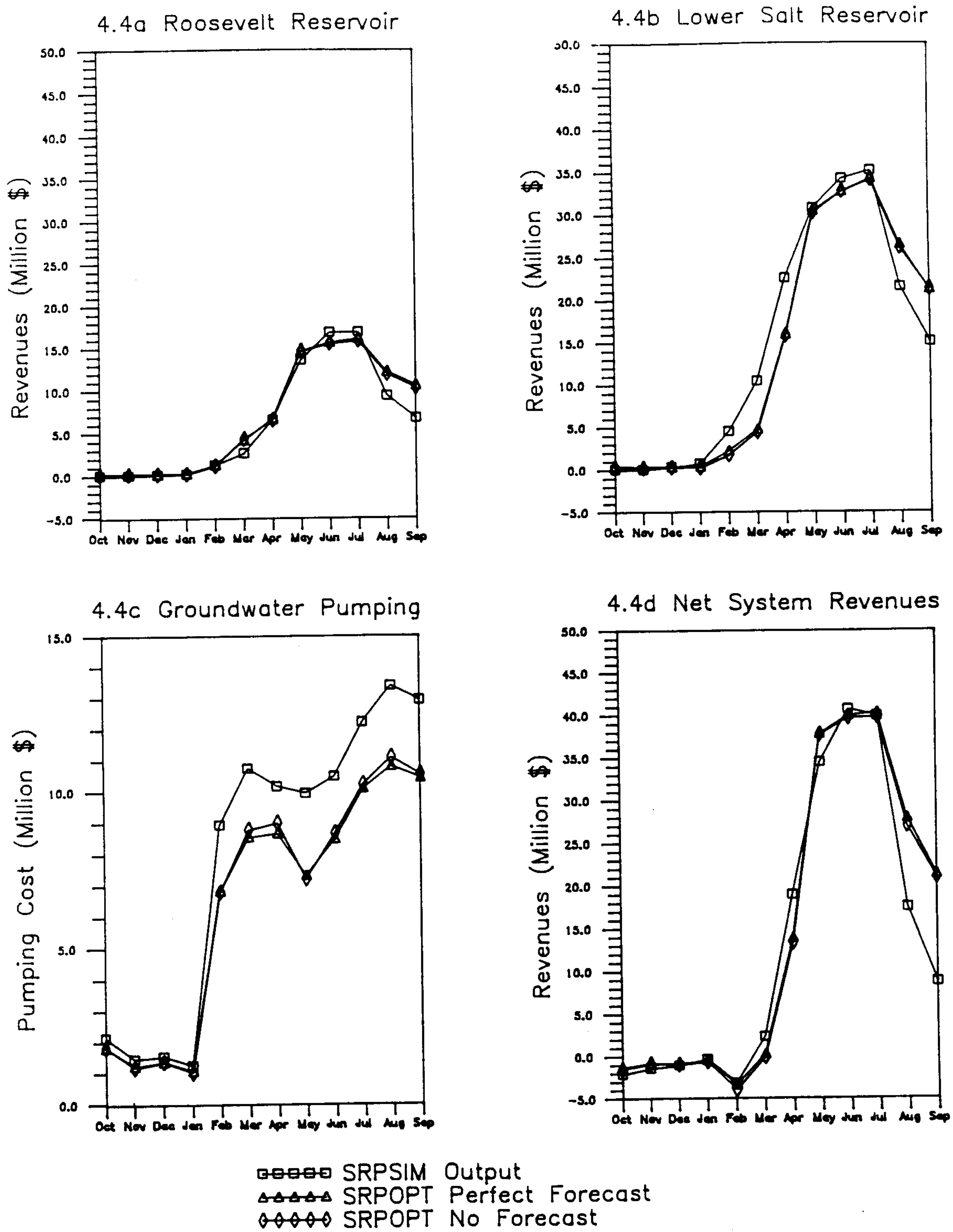


Figure 4.4 Fifty-Year Mean Energy Generation Avoided Cost, Groundwater Pumping Cost, and Net System Energy Generation Avoided Cost given Present Physical Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

Figure 4.5 illustrates the cumulative frequency of storage levels at Roosevelt. Of primary interest is the frequency of capacity storage levels. A slightly lower probability of capacity storage occurred under ELQG control. This translates to a lower frequency of uncontrolled spills at Roosevelt.

4.5.2 Optimal Operation under Reduced Storage Conditions

ELQG control under both forecast scenarios was applied to the SRP reservoir system under a scenario corresponding to a reduction in usable storage at Roosevelt. Maximum storage at Roosevelt was reduced from 1.63 billion cubic meters to 1.21 billion cubic meters, a reduction of nearly 25%, or about a 17% decrease in total system storage. Summary results for this scenario are presented in Table 4.4.

Table 4.4 shows a 93% increase in net avoided cost from hydropower generation given perfect forecast ability and a 70% increase given no forecast ability under ELQG control over the current SRP operating policy. The large difference in avoided cost emphasizes the ineffectiveness of the the current heuristic rule curve operating policy under altered conditions. The same overall trends are seen under the reduced storage scenario as for the present condition scenario, specifically, releases generally exceed system demand under the present operating policy due primarily to overpumping of groundwater. The ELQG operating policy did slightly better job of minimizing spills, and storage levels resulting from the operating policy under ELQG control have a smaller range, with minimum storage levels greater than the storage levels determined under the present operating policy.

