DEVELOPMENT AND APPLICATION OF FRESH:
A FUZZY REASONING SHELL

Shinji Nishida
Richard N. Palmer

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by Shinji Nishida

and

Richard N. Palmer

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December 1989

Environmental Engineering and Sciences
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CHAPTER ONE

Introduction

Expert systems are user-friendly, interactive computer software that use domain specific knowledge, in the form of rules and heuristics, and inference procedures to solve complex problems. Some specific domains for which expert systems have been designed include the area of medical diagnosis, engineering, decision making, management forecast, and geological exploration. These expert systems consist of a domain specific knowledge base and reasoning algorithms known as an inference engine. Some systems consist of an inference engine shell to which various knowledge bases can be added.

The development of knowledge-based expert systems has become a popular topic for scientists and engineers in the late 1970's and 1980's. Among these systems are MYCIN (Shortliffe, 1974), that diagnoses infectious diseases; PROSPECTOR (Duda et al, 1978), that provides consultation to geologists for ore-grade deposits; and XCON (Kraft, 1984), a tool that configurates DEC VAX-11/780 computer systems. The development of powerful and economical personal computers has accelerated the application of expert systems.

In spite of these successes, the development,
maintenance, and enhancement of expert systems are still difficult and complex tasks. Two significant obstacles to the successful development and application of expert systems are knowledge acquisition and the management of uncertainty in the rules and heuristics.

Knowledge acquisition is the process of obtaining knowledge or expertise from human experts and formulating this information into a logic structure that can be interpreted by a computer. This process of knowledge acquisition contains two key technical roles: domain experts and knowledge engineers. A domain expert is a person knowledgeable in a specific field. Domain experts often have obtained a collection of definitions, specialized facts, strategies and heuristics in a narrow domain area. They seldom have experience with expert system technology. A knowledge engineer is the individual who extracts knowledge from domain experts and translates it into expert system rules. Knowledge engineers typically have experience with computer technology, computer languages, and software. They also must have training in interviewing techniques and constructing logic structures for combining knowledge into a decision.

Knowledge acquisition is often cited as an impediment to the development of expert systems because no standard
technique exists to extract knowledge from experts (Buchanan and Shortliffe, 1984). In addition, the interpretation of experts' knowledge by the knowledge engineer is always a potential source of error. Therefore, this process requires the extensive efforts of both domain experts and knowledge engineers in development of successful expert systems.

The management of uncertainty is also important in expert systems development because the human decision making process is based on imprecise, incomplete, or unreliable information and rules (Zadeh, 1983). Certainty factors (representing the confidence that one has in a piece of evidence) and propositional logic are used often in expert system shells to deal with uncertainty.

However, the combination of certainty factors and propositional logic to deal with uncertainty has serious shortcomings. It is difficult to justify their use as a truly accurate model of human thought or human heuristics. In recent years the application of fuzzy logic has been suggested as a framework for managing uncertainty and possibility in expert systems (Zadeh, 1983). It is around this approach that this thesis centers.

This research develops a fuzzy logic based shell, FRESH (Fuzzy Reasoning Shell) that integrates a menu-driven
system, user input screens, graphics, and a fuzzy inference engine. Unlike conventional expert system shells or AI special languages, FRESH allows experts to build a fuzzy reasoning system without extensive computational skills. Therefore, FRESH has a potential to solve the two previously mentioned problems in expert systems development: knowledge acquisition and the management of uncertainty.

FRESH has been used to develop an operational rule base for NWWA (North West Water Authority) reservoir system management in England. The application result shows that FRESH provides an interactive knowledge acquisition environment to construct reservoir operating rules. The proposed operational rule base used here has a very simple production (if-then) rules without forward/backward chaining in rules. However, the present state of fuzzy logic will show that this structure can be extended to more complex settings including forward/backward chaining in the future.

The contents of this thesis are organized as follows: Chapter Two describes two shortcomings of current expert system technology. Chapter Three reviews fuzzy logic principles. Chapter Four compares propagation of uncertainty based on conventional expert systems logic and fuzzy logic. Chapter Five describes the characteristics of FRESH. Chapter Six presents an application result for the
NWUA reservoir system. Chapter Seven contains conclusions and indicates potential further research.
CHAPTER TWO

Limitations of Expert Systems

A conventional knowledge base for an expert system is constructed through the interactions of two groups, domain experts and knowledge engineers. Intuitively, the most direct approach is for an expert to place his or her knowledge directly into a knowledge base. When a knowledge engineer and an expert are not the same person, building a complex knowledge base (one embodying uncertain judgemental knowledge) is a difficult task. Generally, it is very time consuming to construct and debug such a knowledge base system, and the difficulties in constructing such a knowledge base are often underestimated.

One particularly difficult aspect of translating human decision making into an expert system is modeling uncertainty and incomplete information. This problem is fundamental to accurately modeling human decision making and it is an area of research that has many conflicting and competing approaches. Both of the problems, knowledge acquisition and modeling uncertainty, are discussed below.
Knowledge Acquisition

Knowledge acquisition is the most important and most difficult step in developing successful expert systems. This task requires a substantial investment of careful thought and analysis. Because knowledge engineers typically are not experts in the domain in which they are developing an application, the knowledge engineers must overcome communication problems during the process of developing the knowledge base of the expert system. Also, human knowledge or expertise is not always well organized. Thus, the knowledge engineer must be an effective communicator, an organizer, and a logician. The testing and validation of an expert system are also important tasks for knowledge engineers. If the knowledge engineer is unable to adequately extract information from domain experts, the expert system cannot be successful.

There are several strategies to capture knowledge acquisition for expert system development. Table 2-1 summarizes five variations of knowledge acquisition modes suggested by McGraw and Harbison-Briggs (1989).

In the first mode, the knowledge engineer extracts the domain knowledge from human experts and translates it into a knowledge base. In this mode, Parsaye (1985) suggests three major approaches: interviewing experts, learning by
Table 2-1. Variations in Possible Knowledge Acquisition Mode

<table>
<thead>
<tr>
<th>KNOWLEDGE SOURCE</th>
<th>KNOWLEDGE ACQUISITION MODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Human Experts</td>
<td>Knowledge Engineer</td>
</tr>
<tr>
<td>(2) Data</td>
<td>Knowledge Engineer</td>
</tr>
<tr>
<td>(3) Human Experts</td>
<td>Editing Program</td>
</tr>
<tr>
<td>(4) Human Experts</td>
<td>Learning Program</td>
</tr>
<tr>
<td>(5) Texts</td>
<td>Text Understanding Program</td>
</tr>
</tbody>
</table>
observation, and learning by being told. In the first approach, knowledge engineers have formal or informal meeting with domain experts and extract their expertise. This is the most economical and flexible approach. In the second approach, knowledge engineers observe decision processes of domain experts without interference and extract it through various sample scenarios. Learning by observation permits human experts to concentrate on their decision-making process. The third approach requires domain experts to explain their decision-making process to knowledge engineers. This approach will be successful if human experts can express and refine their rules and heuristics precisely. However, the first two approaches require well-trained knowledge engineers, and all approaches require a great amount of time.

The second mode suggests that the knowledge engineer studies more formal sources of information and encodes it into a knowledge base. Textbooks, journal articles, experimental data, videotapes, and other such material are typical examples of where information can be found. This mode does not require the contribution of human experts; however, for many areas of interest, no sources of such information exist. Thus, this mode is often used as a supplement to the first mode.

The combination of the first two modes is often used in
conventional expert system development. However, it requires the attention and efforts of both domain experts and knowledge engineers. The interpretation of the experts' knowledge by the knowledge engineer is always a potential source of error.

The third mode suggests another approach for knowledge acquisition. In this mode, the human experts construct a knowledge base directory using their rules and heuristics with the aid of an editing program. Two types of commercial editing programs are currently available for the personal computer environment: expert system shells, such as INSIGHT2+ (Level Five Research, Inc., 1986) and VP-EXPERT (Paperback Software International, 1987); and AI special languages, such as LISP and PROLOG. In the case of using conventional expert system shells or AI special languages, the human expert must have extensive computational skills to develop useful knowledge-based expert systems. This is the reason why most expert systems are developed by a combination of the first two modes. However, if an acceptably intelligent editing program is available, this approach is the most precise, cost effective, and efficient prototype because it can avoid the intermediate steps that require the knowledge engineers.

One approach to aid constructing a knowledge base is an
induction table. An induction table represents production (IF-THEN) rules in a simple manner. Using this procedure, users can create a knowledge base. Expert system shells that have this feature include VP-EXPERT and EXSYS (Exsys, Inc., 1986). However, experts must learn a considerable amount about programming before such techniques can be used to develop useful knowledge bases.

The fourth mode represents "machine learning." In this mode, the role of human experts is to answer questions based on various scenarios and case studies presented by learning program. Then, the learning program reviews the answers, and helps to construct appropriate rules in the knowledge base. AQUINAS (Boose et al, 1988) is one of the existing systems used at the Boeing Company. AQUINAS helps human experts to organize, analyze, test, and refine their knowledge base through direct interview with human experts.

The final mode suggests that a sophisticated computer reads and understands the texts and builds a knowledge base automatically. Limitations of current computer technology do not allow the development of this type of extraordinarily sophisticated systems.

The last two modes represent a dramatically different approach. These modes require computer technology not currently available. Several laboratory systems exist in
the fourth mode; however, these are in their infancy, not yet completely successful, and not available in the commercial market. Thus, knowledge acquisition is still handled in the first three modes in actual expert system development.

Uncertainty and Approximate Reasoning

Uncertainty in data and reasoning is another serious problem in expert system development. In most instances, facts, rules, and conclusions in a knowledge base contain a degree of uncertainty, and it is critical that expert systems have the capability to quantify this uncertainty and trace its influence on the decision making process.