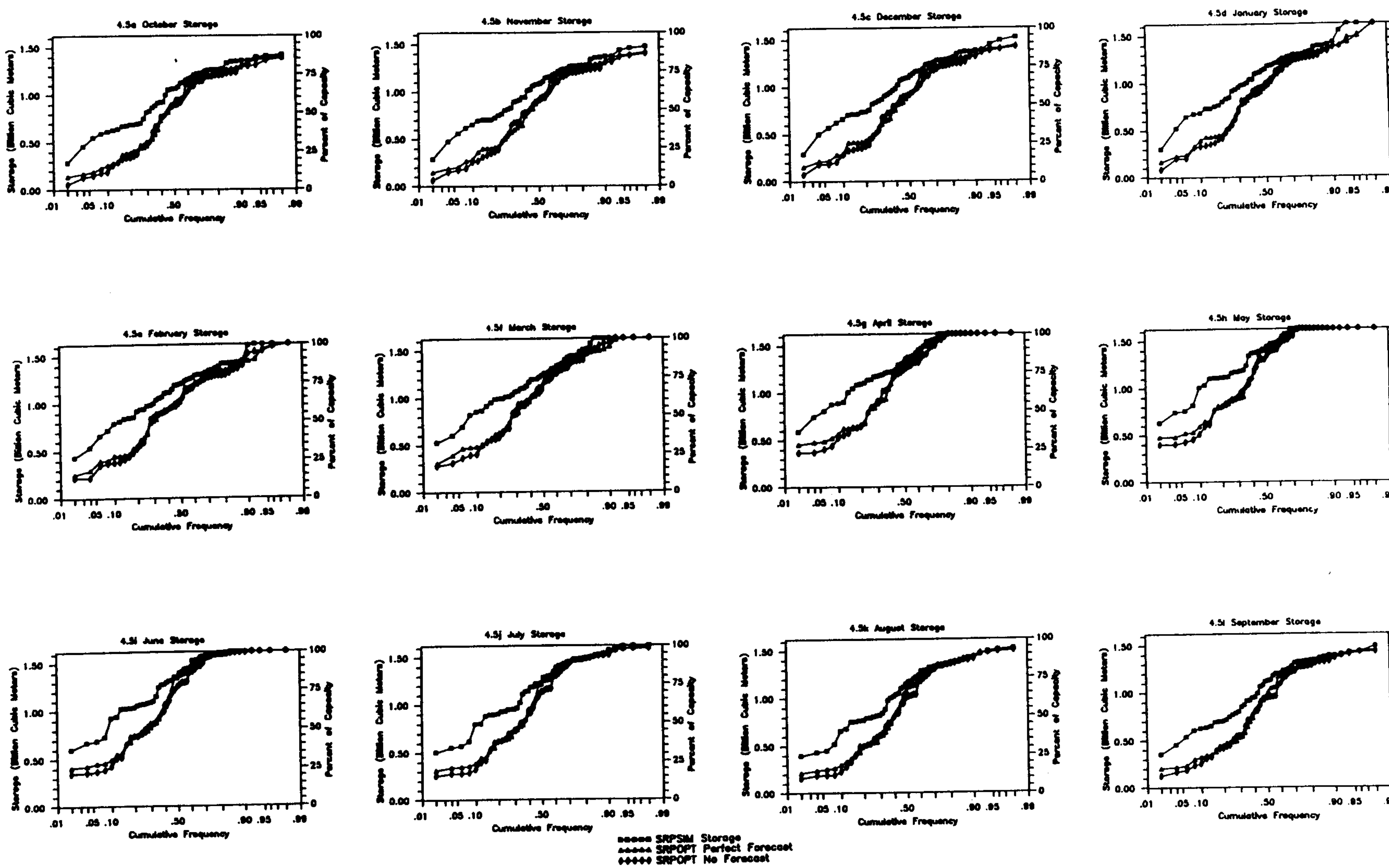


Figure 4.5 Fifty-Year Empirical Cumulative Frequency Distribution Functions for Storage at Roosevelt Reservoir given Present Physical Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

Table 4.4: Mean System Release, Net Energy Production, and Net Avoided Cost for the 50-Year Period 1931 - 1980 Under Reduced Storage Conditions

	SRPSIM			SRPOPT Perfect Forecast			SRPOPT No Forecast			Nominal Demand
Month	Release	GWH	Million$	Release	GWH	Million$	Release	GWH	Million$	$10^9 m^3$
Oct	0.079	-10.8	-2.2	0.076	-6.9	-1.4	0.075	-8.2	-1.6	0.075
Nov	0.040	-7.4	-1.5	0.043	-2.6	-0.5	0.042	-3.5	-0.7	0.037
Dec	0.054	-7.6	-1.5	0.059	-2.3	-0.5	0.056	-4.3	-0.9	0.049
Jan	0.042	-4.3	-0.9	0.050	1.0	0.2	0.047	0.0	0.0	0.037
Feb	0.072	-5.1	-5.1	0.078	0.9	0.9	0.076	1.3	1.3	0.062
March	0.136	1.0	1.0	0.125	1.3	1.3	0.121	1.6	1.6	0.112
April	0.155	18.6	18.6	0.136	15.6	15.6	0.132	17.1	17.1	0.125
May	0.148	20.1	20.1	0.145	38.7	38.7	0.142	37.0	37.0	0.137
June	0.169	20.4	20.4	0.165	37.4	37.4	0.164	34.2	34.2	0.162
July	0.186	25.3	25.3	0.176	35.5	35.5	0.175	31.0	31.0	0.174
Aug	0.165	8.4	8.4	0.152	23.8	23.8	0.147	17.1	17.1	0.149
Sept	0.133	5.7	5.7	0.130	20.2	20.2	0.128	14.7	14.7	0.125
Annual		64.5	88.6		162.8	171.5		138.2	150.9	