To address this problem, most existing expert systems adopt a parameter called a certainty factor. Certainty factors are not purely probabilities, but an informal probability-like measure of confidence or evidence. The first use of certainty factors was in the MYCIN project (Buchanan and Shortliffe, 1984). In MYCIN, certainty factors range from -1 to +1, and rules also have a certainty factor associated with them. The inference engine of MYCIN handles the computation of certainty factors through a combination of methods which are based on predicate logic and certainty factors. The concept of certainty factors
was implemented and tested in MYCIN and became a central element of this system (Buchanan and Shortliffe, 1984).

General concepts of certainty factors (CF) can be explained using the following two simple rules:

**Rule 1**
If: (1) $x$ is A, AND
(2) $y$ is B
Then: (3) $z$ is C with $CF = 0.8$

**Rule 2**
If: (1) $x$ is D, OR
(2) $y$ is E
Then: (3) $z$ is F with $CF = 0.5$

where (1) and (2) are antecedent objects and (3) is a consequence object.

Table 2-2 presents the truth table of these two rules under uncertain information. If antecedents are connected to each other by AND, the minimum CF must be propagated to the consequence. In this case, if the user input answers *false* in any part of the antecedent, the rule fails. On the other hand, if antecedents are connected by OR, the maximum CF can be propagated to the consequence. In this
Table 2-2. Truth Table of Rule 1 and Rule 2
(T: true,  F: false)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Antecedents</th>
<th>Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) CF (2) CF</td>
<td>(3) CF</td>
</tr>
<tr>
<td>1</td>
<td>T 0.4 T 0.6</td>
<td>T ( \text{min}(0.4, 0.6) \times 0.8 = 0.32 )</td>
</tr>
<tr>
<td></td>
<td>F ___ T 1.0</td>
<td>F ___</td>
</tr>
<tr>
<td></td>
<td>T 0.7 F ___</td>
<td>F ___</td>
</tr>
<tr>
<td></td>
<td>F ___ F ___</td>
<td>F ___</td>
</tr>
<tr>
<td>2</td>
<td>T 0.4 T 0.6</td>
<td>T ( \text{max}(0.4, 0.6) \times 0.5 = 0.3 )</td>
</tr>
<tr>
<td></td>
<td>F ___ T 1.0</td>
<td>T 1.0 \times 0.5 = 0.5</td>
</tr>
<tr>
<td></td>
<td>T 0.7 F ___</td>
<td>T 0.7 \times 0.5 = 0.35</td>
</tr>
<tr>
<td></td>
<td>F ___ F ___</td>
<td>F ___</td>
</tr>
</tbody>
</table>
case, the rule may succeed even if the user answered false to the question related to one antecedent of this rule.

It is recognized that the certainty factor model has serious shortcomings, and is hard to justify their use in heuristic rules because much of the uncertainty in expert systems is a function of possibility (Zadeh, 1983) rather than just a measurement of confidence. For example, the following type of rule is often used in water resources management expert systems:

IF: Current storage is less than 50 % of net capacity

THEN: Take an action for drought management with CF = 0.5.

Since the conventional knowledge based systems, which neglect the concept of fuzzy set theory, cannot deal with partial matching, the uncertainty associated with the certainty factor does not affect the antecedent object. For example, this rule succeeds when storage volume is 49.9 % of net capacity, but fails when storage volume is 50.1 %. What is needed is a graduation of action that deals with partial matching through the use of the compositional rule of reasoning.

Another important issue in dealing with uncertainty in knowledge based systems is that of quantifiers such as, very, more or less, most, many, and few, when they are used
in the antecedent objects and/or the consequence object of a rule. Most heuristic rules in expert systems contain these fuzzy quantifiers, and thus the inference engine of an expert system shell should handle these fuzzy propositions.

Approach to Uncertainty Management

This research develops a user friendly environment, (FRESH - Fuzzy Reasoning Shell) that integrates a menu-driven system, user input screens, graphics, and a fuzzy logic based inference engine. FRESH provides an interactive knowledge acquisition environment and a systematic framework for dealing with fuzzy quantifiers. Unlike conventional expert system shells or AI special languages, FRESH allows experts to build fuzzy reasoning systems without extensive computational skills. Therefore, FRESH has a potential to solve the two previously mentioned serious problems in expert systems development: knowledge acquisition and management of uncertainty. Before describing FRESH, a review of fuzzy logic is provided in the Chapter Three.
CHAPTER THREE

Principle of Fuzzy Mathematics

Fuzzy set theory has gained significant recognition from researchers in a wide range of disciplines since its introduction by Zadeh (1965). An important feature of fuzzy set theory is the simplification of complex decision making systems. This is accomplished by incorporating uncertainty into decision making with the use of fuzzy membership functions. These functions are only approximations of complex decision making systems, but their relevance to the original systems is fully maintained.

Many decision making situations cannot be modeled easily, or with any degree of accuracy, using conventional techniques (Zadeh, 1983). This is because the world is full of uncertainties. Since reasoning under uncertainty occupies a central role in decision making, a variety of tools are required to deal with uncertain information. Fuzzy logic is one of these approaches. Indeed, although probability theory is often used to analyze uncertain information, the knowledge or the meaning of certain empirical facts or data is never only probabilistic, but also involves some form of approximate and vague reasoning.

In this chapter, two basic concepts of fuzzy
mathematics, fuzzy sets and fuzzy logic, are briefly described using a concise comparison with classic mathematics. FRESH (Fuzzy Reasoning Shell) incorporates these two principals into a decision making context.

**Fuzzy Sets**

Fuzzy sets are concise mathematical statements of the approximate nature of physical objects or concepts. Such sets underlie much of our ability to summarize, communicate, and make decisions under uncertain or partial information. Indeed, fuzzy sets appear to play an essential role in human cognition, especially in relation to concept formation, pattern classification, and logic reasoning (Kaufmann and Gupta, 1988).

A classic set is defined into two groups; members that certainly belong in the set and nonmembers that certainly do not. A crisp, clear distinction exists between the members and nonmembers of the class or category. Many of the classes and categories we commonly employ, however, such as the class of heavy rain, dry weather, small reservoir storage, or severe drought, do not exhibit this property. Instead, their boundaries seem vague, and the transition from member to nonmember appears gradual rather than abrupt. Thus, the fuzzy set introduces vagueness by eliminating the
crisp boundary dividing members of class from nonmembers.

A fuzzy set can be defined mathematically by assigning a value representing its grade of membership to each possible individual in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. These membership grades are represented by real number values ranging in the closed interval between zero and unity.

For example, let A be a subset of the universe of discourse X. The characteristic function $M_A(x)$ of classical subset A takes its values in the two-element set $(0, 1)$, and is such that $M_A(x) = 1$ if $x \in A$ and $M_A(x) = 0$ otherwise. A fuzzy set A has a characteristic function $M_A(x)$ taking its values in the interval $[0, 1]$, with the grade 1 and 0 representing full membership and nonmembership, respectively. Thus, $M_A(x)$ is called the membership function of A. Since $M_A(x)$ is the grade of membership of $x \in A$, it is also possible to interpret $M_A(x)$ as the degree of possibility (Zadeh, 1983) that $x$ is the value of a parameter fuzzily restricted by A. A is symbolically denoted $\{(x, M_A(x))|x \in A\}$. Other convenient notations are:
\[ A = \int M_A(x)/x \quad \text{when } X \text{ is continuum.} \quad (3-1) \]

\[ A = \frac{M_A(x_1)}{x_1} + \ldots + \frac{M_A(x_n)}{x_n} = \sum_i \frac{M_A(x_i)}{x_i} \quad (3-2) \]

when \( X \) has \( n \) elements \( (|X| = n) \), where + is the union of \( \frac{M_A(x_i)}{x_i} \)'s.

In a fuzzy set, the transition between membership and non-membership is usually gradual rather than abrupt. The shape of membership functions vary. For practical application, however, simple triangular and trapezoidal shapes are the best (Kaufmann and Gupta, 1988) because the use of nonlinear membership function leads to more complex computations. It is important to note that \( X \) is not fuzzy. This implies that \( X \) is a non-fuzzy set of the universe discourse and \( A \) is a fuzzy set derived from that universe. For instance, \( X \) could be the amount of rainfall, and \( A \) could be the linguistic value heavy rain.

Several fundamental structures can be defined on the interval \([0, 1]\), such as fuzzy set union, intersection, and complement operator. These functions coincide with those used in classical set theory. Zadeh (1965) first used the operator \([0, 1], \max, \min, \text{not}\) that are the most frequently employed, i.e.:...
Union : \[ M_{A \cup B}(x) = \max(M_A(x), M_B(x)) \] (3-3)

Intersection: \[ M_{A \cap B}(x) = \min(M_A(x), M_B(x)) \] (3-4)

Complement : \[ \overline{M_A}(x) = 1 - M_A(x) \] (3-5)

Figure 3-1 illustrates these relationships.

**Fuzzy Logic**

Fuzzy logic is a natural expansion of classical logic that every proposition is either true or false. Zadeh (1973) proposed a fuzzy logic in which the antecedent involves a fuzzy conditional proposition. For instance, "If x is A then y is B", with A and B being fuzzy concepts. Such a structure allows the development of simple rules typical of those found in expert systems. However, the fuzzy logic is significantly more complex than that used in most classical propositional logic. This complexity allows the development of much more subtle and accurate management of uncertainty.

**Logic** is the study of the methods and principles of reasoning in all its possible form. One area of classical logic is propositional logic that requires an object to be either true (unity) or false (zero). These values are termed truth values. Propositional logic deals with combinations of objects (or variables) that represent
Figure 3-1. The Basic Mathematics of Fuzzy Sets
arbitrary propositions. As each object represents a hypothetical proposition, it may assume either of the two truth values. Various forms of reasoning rules (or inference rules) can be used for making deductive reasoning.