Mean storage levels, shown in Figure 4.6, were similar under ELQG control and the present operating policy, except at Horseshoe Reservoir. Under the present operating policy, with reduced system storage, the Horseshoe Reservoir was virtually always emptied during August and September, and storage during the remaining months was significantly lower than storage volumes under ELQG control, as shown in Figure 4.6a. Figure 4.7 illustrates a slight difference in operating policy between the current policy and the policy determined under ELQG control: more water was released from the Salt River reservoirs during the summer months under ELQG control, while more water was released during the spring runoff months under the present policy. Figure 4.8 shows the effect of reducing available storage: many more releases in excess of system demand occurred, especially during the spring runoff months. Neither the policy developed under ELQG control nor the present policy performed well in controlling spills Figure 4.9 shows

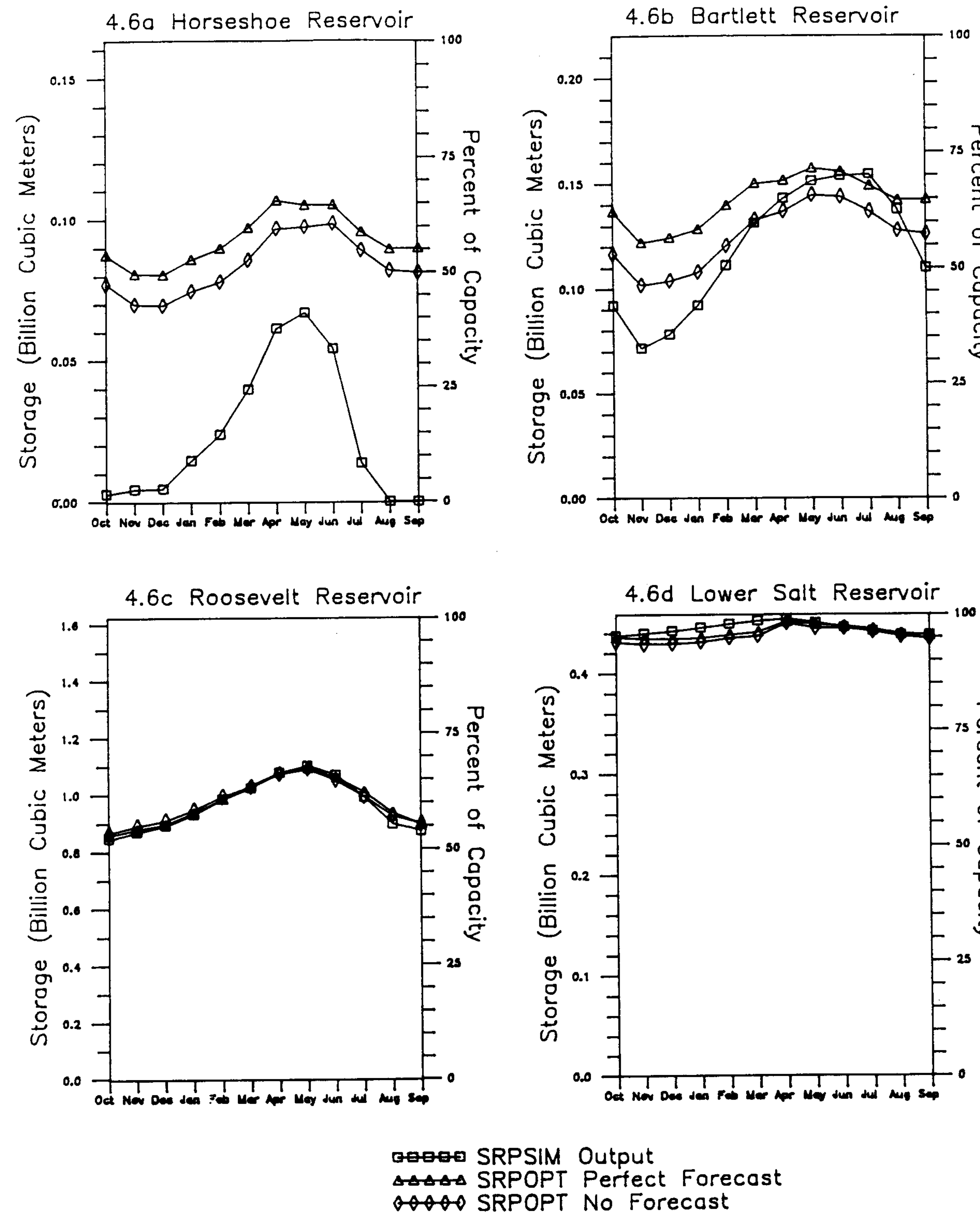


Figure 4.6 Fifty-Year Mean Reservoir Storage at Horseshoe, Bartlett, Roosevelt, and Lower Salt Reservoirs given Reduced Roosevelt Storage Conditions under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

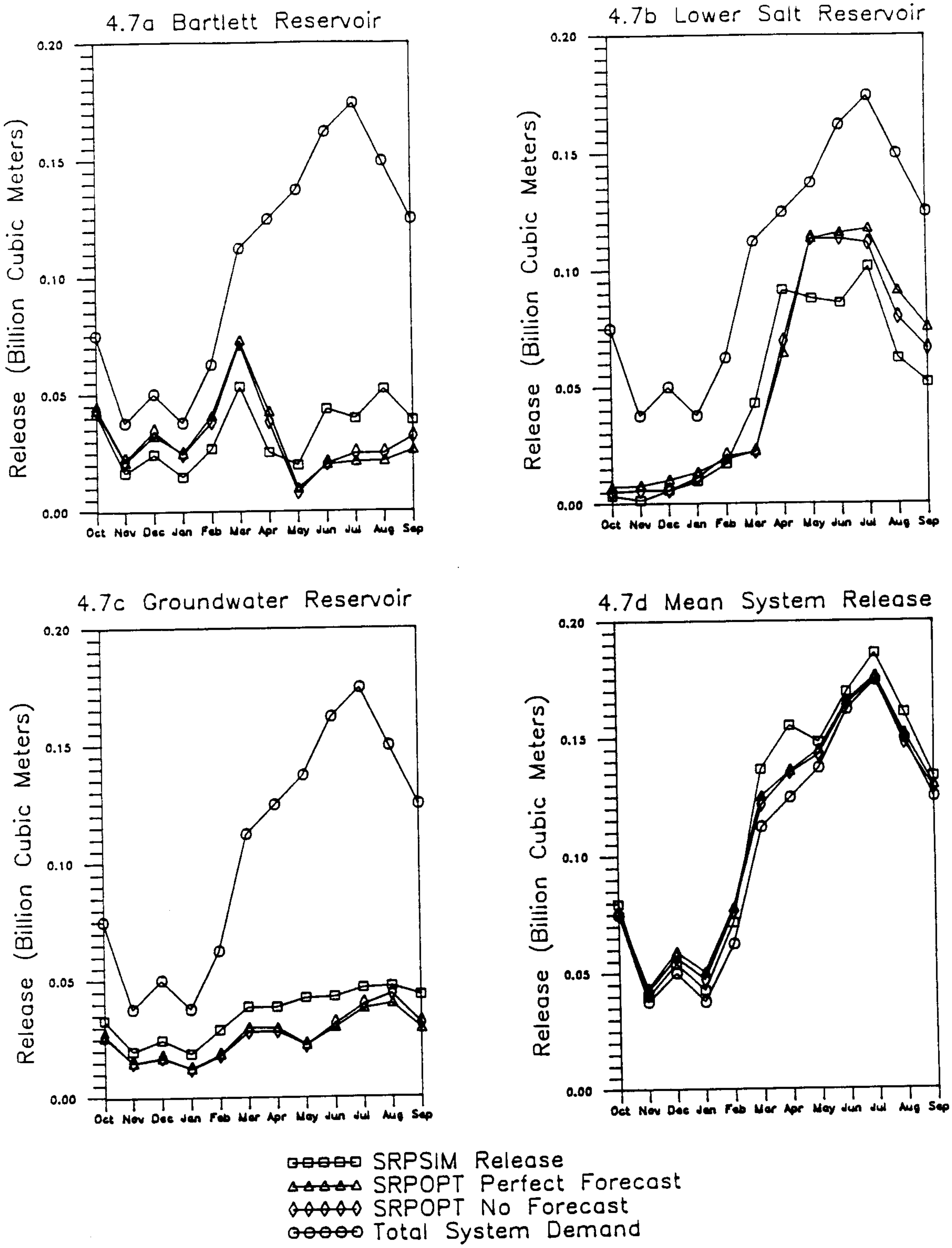


Figure 4.7 Fifty-Year Mean Verde River Release At Bartlett, Salt River Release at Lower Salt, Groundwater Pumping, and System Release at Granite Reef given Reduced Storage Conditions at Roosevelt under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