Modus ponens is one of the most frequently used reasoning rules, i.e.:

\[(A \land (A \Rightarrow B)) \Rightarrow B\]

where \(\land\) is and-function (or conjunction) and \(\Rightarrow\) is implication. Modus ponens states that given two true propositions \(A\) and \(A \Rightarrow B\) (the antecedents), the truth of the proposition \(B\) (the consequence) may be inferred.

The basic assumption of classical logic, that every proposition is either true or false, has long been questioned. This occurs because truth values of certain propositions are essentially indeterminate due to fundamental limitations of measurement. For instance, the following type of rule is often used in water resources management expert systems:

IF current storage is less than 40% of capacity
THEN take an action for drought management.

The antecedent is true if the storage is 39.9%, and is false if the storage is 40.1%. However, near the 40% of
capacity, it is very difficult to distinguish that the antecedent is true or false. Moreover, this is a forced distinction based on a crisp logic.

In order to deal with more flexible propositions, a fuzzy logic was formulated by Zadeh (1973). This flexibility allows the development of much more complex and natural management of uncertainty in propositional logic. The membership grade $M_A(x)$ for $x \in A$ can be interpreted as the truth values of the proposition.

First, we shall consider the following simple form of approximate reasoning that includes a fuzzy conditional proposition:

\begin{align*}
\text{Ant}: & \quad \text{IF current storage is small.} \\
& \quad \text{THEN drought condition is severe is quite true.} \\
\text{Fact}: & \quad \text{Current storage is about 10 \% of capacity.} \\
\text{Cons}: & \quad \text{Drought condition is severe is fairly true.}
\end{align*}

This is a meaningful deductive reasoning. In order to deal with this type of reasoning, fuzzy logic allows the use of fuzzy predicates (small, large, severe, moderate, etc.), fuzzy quantifiers (very, less, many, few, about, etc.), and fuzzy truth values, equivalent to certainty factors in conventional expert systems, (quite true, very true, mostly false, etc.). More generally,
Ant: IF x is A

THEN y is B WITH TV.

Fact: x is A.

------------------------
cons: y is B'.

where x and y are the names of objects; A, A' ("similar to" A), B, and B' ("similar to" B) are the labels of fuzzy subsets in the universes of discourse X, X, Y, and Y, respectively; and TV is the truth value of the antecedent. This reasoning scheme is called generalized modes ponens, and has two major differences from classical modes ponens (Bellman and Zadeh, 1977). First, A' is not required to be identical with A, as it is in the classical case. Second, A, A', and B are not required to be crisp.

The implied relation between the two objects x and y is expressed in terms of the cartesian product of the two fuzzy subsets A and B. The cartesian product, characterized by a function $M_R(x,y)$ by which each pair $(x,y)$ is assigned a number in $[0, 1]$ indicating the extent to which the relation R is true for $(x,y)$, of two fuzzy subset A and B is denoted as $A\ast B$. Equations (3-6) and (3-7) illustrate two different definitions of cartesian product; the logical-product (Mamdani, 1977) and the algebraic-product (for example, Dubois and Prade, 1980):
Logical-Product

\[ A \ast B = \sum_i \sum_j \min \{M_A(x_i), M_B(y_j)\} \] (3-6)

Algebraic-Product

\[ A \ast B = \sum_i \sum_j \{M_A(x_i) \ast M_B(y_j)\} \] (3-7)

where \( \Sigma \) stands for union; \( x_i \) and \( y_j \) are generic elements of the universes of discourse \( X \) and \( Y \), respectively; \( i = 1, 2, \ldots, m \); and \( j = 1, 2, \ldots, n \).

A typical reservoir operation rule that is previously mentioned can be written as:

**IF** current storage is small  
**THEN** drought condition is severe.

Equations (3-6) and (3-7) can be used to infer the fuzzy subset of the second object given a membership function, or a value, for the first. For example, "Current storage is 10% of capacity. What is the drought condition?"

Suppose we denote the relationship between two objects by \( R_{AB} \) \((A \Rightarrow B)\). Then, 10% of capacity is the given membership function of current storage \( A' \) and the membership function of drought condition \( B' \) is inferred by:

\[ B' = A' \circ R_{AB} \] (3-8)
where \( o \) is composition. Composition is performed by the maximum-product of \( A' \) and \( R_{AB} \) (See Appendix A for more details). In the present application, \( A' \) is chosen to be nonfuzzy vectors with only one element equal to unity, all the rest being zero. Figure 3-2 shows the reasoning results by both the logical-product and the algebraic-product. The difference between these two methods is the shape of the membership function (or possibility distribution) of the reasoning results. The result by the logical-product shows that the upper portion of the consequence membership function is truncated by the grade of the antecedent membership function associated with the given fact, and the shape of membership function is changed to trapezoidal. On the other hand, the result using the algebraic-product illustrates that the consequence membership function is algebraically multiplied by the grade of the antecedent membership function associated with the given fact, and the shape of membership function is maintained triangle.

Sugeno (1984) suggests that using the algebraic-product can improve reasoning results because of its property that maintain the original shape of the consequence membership function. Therefore, the algebraic-product is adopted as a fuzzy reasoning operator for this research.
Figure 3-2. Reasoning Results under the Fact, Current Storage is 10% of Capacity
The implementation of truth values (TV) to fuzzy conditional propositions is accomplished in same manner as the implementation of certainty factors (CF) in conventional expert systems. Extending the previous example mentioned before:

IF current storage is small
THEN drought condition is severe WITH TV = 0.5.

The reasoning is "current storage is 10 % of capacity. what is drought condition ?". Figure 3-3 shows the reasoning result with TV using the algebraic-product. First, the grade of the membership function of the consequence object is multiplied by the truth value (TV = 0.5). Second, the reasoning result of the consequence membership function is multiplied again by the grade of the antecedent membership function associated with the given fact, the current storage is 10 % of capacity.

More complex rules are developed by use of two fuzzy antecedent objects, x1 and x2, that are connected by AND.

Ant : IF x1 is A AND x2 is B
THEN y is C.

Fact1: x1 is A'.
Fact2: x2 is B'.

Cons : y is C'.
Figure 3-3. Reasoning Result with TV under the Fact, Current Storage is 10% of Capacity
For instance,

IF current storage is small
AND demand is large
THEN drought condition is severe.
Current storage is 10% of capacity.
Demand is 120% of base demand.

What is Drought condition?

In this example, the algebraic-product and composition can be defined as:

\[ A \times C \text{ AND } B \times C = \sum_i \sum_k (M_A(x_{1i}) \times M_C(y_k)) \]
\[ \cap \sum_j \sum_k (M_B(x_{2j}) \times M_C(y_k)) \quad (3-9) \]

\[ C' = (A' \circ R_{AC}) \cap (B' \circ R_{BC}) \quad (3-10) \]

Equation (3-10) implies that this example can be divided into the two simple propositions previously mentioned. Thus, each composition is performed by the maximum-product of the antecedents, \( A' \) and \( R_{AC} \), and \( B' \) and \( R_{BC} \), respectively. Then, the AND operator is interpreted as the minimum (or intersection) operation, and connects these two separate propositions. Figure 3-4 shows the reasoning result by the algebraic-product.
Figure 3-4. Reasoning Result under the Facts, Current Storage is 10% of Capacity AND Demand is 120% of Base Demand
Finally, the OR operator is considered with the AND operator.

\[ \text{Ant} : \ IF \ (x_1 \ is \ A \ AND \ x_2 \ is \ B) \ OR \ (x_3 \ is \ C) \]
\[ \quad \text{THEN} \ y \ is \ D. \]

\[ \text{Fact1:} \ x_1 \ is \ A'. \]
\[ \text{Fact2:} \ x_2 \ is \ B'. \]
\[ \text{Fact3:} \ x_3 \ is \ C'. \]
\[ \text{Cons:} \ y \ is \ D'. \]

This form of fuzzy reasoning can be divided into the following two sets of fuzzy conditional propositions.

\[ \text{Ant 1:} \ IF \ x_1 \ is \ A \ AND \ x_2 \ is \ B \]
\[ \quad \text{THEN} \ y \ is \ D. \]

\[ \text{Fact1:} \ x_1 \ is \ A'. \]
\[ \text{Fact2:} \ x_2 \ is \ B'. \]
\[ \text{Cons1:} \ y \ is \ D'. \]

OR

\[ \text{Ant 2:} \ IF \ x_3 \ is \ C \]
\[ \quad \text{THEN} \ y \ is \ D. \]

\[ \text{Fact3:} \ x_3 \ is \ C'. \]
\[ \text{Cons2:} \ y \ is \ D'. \]

Thus,

\[ \text{Cons:} \ \text{Cons1} \ OR \ \text{Cons2}. \]
For instance,

IF (current storage is small AND demand is large) 
    OR (predicted storage is small) 
THEN drought condition is severe.

Current storage is 10 % of capacity.
Demand is 120 % of base demand.

Predicted storage is 20 % of capacity.

What is drought condition?

The algebraic-product and composition is defined as:

\[(A\ast D \text{ AND } B\ast D) \text{ OR } C\ast D\]
\[= [\Sigma_i \Sigma_l (M_A(x_{1i}) \ast M_D(y_l)) \cap \Sigma_j \Sigma_l (M_B(x_{2j}) \ast M_D(y_l))] \cup [\Sigma_k \Sigma_l (M_C(x_{3k}) \ast M_D(y_l))]
\]
\[D' = [(A' \circ R_{AD}) \cap (B' \circ R_{BD})] \cup [(C' \circ R_{CD})]
\]

Composition and the AND operator are performed in same manner. Then, the OR operator is interpreted as the maximum (or union) operation, and connects these two sets of fuzzy conditional propositions. Figure 3-5 illustrates the reasoning result.