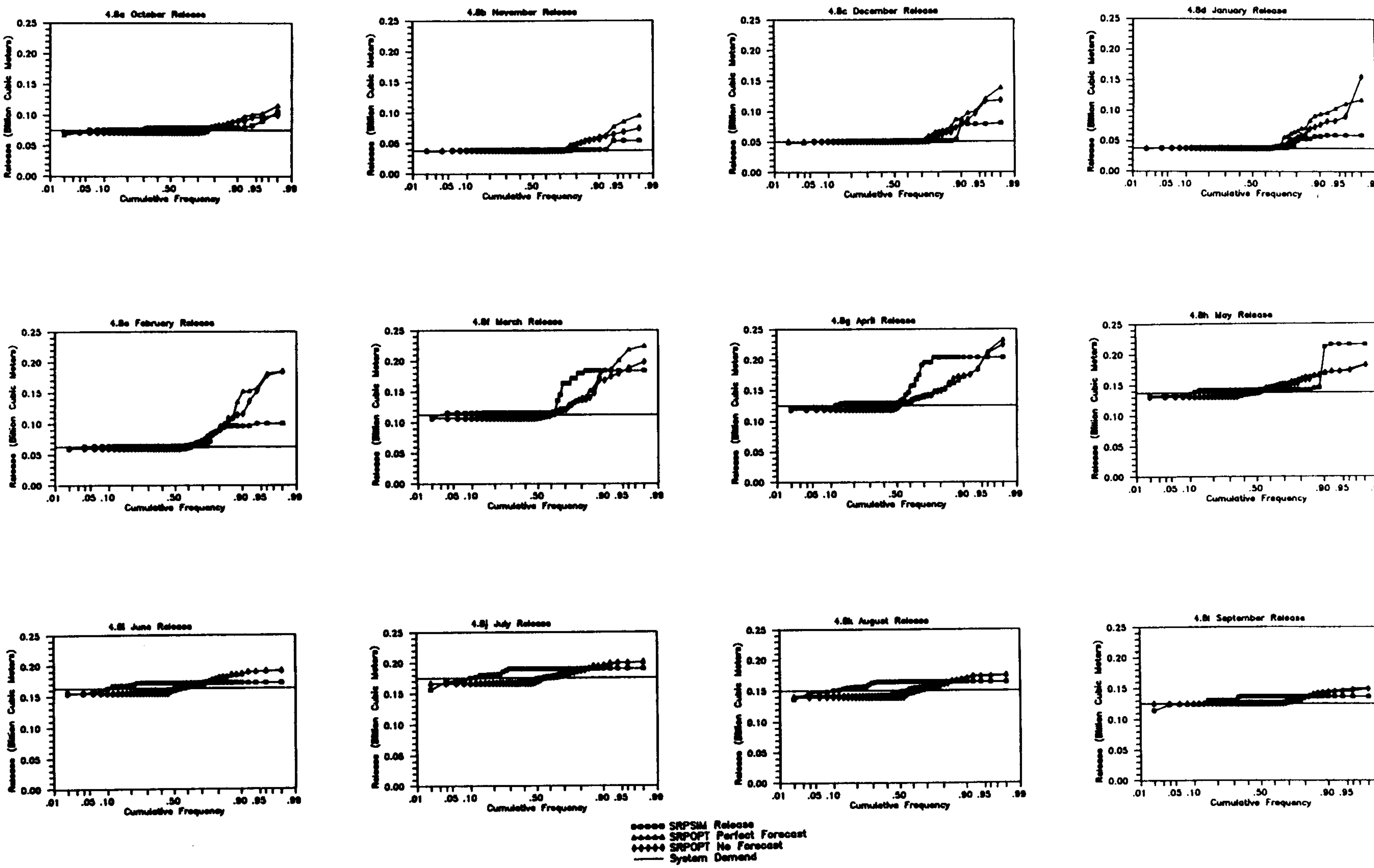


Figure 4.8 Fifty-Year Empirical Cumulative Frequency Distribution Functions for System Release at Granite Reef given Reduced Storage Conditions at Roosevelt under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

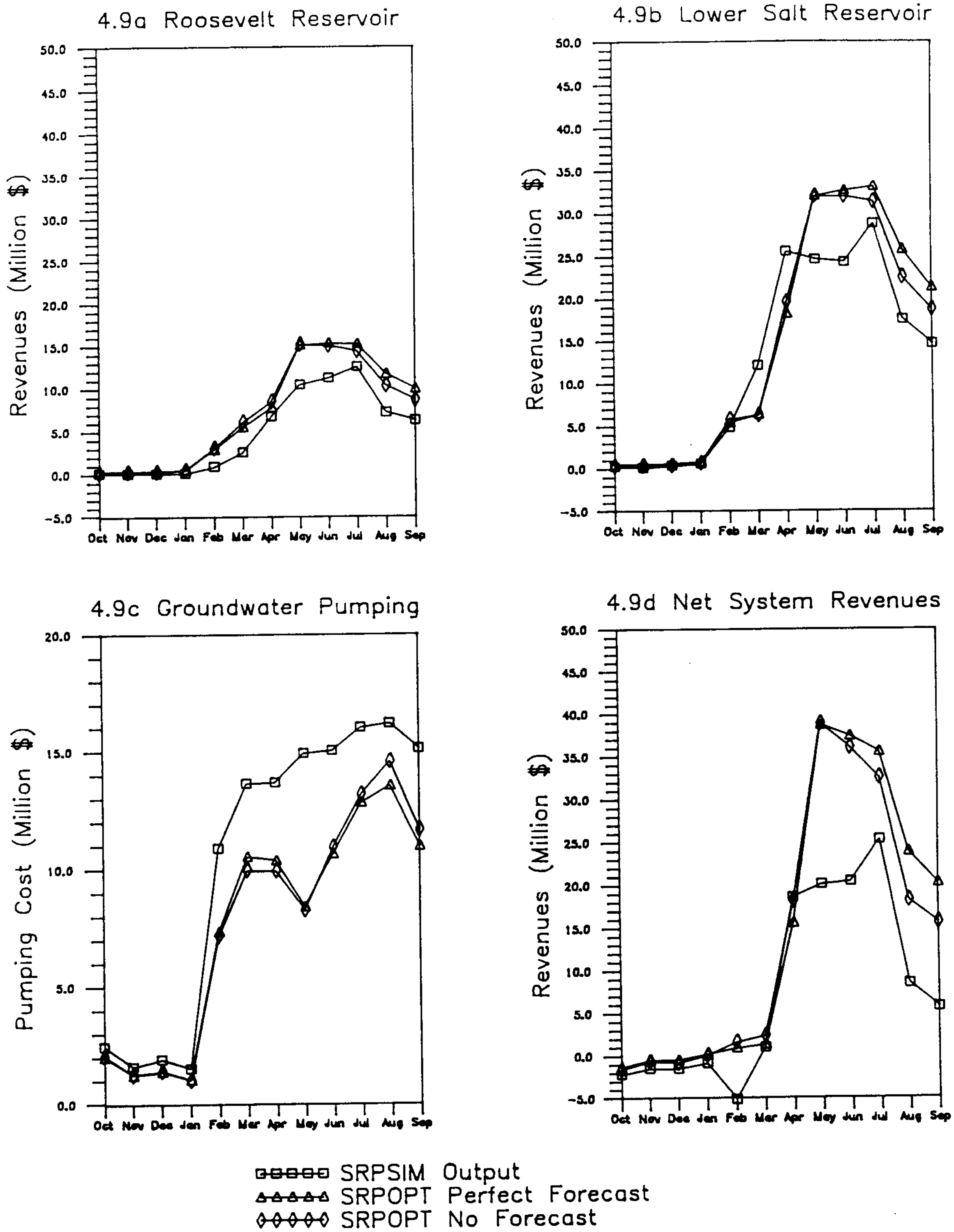


Figure 4.9 Fifty-Year Mean Energy Generation Avoided Cost, Groundwater Pumping Cost, and Net System Energy Generation Avoided Cost given Reduced Storage Conditions at Roosevelt under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

that under ELQG control net avoided cost increased, primarily due to lower groundwater pumping costs, and reduction of spill volume during the late summer months. Figure 4.10 shows that storage at Roosevelt was slightly less variable under ELQG control, more extreme low volumes occured under the current policy.