In general, we shall consider the following multiple fuzzy conditional propositions that have several different fuzzy propositions combined with the OR operator, such as:
Figure 3-5(1). Reasoning Result under the Facts, Current Storage is 10 % of Capacity AND Demand is 120 % of Base Demand AND Predicted Storage is 20 % of Capacity
Membership Function of Predicted Storage is Small

Membership Function of Drought Condition is Severe

Membership Function of Drought Condition is Severe

Figure 3-5(2). Reasoning Result under the Facts, Current Storage is 10% of Capacity AND Demand is 120% of Base Demand AND Predicted Storage is 20% of Capacity
Proposition 1: A1 AND B1 AND C1 => X1 OR
Proposition 2: A2 AND C2 => X2 OR

.............................

Proposition n: Jn AND Ln => Xn.

Fact: A', B', C', ..., and L'.

Consequence: X'.

The consequence X' by given facts is interpreted as the union of X1', X2', ..., and Xn'. For instance,

Storage is very small AND demand is high
=> drought is very severe.

Storage is small AND inflow is small
=> drought is severe.

.............................

Storage is medium AND month is July
=> drought is moderate

In expert system applications, the interest in fuzzy reasoning is that under the given condition, what is the most relevant consequence? In this example, what is the drought condition, very severe, severe, or moderate? To accomplish this purpose, combined reasoning result X' must be compared with pre-defined membership functions of consequence X1, X2, and Xn, such as very severe, severe, and moderate. One can decide then which consequence is the most relevant. The following matching degree is proposed
for this research to measure relevance of consequences.

\[
\text{Matching degree} = \frac{\text{Intersection}}{\text{Union}} \quad (3-13)
\]

Figure 3-6 illustrates the definition of the matching degree. Under this criterion, perfect match must be 1.0 because such a case the union and intersection is identical.

Summary

The principals of fuzzy logic have been discussed in this chapter. The application of the principals to a simple water resource example also has been demonstrated. In addition, the development of simple rule based logic similar to that used in expert systems has been described and the use of fuzzy reasoning operators has been explained. This logic allows a single conceptual framework to deal with different types of uncertainty. Probability theory was long thought to be the only numerical approach to deal with uncertain information. However, an extensive amount of recent applications of fuzzy logic shows that this logic has changed the perceptions of many scientific researchers.

The following chapter describe differences between propagation of uncertainty using certainty factors in conventional expert systems and membership functions using
fuzzy logic.
Pre-defined Membership Reasoning Result

$M_A(x)$ $M_{A'}(x)$

Matching Degree = \frac{\text{Intersection}}{\text{Union}}

Figure 3-6. Definition of Matching Degree
CHAPTER FOUR

Comparison Between Conventional Expert System's Logic and Fuzzy Logic

Interest in expert systems that are based on fuzzy set theory and fuzzy logic has recently increased. The differences between the propagation of uncertainty using certainty factors in expert systems and membership functions using fuzzy logic are of particular interest. Few comparative studies exist that evaluate this important topic.

This chapter describes the conceptual differences of these two techniques for uncertainty propagation. It also describes the propagation of uncertainty using a simple multi-layer reasoning (chaining of the rules) example.

Conceptual Differences Between Certainty Factor Model and Fuzzy Logic

Zadeh (1983) suggests that the most important issues associated with uncertainty information and rules in expert systems are 1) the fuzziness of antecedents and/or consequences in rules, and 2) the natural way of partial
matching between the antecedent of rule and a fact supplied by the user. These are discussed below.

1) The fuzziness of antecedents and/or consequence in rules.

   IF \( x \) is \( A \) THEN \( y \) is \( B \) WITH \( TV = \alpha \).

where the antecedent (\( x \) is \( A \)), and the consequence (\( y \) is \( B \)) are fuzzy propositions and \( \alpha \) is a truth value, \( TV \) (equivalent to a certainty factor, \( CF \), in conventional expert systems). For example, the following type of rule is often used in water resources management expert systems:

   IF storage is about less than 30 % of net capacity
   THEN drought condition is severe
   WITH \( TV = 0.7 \),

in which the antecedent "storage is about less than 30 % of net capacity" and the consequence "drought condition is severe" are fuzzy propositions because these objects involve the concept of fuzzy set theory.

Perhaps the best known and documented expert system is MYCIN (Buchanan and Shortliffe, 1984). These researchers chose not to use fuzzy logic to manage uncertainty propagation, despite their familiarity with the fuzzy approaches. They suggest that, "Although an analogy
between our diagnostic problem and fuzzy set theory can be made, the statement of diagnostic decision criteria in terms of set membership does not appear to be a natural concept for the experts who must formulate our rules." However, this does not lead necessarily to the conclusion that fuzzy set theory cannot contribute in expert system applications.

2) The natural way of partial matching between the antecedent of rule and a fact supplied by the user.

    Often there are situations in which a fact such as "x is A'" may not match exactly the antecedent of any rules of the form "IF x is A THEN y is B WITH CF = a", where A' can also be a membership function in fuzzy logic. Expert systems using fuzzy set theory can deal naturally with partial matching using continuous membership functions.

    The certainty factor model in conventional expert systems emulates partially the fuzzy set approach, but has several limitations. For example the user can imply uncertainty to the statement "storage is somewhat less than 30% of net capacity" by assigning a certainty factor of 0.5 to be associated with the rule "IF storage is somewhat less than 30% of net capacity THEN drought condition is severe". This rule is a fuzzy proposition. However, in this example, expertise is shared by the users as well as the
expert system because the user must assume the responsibility in answering this question using a fuzzy set concept (under a given situation, the user must imagine the shape of membership functions and enter a CF according to his expertise). Also, the certainty factor model does not deal with membership as input by the user. This requires that the propagation of uncertainty in the certainty factor model be done with a single number rather than the entire shape of membership function (distribution of possibility).

Comparison of Propagation of Uncertainty in Multi-Layer Reasoning

The propagation of uncertainty in multi-layer reasoning (chaining of rules) using fuzzy logic is an active area of research (Dubois and Prade, 1989). There are several procedures available to define the functions of union and intersection as well as combining rules for OR and AND in fuzzy logic. In this research, the procedures suggested by Mamdani (1977) are used to define the OR and AND operators.

The fundamental techniques described in the previous chapter have been implemented in a simple two-layer forward chaining over propositions. In this section, an example of its application and the results concerning an efficient
procedure for a generalized modus ponens are compared with the certainty factor model. The objectives for presenting this example are to demonstrate: 1) the difference between fuzzy logic and certainty factors in propagating uncertainty, and 2) how certainty factor models can emulate fuzzy logic.

The illustrative example concerns determination of release during drought. The following rules are represented by the tree structure in Figure 4-1.

Rule 1:

IF predicted rainfall is small
THEN future inflow is small
WITH TV = 1.0.

Rule 2:

IF predicted rainfall is large
THEN future inflow is large
WITH TV = 1.0.

Rule 3:

IF future inflow is small and storage is small
THEN release is small
WITH TV = 1.0.
Rule 4:

IF future inflow is large and storage is small
THEN release is medium
WITH TV = 1.0

Rule 5:

IF future inflow is small and storage is large
THEN release is medium
WITH TV = 0.5

Rule 6:

IF future inflow is large and storage is large
THEN release is large
WITH TV = 1.0

Figure 4-1  Tree Structure of Rules
**Fuzzy Logic Approach**

In the application of fuzzy logic, one must define the shape of membership functions for each object, predicted rainfall, future inflow, storage, and release. These membership functions can be continuous functions, however, in this example they are treated as discrete functions for simplicity. The shape of membership functions for each object are defined in Figure 4-2. Each object takes its value from zero to 100 with 25 intervals. This is relative scale and values have no units.

Using the notation for membership functions and cartesian products described in Chapter Three, the rules can be expressed in the following matrices:

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>Membership Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Predicted Rainfall is small:</strong> ( M_{A1} )</td>
<td>[1.0 0.6 0.3 0.1 0.0]]</td>
</tr>
<tr>
<td><strong>Predicted Rainfall is large:</strong> ( M_{A2} )</td>
<td>[0.0 0.1 0.3 0.6 1.0]]</td>
</tr>
<tr>
<td><strong>Future Inflow is small:</strong> ( M_{B1} )</td>
<td>[1.0 0.8 0.5 0.2 0.0]]</td>
</tr>
<tr>
<td><strong>Future Inflow is large:</strong> ( M_{B2} )</td>
<td>[0.0 0.2 0.5 0.8 1.0]]</td>
</tr>
<tr>
<td><strong>Storage is small:</strong> ( M_{C1} )</td>
<td>[1.0 0.9 0.5 0.1 0.0]]</td>
</tr>
<tr>
<td><strong>Storage is large:</strong> ( M_{C2} )</td>
<td>[0.0 0.1 0.5 0.9 1.0]]</td>
</tr>
<tr>
<td><strong>Release is small:</strong> ( M_{D1} )</td>
<td>[0.5 1.0 0.5 0.0 0.0]]</td>
</tr>
<tr>
<td><strong>Release is medium:</strong> ( M_{D2} )</td>
<td>[0.0 0.5 1.0 0.5 0.0]]</td>
</tr>
<tr>
<td><strong>Release is large:</strong> ( M_{D3} )</td>
<td>[0.0 0.0 0.5 1.0 0.5]]</td>
</tr>
</tbody>
</table>

value: 0 25 50 75 100
Predicted Rainfall \((w)\)

Small (A1)  

Large (A2)  

Future Inflow \((x)\)

Small (B1)  

Large (B2)  