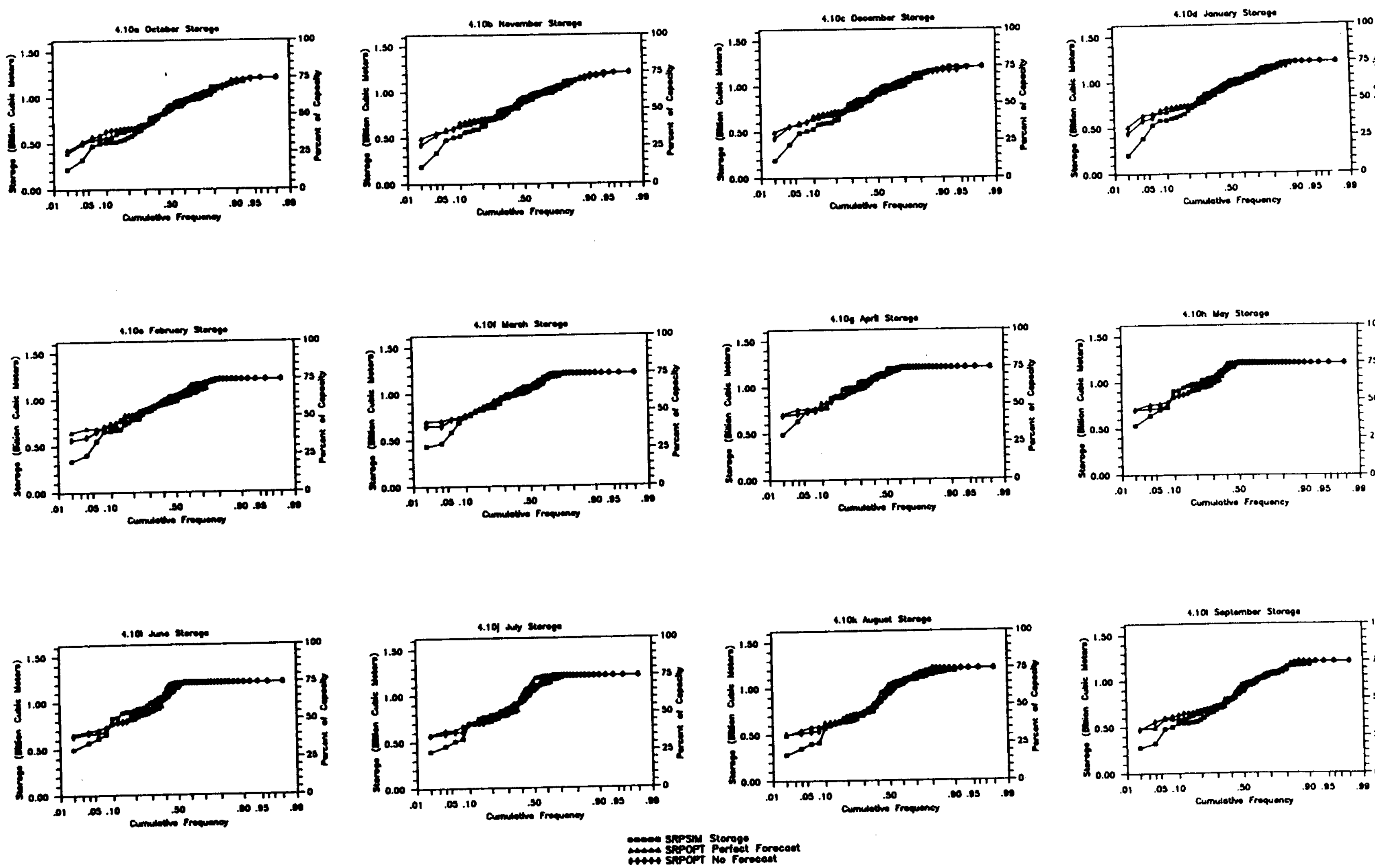


Figure 4.10 Fifty-Year Empirical Cumulative Frequency Distribution Functions for Storage at Roosevelt Reservoir given Reduced Storage Conditions at Roosevelt under the Present Operating Policy with Perfect Forecast Ability, and the Optimal Operating Policy with both Perfect and No Forecast Ability

CHAPTER 5: SUMMARY AND CONCLUSIONS

5.1 Summary of Results

Extended linear quadratic Gaussian (ELQG) control, a non-linear stochastic control method for optimal control of multi-objective reservoir systems, was applied to the Salt River Project (SRP) reservoir system. ELQG control is a trajectory-iteration algorithm. When given an initial release trajectory, it will move, according to values obtained from a penalty function based objective function, along a Newton's direction, to an optimal release trajectory.

The method was used to determine an operating policy for the SRP reservoir system under two system configurations. These were the present configuration, and a reduced storage configuration that mimicked the reduction in usable storage at Roosevelt Lake that will be necessary during the construction period for the planned raising of Roosevelt Dam. The ELQG optimal operating policy was compared using simulation to the present operating policy (derived from rule curve analysis) for both system configurations. The basis for the comparisons was the net hydropower avoided cost, computed by applying a seasonally varying avoided cost (essentially the cost that would be incurred through generation of power from the next most costly generating resource) to the net hydropower generation (turbine generation less groundwater pumping) for each month. The ability of the two operating policies to meet water supply requirements, which were treated as a constraint in the optimization model, was also compared.

Under present conditions, ELQG policy was found to result in an increase in hydropower avoided cost of about 8 percent for no forecast, and 10 percent for a perfect forecast. The major difference between the two operating policies was reduced groundwater pumping under ELQG control. The operation of the Verde River reservoirs represented another major difference between the two operating policies. Under the present policy, Horseshoe Reservoir was virtually always empty during the summer months of August and September; the ELQG policy generally resulted in higher summer storage. In addition, under ELQG control there were also fewer extremely low storage volumes at Roosevelt Reservoir.

For the reduced storage configuration, ELQG resulted in a 70 percent increase in hydropower avoided cost for no forecast, and an increase of over 90 percent for a perfect forecast. The present operating policy resulted in much more frequent, and larger, spills than did ELQG control. This represents the major difference between the two operating policies. Under the present operating policy, variations in storage were much greater than under ELQG control, and when water supply shortfalls occurred, they tended to be more severe. It is likely that a better heuristic operating policy could be found for the reduced storage configuration given the much smaller difference in hydropower avoided cost for the present configuration. If such an effort is undertaken, the ELQG policy should provide some guidance as to the direction in which improvements could best be achieved.

5.2 Further Research Recommendations

One area in which improvements in the ELQG algorithm could likely be improved is replacement of the quadratic penalty function for storage. The quadratic penalty function tends to discourage deviations in storage from the target value, while in fact it is only necessary that the storage lie in the range from empty to full. Georgakakos (1989) has employed a barrier function in place of the quadratic penalty function which penalizes the objective function value only as bounds are approached. The function is shown below:

$$C(s) = \left[\frac{s_j^{max}(k) - s_j(k)}{s_j^{max}(k) - s_j^{min}(k)}\right]^{-\mu} \cdot \left[\frac{s_j(k) - s_j^{min}(k)}{s_j^{max}(k) - s_j^{min}(k)}\right]^{-v} \qquad 5.1$$

Figure 5.1 shows values for the barrier function given different values of μ and v. One problem with use of the barrier function is that it is not defined beyond the bounds, which can present problems for the numerical search procedure. Given the extreme variation in historic inflows to the SRP system, simulation of the system given historical inflows usually results in storage violations during the calculations of initial storage trajectories. Therefore, the use of a barrier function of this type for the SRP system is limited. A different form of the barrier penalty function which avoids the numerical problems associated with storage limits is:

$$C(s) = {s^*}^{\beta} \cdot e^{ks^*} \qquad 5.2$$

where β and k are positive integers, and

$$s^* = \frac{s - s_T}{s_{max} - s_T} \qquad \text{for } s_T < s < s_{max}$$

$$s^* = 1 - \frac{s - s_{min}}{s_T - s_{min}} \qquad \text{for } s_T > s > s_{min}$$

Values of this barrier function for different values of k and β are shown in Figure 5.2. Two year simulations of the SRP system under this form of a penalty function were tested.