Figure 4-2(1). Membership Functions for Each Object
Figure 4-2(2). Membership Functions for Each Object
| Rule 1: \( R_{A1B1} \) | 1.0 0.6 0.3 0.1 0.0 |
| | 0.8 0.6 0.3 0.1 0.0 |
| | 0.5 0.5 0.3 0.1 0.0 |
| | 0.2 0.2 0.2 0.1 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |

| Rule 2: \( R_{A2B2} \) | 0.0 0.0 0.0 0.0 0.0 |
| | 0.0 0.1 0.2 0.2 0.2 |
| | 0.0 0.1 0.3 0.5 0.5 |
| | 0.0 0.1 0.3 0.6 0.8 |
| | 0.0 0.1 0.3 0.6 1.0 |

| Rule 3-1 \( R_{B1D1} \) | 0.5 0.5 0.5 0.2 0.0 |
| | 1.0 0.8 0.5 0.2 0.0 |
| | 0.5 0.5 0.5 0.2 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |

| Rule 3-2 \( R_{C1D1} \) | 0.5 0.5 0.5 0.1 0.0 |
| | 1.0 0.9 0.5 0.1 0.0 |
| | 0.5 0.5 0.5 0.1 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |

| Rule 4-1 \( R_{B2D2} \) | 0.0 0.0 0.0 0.0 0.0 |
| | 0.0 0.2 0.5 0.5 0.5 |
| | 0.0 0.2 0.5 0.8 1.0 |
| | 0.0 0.2 0.5 0.5 0.5 |
| | 0.0 0.0 0.0 0.0 0.0 |

| Rule 4-2 \( R_{C1D2} \) | 0.0 0.0 0.0 0.0 0.0 |
| | 0.5 0.5 0.5 0.1 0.0 |
| | 1.0 0.9 0.5 0.1 0.0 |
| | 0.5 0.5 0.5 0.1 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |

| Rule 5-1 \( R_{B1D2} \) | 0.0 0.0 0.0 0.0 0.0 |
| | 0.5 0.5 0.5 0.2 0.0 |
| | 1.0 0.8 0.5 0.2 0.0 |
| | 0.5 0.5 0.5 0.2 0.0 |
| | 0.0 0.0 0.0 0.0 0.0 |
Rule 5-2 \( R_{C2D2} = \)
\[
\begin{array}{cccccc}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5 & \\
0.0 & 0.1 & 0.5 & 0.9 & 1.0 & \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5 & \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & \\
\end{array}
\]

Rule 6-1 \( R_{B2D3} = \)
\[
\begin{array}{cccccc}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.8 & 0.0 & 0.0 & 0.0 & \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5 & \\
0.0 & 0.2 & 0.5 & 0.8 & 1.0 & \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5 & \\
\end{array}
\]

Rule 6-2 \( R_{C2D3} = \)
\[
\begin{array}{cccccc}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5 & \\
0.0 & 0.1 & 0.5 & 0.9 & 1.0 & \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5 & \\
\end{array}
\]

Modus ponens to infer the consequence can be written as:

\[
M_B' = \{ (A' \circ R_{A1B1}) \cup (A' \circ R_{A2B2}) \mid TV = 1.0 \}
\]

\[
M_D' = \{ (M_B' \circ R_{B1D1}) \cap (C' \circ R_{C1D1}) \mid TV = 1.0 \}
\]

\[
U \{ (M_B' \circ R_{B2D2}) \cap (C' \circ R_{C1D2}) \mid TV = 1.0 \}
\]

\[
U \{ (M_B' \circ R_{B1D2}) \cap (C' \circ R_{C2D2}) \mid TV = 0.5 \}
\]

\[
U \{ (M_B' \circ R_{B2D3}) \cap (C' \circ R_{C2D3}) \mid TV = 1.0 \}
\]

where \( \circ \) is composition, \( \cup \) is OR (maximum) operator, and \( \cap \) is AND (minimum) operator. Since linear relationships exist between the cartesian product and the truth value, the composition with the truth value can be performed by simple multiplication of truth value by the membership functions and the cartesian products.
As described earlier, the user response can be a membership function. To simplify the comparison with a certainty factor model in this example, however, the user's responses are chosen to be nonfuzzy vectors with only one element equal to unity and the remaining equal to zero. Fuzzy logic based expert systems convert a value to a nonfuzzy vector; therefore, the user specifies one value for each object.

For example, the conclusion that results from the given facts, predicted rainfall is 25 and storage is 25, is calculated as:

\[
M_B' = \{ (A' \circ R_{A1B1}) \cup (A' \circ R_{A2B2}) \mid TV = 1.0 \} \\
= [0.0 \ 0.1 \ 0.0 \ 0.0 \ 0.0] \circ \\
\begin{array}{ccccc}
1.0 & 0.6 & 0.3 & 0.1 & 0.0 \\
0.8 & 0.6 & 0.3 & 0.1 & 0.0 \\
0.5 & 0.5 & 0.3 & 0.1 & 0.0 \\
0.2 & 0.2 & 0.7 & 0.1 & 0.0 \\
0.0 & 0.2 & 0.0 & 0.0 & 0.0 \\
\end{array} \\
\cup \\
\begin{array}{ccccc}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.1 & 0.2 & 0.2 & 0.2 \\
0.0 & 0.1 & 0.3 & 0.5 & 0.5 \\
0.0 & 0.1 & 0.3 & 0.6 & 0.8 \\
0.0 & 0.1 & 0.3 & 0.6 & 1.0 \\
\end{array} \\
= [0.6 \ 0.6 \ 0.5 \ 0.2 \ 0.0] \cup [0.0 \ 0.1 \ 0.1 \ 0.1 \ 0.1] \\
= [0.6 \ 0.6 \ 0.5 \ 0.2 \ 0.1]
\]

\[
M_D' = \{ (M_B' \circ R_{B1D1}) \cap (C' \circ R_{C1D1}) \mid TV = 1.0 \} \\
\cup \{ (M_B' \circ R_{B2D2}) \cap (C' \circ R_{C1D2}) \mid TV = 1.0 \} \\
\cup \{ (M_B' \circ R_{B1D2}) \cap (C' \circ R_{C2D2}) \mid TV = 0.5 \} \\
\cup \{ (M_B' \circ R_{B2D3}) \cap (C' \circ R_{C2D3}) \mid TV = 1.0 \}
\]
\[
\begin{align*}
\mathbb{E} &= \{ [0.6 \ 0.6 \ 0.5 \ 0.2 \ 0.1] \circ \\
&\quad \begin{bmatrix}
0.5 & 0.5 & 0.5 & 0.2 & 0.0 \\
1.0 & 0.8 & 0.5 & 0.2 & 0.0 \\
0.5 & 0.5 & 0.5 & 0.2 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \\
\cap [0 \ 1 \ 0 \ 0 \ 0] \circ \\
&\quad \begin{bmatrix}
0.5 & 0.5 & 0.5 & 0.1 & 0.0 \\
1.0 & 0.9 & 0.5 & 0.1 & 0.0 \\
0.5 & 0.5 & 0.5 & 0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \\
U \{ [0.6 \ 0.6 \ 0.5 \ 0.2 \ 0.1] \circ \\
&\quad \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5 \\
0.0 & 0.2 & 0.5 & 0.8 & 1.0 \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \\
\cap [0 \ 1 \ 0 \ 0 \ 0] \circ \\
&\quad \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.5 & 0.5 & 0.5 & 0.1 & 0.0 \\
1.0 & 0.9 & 0.5 & 0.1 & 0.0 \\
0.5 & 0.5 & 0.5 & 0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \\
U \{ [0.30 \ 0.30 \ 0.25 \ 0.10 \ 0.05] \circ \\
&\quad \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.25 & 0.25 & 0.25 & 0.10 & 0.00 \\
0.50 & 0.40 & 0.25 & 0.10 & 0.00 \\
0.25 & 0.25 & 0.25 & 0.10 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00
\end{bmatrix} \\
\cap [0 \ 0.5 \ 0 \ 0 \ 0] \circ \\
&\quad \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.05 & 0.25 & 0.25 & 0.25 \\
0.00 & 0.05 & 0.25 & 0.45 & 0.50 \\
0.00 & 0.05 & 0.25 & 0.25 & 0.25 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00
\end{bmatrix} \\
U \{ [0.6 \ 0.6 \ 0.5 \ 0.2 \ 0.1] \circ \\
&\quad \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5 \\
0.0 & 0.2 & 0.5 & 0.8 & 1.0 \\
0.0 & 0.2 & 0.5 & 0.5 & 0.5
\end{bmatrix} \\
\cap [0 \ 1 \ 0 \ 0 \ 0] \circ \\
&\quad \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5 \\
0.0 & 0.1 & 0.5 & 0.9 & 1.0 \\
0.0 & 0.1 & 0.5 & 0.5 & 0.5
\end{bmatrix}
\end{align*}
\]
= \{[0.50 0.60 0.50 0.00 0.00] \land [0.50 0.90 0.50 0.00 0.00]\} \
U \{[0.00 0.50 0.50 0.50 0.00] \land [0.00 0.50 0.90 0.50 0.00]\} \
U \{[0.00 0.25 0.30 0.25 0.00] \land [0.00 0.05 0.05 0.05 0.00]\} \
U \{[0.00 0.00 0.50 0.50 0.50] \land [0.00 0.00 0.10 0.10 0.10]\}

= [0.50 0.60 0.50 0.00 0.00] \
U [0.00 0.50 0.50 0.50 0.00] \
U [0.00 0.05 0.05 0.05 0.00] \
U [0.00 0.00 0.10 0.10 0.10]

= [0.50 0.60 0.50 0.50 0.10]

Figure 4-3 illustrates the membership function of the consequence object, release, using fuzzy reasoning.