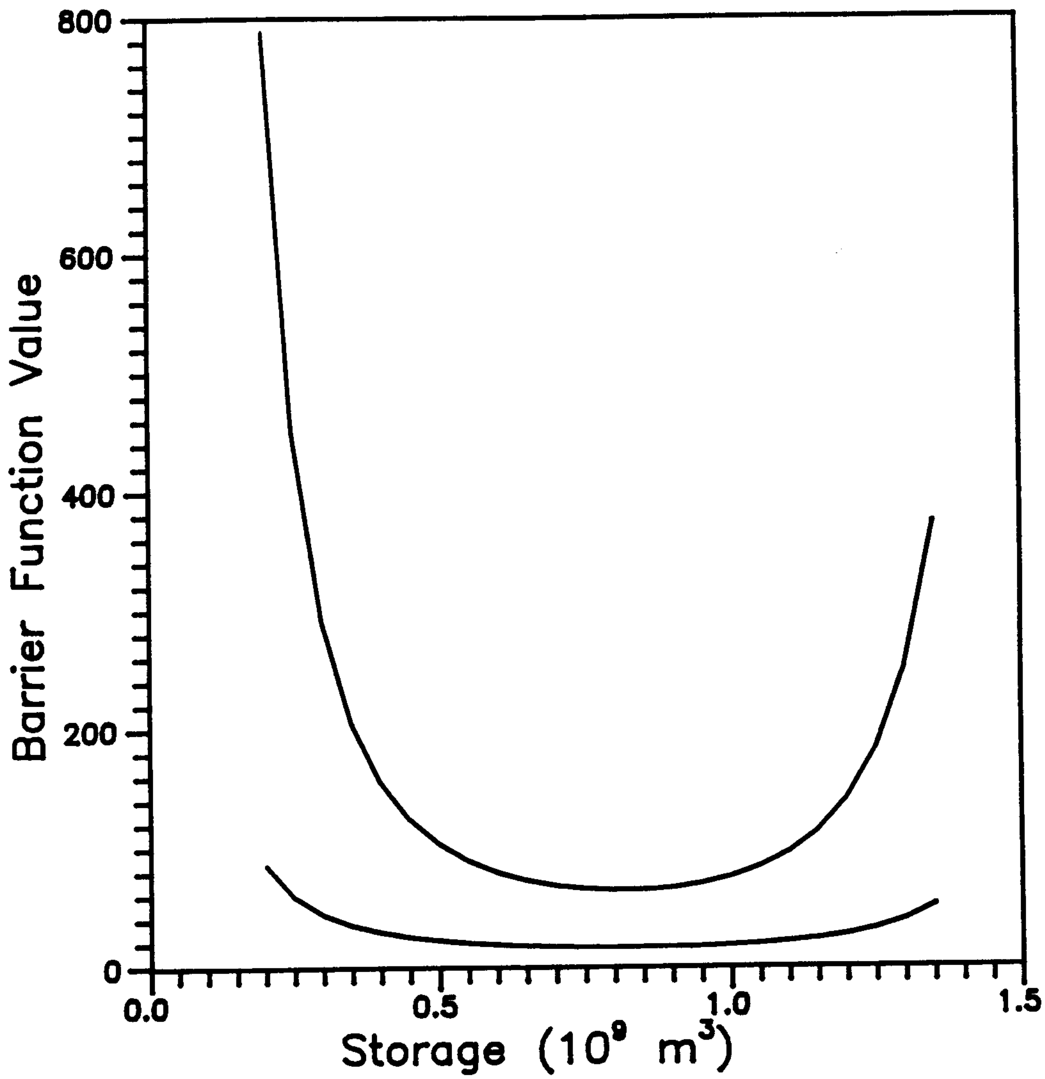


Figure 5.1 Barrier Function Values for Introducing Storage Constraints at Roosevelt Reservoir

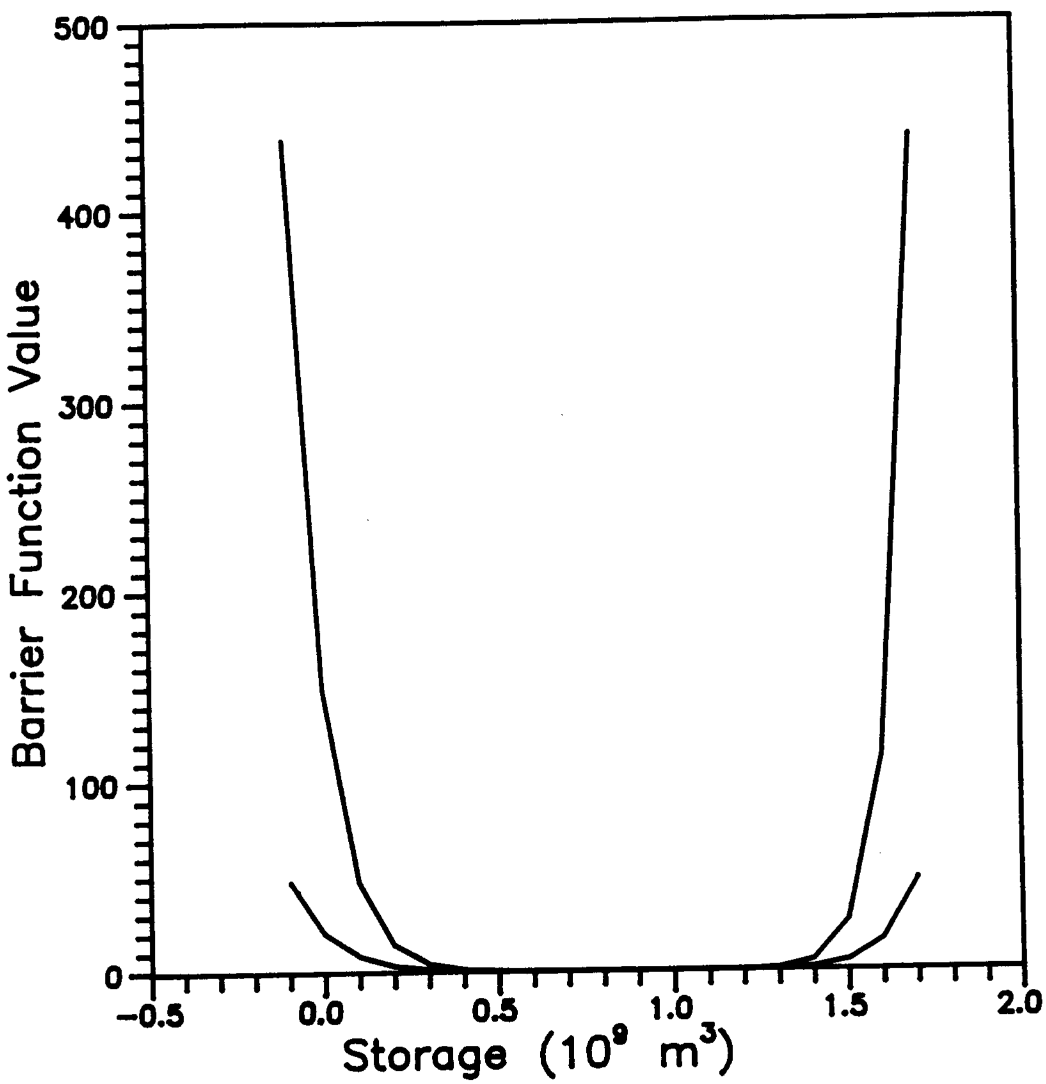


Figure 5.2 Alternative Barrier Function Values for Introducing Storage Constraints at Roosevelt Reservoir

Convergence times were on the order of five to ten times more iterations than were required under the quadratic penalty function, because the function is less sensitive to storage deviations from the target. Additional work needs to be done to determine the sensitivity of the operating policy to the penalty function.

Another area in which additional work should be done is integration of operational forecasts. SRP currently obtains long-term runoff forecasts from several sources. In the work reported here, we tested only a perfect and no forecast case. One problem is that ELQG assumes that the probability distribution of forecast errors in Gaussian, while in fact forecast errors for the SRP system are expected to have strong positive skew. Sensitivity tests need to be performed to determine the importance of this assumption. If necessary, additional model development may be necessary to relax the Gaussian forecast error assumption. Once a method of integrating operational forecasts has been developed, it will be possible to use the model to evaluate the worth of alternative forecasting algorithms and forecast data collection networks.

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