Membership Function of the Consequence, Release

![Membership Function of the Consequence, Release](image.png)

Figure 4-3. Fuzzy Reasoning Result
We can interpret this reasoning result in several ways. First, the simplest implication is that the release is 25 because that value is associated the maximum membership function. Second, the release could be established at 40 based on a calculation of the horizontal centroid of the membership function. The horizontal centroid (HG) can be calculated by:

\[
HG = \frac{0\times0.5 + 25\times0.6 + 50\times0.5 + 75\times0.5 + 100\times0.1}{0.5 + 0.6 + 0.5 + 0.5 + 0.1} = 40
\]

These two implications are often used in the applications of dynamic system's controller. Finally, an implication can be drawn based on the calculation of the matching degree as defined in Chapter Tree. In this example, the calculation results of matching degree for each consequences are:

- Release is small with matching degree 62 %.
- Release is medium with matching degree 56 %.
- Release id large with matching degree 32 %.

These fuzzy matrix manipulations appear complicated. However, they are performed by maximum and minimum operators, and are not computationally intensive.
Certainty Factor Model Approach

The certainty factor model is now examined using the same problem. Several approaches can be taken to deal with quasi-fuzzy logic in the certainty factor model applications; however, the following simple assumptions are incorporated:

1) A fuzzy set is used to represent each object; rainfall, inflow, storage, and release.

2) The rules and the tree structure presented in Figure 4-1 are maintained.

3) The goal of this system is to determine release.

4) The user has the responsibility to enter appropriate certainty factors into the system.

5) The conclusion of the system is a qualitative expressions, such as "Release is small with \( CF = 0.8 \)", etc.

6) The system has the capability to deal with multi-attribute objects.

These conditions are similar to those in the example of fuzzy logic; thus, predicted rainfall is 25 and storage is 25. In this example, the user cannot enter these two values directly. Instead, the user must answer the
following questions with certainty factors.

Q1: Is predicted rainfall small?
A1: True with $CF = 0.6$.

Q2: Is predicted rainfall large?
A2: True with $CF = 0.1$.

Q3: Is storage small?
A3: True with $CF = 0.9$.

Q4: Is storage large?
A4: True with $CF = 0.1$.

Based on these user inputs, the conclusions of future inflow and release are:

Rule 1: Future inflow is small with $CF = 0.6$.
Rule 2: Future inflow is large with $CF = 0.1$.

Rule 3: Release is small with $CF = 0.6$.

$$CF = \min(0.9, 0.6) \times 1.0.$$ 

Rule 4: Release is medium with $CF = 0.1$.

$$CF = \min(0.9, 0.1) \times 1.0.$$ 

Rule 5: Release is medium with $CF = 0.05$.

$$CF = \min(0.1, 0.6) \times 0.5.$$ 

Rule 6: Release is large with $CF = 0.1$.

$$CF = \min(0.1, 0.1) \times 1.0.$$
If Rule 4 and Rule 5 are connected with OR, the final conclusions are:

- Release is small with $CF = 0.6$.
- Release is medium with $CF = 0.1$.
- Release is large with $CF = 0.1$.

It is obvious from these conclusions that the certainty factor model does not capture the same type of information as the fuzzy logic. The certainty factor model can emulate fuzzy logic only in the most certain conclusion ($CF$ of "release is small" is the same as the maximum membership in Figure 4-3). However, the certainty factor model cannot emulate the distribution of possibility. In this example, the certainty factor of "release is medium" is the same as the certainty factor of "release is large". This conclusion is intuitively contradictory.

One approach to emulate the possibility distribution in rule based systems with certainty factors is to develop a series of rules that replicate the membership function. For instance, Rule 3 can be divided into the following five rules.
Rule 3-1:

IF future inflow is small and storage is small
THEN release is 0
WITH $CF = 0.5$.

Rule 3-2:

IF future inflow is small and storage is small
THEN release is 25
WITH $CF = 1.0$.

Rule 3-3:

IF future inflow is small and storage is small
THEN release is 50
WITH $CF = 0.5$.

Rule 3-4:

IF future inflow is small and storage is small
THEN release is 75
WITH $CF = 0$.

Rule 3-5:

IF future inflow is small and storage is small
THEN release is 100
WITH $CF = 0$.

If each rule that has the same consequence is connected to the other rules with OR, the final conclusions are:
Release is 0 with CF = 0.3.
Release is 25 with CF = 0.6.
Release is 50 with CF = 0.1.
Release is 75 with CF = 0.1.
Release is 100 with CF = 0.05.

This approach also cannot emulate fuzzy logic correctly because of the technique used to propagate uncertainty. In fuzzy logic, uncertainty is propagated by membership functions. By contrast in the uncertainty factor model, it is performed based on one value. Therefore, even though rules are divided into small rules, the certainty factor model cannot capture fuzzy information precisely.

Summary

This chapter discusses the differences between fuzzy logic and the certainty factor model in terms of propagation of uncertainty. The important conclusions are:

1) The concept of fuzzy sets is useful for water resources management expert systems because rules such as, "Storage is Small", are often used in practical applications.
2) The certainty factor model of conventional expert systems can emulate fuzzy propositions partially; however, expertise must be in the user's hand as well as the expert system.

3) Propagation of uncertainty cannot be performed correctly by the certainty factor model, even if rules are divided into small rules.

Table 4-1 summarizes the differences between the certainty factor model and fuzzy logic.
<table>
<thead>
<tr>
<th>Subject</th>
<th>Certainty Factor Model</th>
<th>Fuzzy Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositions</td>
<td>Can be Fuzzy</td>
<td>Fuzzy</td>
</tr>
<tr>
<td>Antecedent</td>
<td>Can be Fuzzy</td>
<td>Fuzzy</td>
</tr>
<tr>
<td>Consequence</td>
<td>Not Fuzzy But Uncertain</td>
<td>Fuzzy Membership</td>
</tr>
<tr>
<td>User Input</td>
<td>True with CF</td>
<td>Value or Membership (can be in system)</td>
</tr>
<tr>
<td>Expertise</td>
<td>System and User</td>
<td>System</td>
</tr>
<tr>
<td>Vehicle of Uncertainty</td>
<td>One Value</td>
<td>Membership Function</td>
</tr>
<tr>
<td>Form of Rules</td>
<td>Wordy</td>
<td>Concise</td>
</tr>
</tbody>
</table>
CHAPTER FIVE

Development of FRESH (Fuzzy Reasoning Shell)

FRESH, (Fuzzy Reasoning Shell) provides an interactive knowledge acquisition environment and a fuzzy logic based framework for the management of uncertainty. FRESH integrates a menu-driven system, user input screens, graphics, and a fuzzy logic based inference engine. With FRESH, a user develops fuzzy logic based decision making systems (or dynamic system controllers) that use a knowledge base, and provide solutions to problems that require approximate reasoning.

FRESH is written in Turbo C (Borland International, 1988) for the IBM Personal Computer and its compatibles. The C language was chosen because it provides a structured skeleton without limiting the creativity of the programmer, while C compilers consistently produce extremely fast and efficient executable program. C also provides powerful string manipulation capabilities. Most currently advanced computer software including word processors, expert system shells, and UNIX are written in C.

Since FRESH is an executable system, the programmer can link FRESH with another system that was written in an AI
language. This means that it is possible to develop fuzzy logic based expert systems that incorporate backward chaining, database management, and fuzzy reasoning capabilities in the future.

The remainder of this chapter provides an introduction to the organization and functions of FRESH. Appendix B describes the installation and hardware requirements of FRESH.

FRESH Functions

The use of FRESH is simple. Since FRESH uses a menu-driven system, the user can begin by using the sample knowledge base provided that is explained in Chapter Five. After using this sample knowledge base, the user can easily create his/her knowledge bases to run under FRESH.

All of the major functions of FRESH are accessed from a main menu that lists five primary options. These options include 1) editing membership functions and fuzzy propositions, 2) graphics of membership functions, 3) graphics of fuzzy conditional propositions, 4) fuzzy inference engine including graphics presentation of fuzzy reasoning results, and 5) print a list of membership functions and fuzzy propositions.
With menu screens, the user selects the desired FRESH function by either a pointer (using the upward and downward cursor arrow keys followed by the <ENTER> key) or the first red colored letter of the entries, called hot keys.

FRESH has on-line HELP available from every menu screen. HELP is restricted to provide information pertinent to the specific context from which HELP was called. When HELP is activated by pressing <? > key, a pop-up screen of topics is presented that explains the feature of FRESH. To exit HELP, pressing any key will return the user to the previously active screen. Figure 5-1 shows the main menu of FRESH.

**FRESH Editor**

Choose "Edit Membership Functions or Propositions" from the main menu with either the cursor allow followed by <ENTER> key or the hot key. FRESH shows the next menu and asks the user whether to "Edit New File" or "Edit Existing File" (Figure 5-2). The FRESH editor allows the user to create new membership functions and fuzzy conditional propositions (the equivalent to a knowledge base of conventional expert systems). Also, the user can modify, add, and delete existing membership functions and fuzzy propositions using this editor.
Options Menu

E Edit Membership Functions and Conditional Propositions
M View User Defined Membership Functions
F View User Defined Fuzzy Conditional Propositions
R Run Fuzzy Reasoning Based on User Input
P Print a List
Q Quit Program

Press desired hot key or use arrow keys and press Enter; ? for help

Figure 5-1. Options Menu

File Menu

E Edit Existing File
N Edit New File
R Return to Option Menu
Q Quit Program

Press desired hot key or use arrow keys and press Enter; ? for help

Figure 5-2. File Menu
If the user chooses "Edit Existing File", FRESH automatically accesses 'C:\FRESH' directory and shows data files that are currently available on screen (Figure 5-3). In this screen, three hot keys (N, P, and C) are used to choose the desired file. After obtaining the input data file name using the combination of hot keys, the FRESH editor provides the next edit menu that allows the user to edit membership functions or fuzzy conditional propositions (Figure 5-4).

The user can define only one consequence object, but up to nineteen antecedent objects. Each object can have up to ten different membership functions. For example, the user can define fuzzy quantifiers, such as very small, small, medium, large, and very large, for the object, reservoir storage.

If the selection is "Modify Membership Function of Existing Object", FRESH requires the user to select the object name using three hot keys (N, P, and C) that are shown in Figure 5-5. After obtaining the object name, FRESH will ask the user to enter the number of membership functions (Figure 5-6) and the minimum and the maximum values of the object (Figure 5-7). Then, the FRESH editor requests the user to enter the name and the shape of membership functions (Figure 5-8).
Existing File Name Selection Screen

Choose desired filename to store membership functions and propositions using the following three HOT characters.

N: Next file,     P: Previous file,     C: Choose file

The files shown in the window below are available at present. NOTE: Only the first 15 files in the 'C:FRESH' directory can be seen.

NWFA_JAN.DAT

NWFA_JAN.DAT  NWFB_FEB.DAT  NWFC_MAR.DAT  NWFD_APR.DAT
NWFE_MAY.DAT  NWFF_JUN.DAT  NWFG_JUL.DAT  NWFH_AUG.DAT
NWFI_SEP.DAT  NWFJ_OCT.DAT  NWFK_NOV.DAT  NWFL_DEC.DAT

Figure 5-3. Existing File Mane Selection Screen

Edit Menu
1. Modify Membership Function of Existing Object
2. Add New Object
3. Delete Existing Object
4. Edit Fuzzy Conditional Propositions
5. Change Fuzzy Reasoning Accuracy
6. Draw Existing Membership Functions
7. Draw Existing Conditional Propositions
8. Return to Option Menu
9. Quit Program

Press desired hot key or use arrow keys and press Enter; ? for help

Figure 5-4. Edit Menu
Object Name Selection Screen

Choose desired object name to modify membership functions using the following three HOT keys.
N : Next object,   P : Previous object,   C : Choose object
The following objects shown in the window are available at present.

Operating_policy

Operating_policy  Storage_in_Jan.(ML)  Sim_Result(% of Cap)
Lake_Dist(%fall/wk)

Figure 5-5. Object Name Selection Screen

Number of Membership Functions of Operating_policy
Please enter the number of membership functions as an INTEGER value.
Maximum number of membership functions is 10.

4

Figure 5-6. Number of Membership Functions Entering Screen
Input Screen for Operating_policy

Please enter the minimum and the maximum value of this object.

The minimum value : 0.00000
The maximum value : 1.00000

Figure 5-7. Minimum and Maximum Values of Objects Entering Screen

Input Screen for Operating_policy

Please enter the name of this membership function (maximum length of characters is 20) and four x-axis points.

The minimum value : 0.00000 The maximum value : 1.00000

The name of this membership function : Normal Operation
The x-axis of bottom left : 0.00000
The x-axis of top left : .200000
The x-axis of top right : .400000
The x-axis of bottom right : .400000

Figure 5-8. Name and X-Axis Points of Membership Functions Entering Screen
Membership functions must be piecewise-linear in FRESH. However, they can be non-symmetric triangle or trapezoidal. Bottom left, top left, top right, and bottom right points of membership functions are specified by the user. Ordinates of bottom and top of membership functions are always zero and unity, respectively. Thus, the user's input is only these four x-axis. Figure 5-9 illustrates the location of these four points. If the x-axis of top left is equal to the x-axis of top right, the membership function is triangle.

When the user selects "Add New Object" in Figure 5-4, FRESH requires the user to enter the new object name. The user must type it in response to the questions by FRESH. FRESH then evaluates the user's responses to avoid errors in the knowledge base. In this case, the user cannot choose the name of existing objects. The result for this is "You cannot redeclare the name of existing objects. Please enter new object name!". After obtaining the new object name, FRESH will ask the same questions as the previous example.

If the user chooses "Delete Existing Object" in Figure 5-4, FRESH requires the user to select the desired object name using three hot keys. After obtaining the desired object name, FRESH will present the influence of this action and confirm it (Figure 5-10).
Figure 5-9. Location of Four X-Axis Points of Membership Functions

You will lose 14 propositions that contain this object. Are you sure? (Y/N)

Object Name Selection Screen
Choose desired object name to modify membership functions using the following three HOT keys:
N : Next object,   P : Previous object,   C : Choose object
The following objects shown in the window are available at present.

Storage_in_Jan.(M1)

Operating_policy  Storage_in_Jan.(M1)  Sim_Result(% of Cap)
Lake_Dist(%full/wk)

Figure 5-10. Confirmation Screen of "Delete Existing Object"
The user can define up to one hundred fuzzy conditional propositions. Each fuzzy proposition can have up to three antecedent objects. Therefore, one hundred fuzzy conditional propositions, or rules, can be connected by OR operators, and three antecedent objects can be connected by AND operators in each fuzzy conditional proposition.

When the user's selection is "Edit Fuzzy Conditional Propositions" in Figure 5-4, FRESH shows the editing screen (Figure 5-11). In this screen, the user uses six hot keys (N, P, M, A, D, and R) to choose the desired editing functions. Instructions for these hot keys are available from FRESH. The user can edit propositions with the hot keys. Figures 5-12 and 5-13 show this environment. The user can select appropriate object names and membership function names using three hot keys (N, P, and C). The bottom window shows available object names and membership function names.

FRESH uses truth values (TV), from zero to unity, for each fuzzy proposition. The truth values are equivalent to certainty factors (CF) of conventional expert systems. For example, the user can define the following two rules that have different TV:
IF Current storage is Small AND Demand is Large
THEN Drought condition is Severe
WITH TV = 1.0.

OR

IF Predicted storage is Small
THEN Drought condition is Severe
WITH TV = 0.8.

These two rules imply that the consequences of rules are the same, Drought condition is Severe, but the confidence or certainty of rules is different. This capability exists because users commonly have more confidence in some rules than others.

If the user selects "Change Fuzzy Reasoning Accuracy" in Figure 5-4, FRESH will request for input values of reasoning accuracy (Figure 5-14). This is necessary because FRESH calculates matching degree using numerical integration. FRESH asks the user for the number of subintervals between minimum and maximum values of consequence object.
Figure 5-11. Fuzzy Conditional Propositions Editing Screen

```
Fuzzy Conditional Proposition Editing Screen

Choose desired option using the following HOT keys.
N: Next, P: Previous, M: Modify, A: Add, D: Delete, R: Return

Proposition Name : Rule Normal_2
IF Storage_in_Jan.(Ml) IS Zone_B
AND Sim_Result(% of Cap) IS More_than_15%
AND
THEN Operating_policy IS Normal_Operation WITH TV = 1.00.
```

Figure 5-12. Propositions Editing Environment (1)
Choose desired option using the following HOT keys.
N: Next, P: Previous C: Choose

Proposition Name: Rule_Normal_2
IF Storage_in_Jan.(H1) IS Zone_B
AND Sim_Result(%_of_Cap) IS More_than_15%
AND IS
THEN Operating_policy IS Normal_Operation WITH TV = 1.00.

Figure 5-13. Propositions Editing Environment (2)

Accuracy Input Screen
Enter desired accuracy of Fuzzy Reasoning as an INTEGER value.
The accuracy is the number of subintervals for numerical integration.
The value should be at least 10 and less than 100.
NOTE: High accuracy leads to long execution time. 20 is recommended.

Figure 5-14. Accuracy Input Screen
FRESH Graphics

When the user selects "View User Defined Membership Functions" or "View User Defined Fuzzy Conditional Propositions" from the main menu FRESH automatically accesses 'C:\FRESH' directory and provides the data files that are currently available on screen (Figure 5-3). After a data file name is selected, the FRESH graphics will plot this information.

The user can also access the FRESH graphics from the edit menu (Figure 5-4). The user can, therefore, modify membership functions or fuzzy conditional propositions using an interactive environment.

FRESH automatically determines whether the computer has a EGA or VGA color monitor and presents the appropriate graphics. Since the C language allows good video graphics image on color display, a VGA or high resolution EGA color monitor is recommended.

FRESH Inference Engine

When the user selects the "Run Fuzzy Reasoning Based on User Input" from the main, FRESH automatically accesses 'C:\FRESH' directory and presents the data files that are currently available on screen (Figure 5-3). After
selection of the appropriate file name, FRESH requests the user to enter values of antecedent objects (Figure 5-15). The user can, then, choose "Show Reasoning Results for Each Conditional Propositions", "Show Matching Result and Summary of Fuzzy Reasoning", or both from menu screen (Figure 5-16). The FRESH graphics has the capability to present these results during a fuzzy reasoning session. The FRESH inference engine, then, performs fuzzy reasoning to solve the specified problem.

With the reasoning results for each conditional fuzzy proposition, the user reviews the rules of the knowledge base, the input provided to FRESH, and the reasoning results. This facility provides transparency to the knowledge base; that is, the user can review the rule base, logic, and truth values. During the development of a knowledge base, this capability is a powerful debugging tool that enables the user to test the results of different input.

The FRESH inference engine computes the matching degrees of pre-defined membership functions of consequence object and graphs the results. FRESH, then, provides the number of active rules under given conditions, and suggests
Input Screen for Storage_in_Jan.(M1)

Please enter the antecedent object value to use in Fuzzy Reasoning.

The minimum value of this object: 0.00000
The maximum value of this object: 4355.00
4355.00

Figure 5-15. Antecedent Object Value Input Screen

Results of Fuzzy Reasoning Menu

B Show Both Results Listed Below
P Show Reasoning Results for Each Conditional Proposition
S Show Matching Result and Summary of Fuzzy Reasoning
R Return to Option Menu
Q Quit Program

Press desired hot key or use arrow keys and press Enter; ? for help

Figure 5-16. Results of Fuzzy Reasoning Menu
the best three matched consequence in the summary of fuzzy reasoning.

Summary

FRESH is an interactive knowledge acquisition environment and a fuzzy logic based shell. FRESH provides a menu-driven system to allow a user to create his/her knowledge bases easily. Syntax error evaluation is also provided. Figure 5-17 summarizes the structure of FRESH. FRESH is written in the C language and designed to run on IBM Personal Computer and its compatibles.

The following chapter illustrates the application of FRESH to a North West Water Authority reservoir management system.
Figure 5-17. Structure of FRESH
CHAPTER SIX

Application of FRESH

to North West Water Authority Reservoir Operation

This chapter describes the application of FRESH to the development of reservoir operating policies. In this example, operating policies are developed for the North West Water Authority (NWWA) of Warrington, England. The operating policy is implemented as an interpreter of a set of rules expressed as fuzzy conditional propositions. Since rules used in practice contain linguistic vagueness, fuzzy propositional logic is used to synthesize the linguistic operational rules of an expert. This chapter begins with a brief introduction to the NWWA and their development of operating policies followed by the application. Much of this introductory material was abstracted from Slaughterbeck (1989).

North West Water Authority

NWWA is one of the largest water supply authorities in Great Britain. The NWWA was established by the Water Act 1973 that required aggregation of 231 small public water supply systems in northwest portion of England.
The NWWA supplies water to 7,000,000 people stretching from the Scottish border in the north to Chesire in the south. The service area, approximately 14,500 km², is divided into 7 districts that have responsibility for water supply, sewage, and recreation, and receives about 1,200 mm of precipitation per year. The total supply is approximately 2,500 megalitres per day (ML/d). The NWWA operates about 120 reservoirs as well as pumped transfer systems of lake and river water.

The most severe drought of record occurred in England in 1976. This drought led the Drought Act of 1976 that allowed the NWWA to prohibit or limit water usage, such as recreation grounds, golf courses, lawns, swimming pools, washing cars, and cleaning buildings (Parker, 1980).

Water has been inexpensive and abundant in England and this has resulted in a high per capita use (Parker, 1980). Until the 1976 drought, most people had not been aware that water is finite resource and that drought could negatively impact their lifestyles.

Project Area and Operating Policies

The case study in this thesis deals with three reservoirs; Clowbridge, Cloughbottom, and Haslingden Grain,
located in the Rossendale District. These three reservoirs provide water to Manchester and the surrounding area. They are operated as one system by the NWWA; therefore, this system can be conceptualized from the multi-site reservoir system to a single reservoir system. Active storages of these reservoirs are 1229 megalitres (Ml) at Clowbridge, 785 Ml at Cloughbottom, and 1876 Ml at Haslingden Grane, respectively. They supply approximately 28 Ml/d under normal condition.

Like other water authorities in England, the NWWA uses control curves as their primary approach to operate reservoir systems. The NWWA defines a reservoir control curve as the set of values of storage required at the beginning of each month to meet a specified demand during a design drought without failure (Walsh, 1971). These control curves are determined without directly considering economic efficiency but by restricting the probability of reservoir storage being exhausted.

The derivation of control curve ordinates can be divided into two steps. First, the inflow that might occur during the design drought are calculated from a probabilistic analysis. Second, these probabilistic
inflows are converted to values of storage required by simple continuity equations balancing inflow and release.

The minimum historic and probabilistic (1, 2, 5, 10, and 20 %) inflows for periods of 1 through 36 consecutive months starting in each month of the calendar year are calculated based on Stocks reservoir (a pilot reservoir of the NWWA) inflow data. These 432 (12 calendar month * 36 different consecutive months) sets of data are examined and fit to a 2-parameter log-normal distribution. Based on the continuity equation, the storage requirement at the beginning of each calendar month is calculated (Walsh, 1971).

The reservoir operational rules at the NWWA are simple. Two control curves are used to make decisions. The upper control curve is calculated based on minimum historic records and safe yield for the reservoir. They assume that the safe yield is a constant value throughout the year. The maximum ordinate of this curve is equal to active storage capacity, called top water level in the NWWA. The lower curve is calculated based on minimum historic events and 85 % of the safe yield value. Figure 6-1 and Table 6-1 show the control curves at three sites and conceptualized reservoir.
Figure 6-1. Control Curves for Three Sites and Conceptualized Reservoir
Table 6-1. Ordinates of Control Curves
for Three Sites and Conceptualized Reservoir

<table>
<thead>
<tr>
<th>Month</th>
<th>Clowbridge</th>
<th>Cloughbottom</th>
<th>Haslingden Grain</th>
<th>Hypothetical Reservoir</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>upper</td>
<td>lower</td>
<td>upper</td>
<td>lower</td>
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<td>536</td>
<td>368</td>
</tr>
<tr>
<td>11</td>
<td>1166</td>
<td>953</td>
<td>588</td>
<td>420</td>
</tr>
<tr>
<td>12</td>
<td>1175</td>
<td>981</td>
<td>655</td>
<td>458</td>
</tr>
</tbody>
</table>

Top Water Level: 1376 872 2107 4355
Three simple reservoir conditions are derived from these control curves. If the storage level remains above the upper control curve, the reservoir is healthy (Zone A). If the storage is between the two control curves, the reservoir is satisfactory (Zone B). If the storage level is below the lower control curve, the reservoir is concern (Zone C).

When reservoir stays in Zone A, supplemental releases can be made. This implies that abstraction is higher than the safe yield. Further analysis is unnecessary in this position.

When reservoir stays in Zone B, only the safe yield can be released. Further analysis is unnecessary in this position. Instead, bringing water from other sources or other districts (Lake District in this case study) might be considered (called rezoning).

If the storage falls below Zone C, further analysis is required to determine if drought actions are necessary. Abstraction already equals to the safe yield. If simulation results, based on the safe yield and minimum historic inflows, indicate failure, the abstraction is reduced to the critical yield calculated by the simulation program. In this case, bringing water from other district
is required, and/or the other drought actions may be necessary, such as advertisement of urgent restraint in water use, compensation water reductions, and hosepipe bans.

However, these three simple rules do not govern all of practical situations. The other important and practical operating policies are (Slaughterbeck, 1989):

1) The operating policy based on control curves is not strict. In some cases, overdraft will continue against the operating policy because rezoning, for example, will cause water quality problems and field work.

2) If the storage of Lake District is less than 30 % of net capacity and falling more than 4 % per week, bringing in water from this source may not be possible.

3) If the simulation result shows that the future minimum storage will be less than 15 % of active storage, the manager starts to be concerned this situation is "failure".

These practical rules contain linguistic vagueness, such as some cases, may not be possible, and starts to be concerned, and implies the necessity of approximate reasoning.
Approximate Reasoning Using Linguistic Synthesis by FRESH

The linguistic synthesis approach is applied to make decisions concerning drought management policy for the conceptualized reservoir in the NWWA. The point of view adopted here is that objects are associated with universes of discourse that are nonfuzzy sets. These objects take on specific linguistic values that are expressed as fuzzy subsets of the corresponding universes. For instance, an universe of discourse is storage volume, an object is current storage, and a linguistic value is Zone A.

The rules obtained from the expert in this case study consist of one consequence object and three antecedent objects. For instance, if a rule can be stated as "IF A AND B AND C THEN D", the antecedent objects are A, B, and C and the consequence object is D. The consequence object is Operating Policy that has four linguistic values: 1) Normal Operation, 2) Draw from Lake District, 3) Consider Bans, and 4) Need Bans. The three antecedent objects are 1) Current Storage, 2) Simulated Future Minimum Storage, and 3) Condition of Lake District (storage falling rate per week). They also have linguistic values like the consequence object. As previously mentioned, the transitions of these
linguistic values are gradual rather than abrupt. The operating policy consists of a set of rules in terms of specific linguistic values of these objects.

A list of fuzzy subsets (including their linguistic values) and rules for June by FRESH is shown in Appendix C. FRESH does not have a capability to deal with dimensional arrays; therefore, a user must make twelve different data files associated with each calendar month in this application. These results have been obtained after several modifications of the fuzzy subsets and operating rules. The trial and error method was used in order to obtain an appropriate set of rules. These rules are in the form of fuzzy conditional propositions and can be expressed by fuzzy relations that are discussed in Chapter Three.

Figure 6-2 illustrates the shape of membership functions of each object by FRESH. For instance, the consequence object, Operating Policy, has four membership functions (linguistic values). They overlap each other and the transitions are gradual. For this object, the conceptual meaning of $x$-axis can be interpreted as potential of drought. Small $x$ value means drought potential is minor, and large $x$ means drought potential is severe.
USER DEFINED MEMBERSHIP FUNCTIONS

Consequence Object
Need_Bans
Consider_Bans
Draw_from_Lake_Dist.
Normal_Operation

Operating_policy

Antecedent Object
Zone_A
Zone_B
Zone_C

Storage_in_June(ML)

Antecedent Object
More_than_15%
Less_than_15%
Failure

Sim_Result(% of Cap)

Antecedent Object
More_than_4/week
Less_than_4/week
Healthy

Lake_Dist(\$/day/uk)

(Press any key to go to next screen)

Figure 6-2. The Shape of Membership Functions of Each Object by FRESH
The other example is the first antecedent object, Current Storage. This object has three membership functions: 1) Zone A, 2) Zone B, and 3) Zone C. The crisp operating policy says that if Current Storage is 3093 then it is in Zone B, and if Current Storage is 3091 then it is in Zone C. As previously mentioned, however, this crisp rule is not accurate because if Current Storage is around 3092 (the lower control curve) it is sometimes Zone B, and it is sometimes Zone C. The implementation of this vague condition by fuzzy set theory is that:

1) If Current Storage is 3092 then the possibility of "in Zone C" is 0.5, and the possibility of "in Zone B" is 0.5.

2) If Current Storage is 2992 then the possibility of "in Zone B" is 0.25, and the possibility of "in Zone C" is 0.75.

3) If Current Storage is 2892 then the possibility of "in Zone B" is zero, and the possibility of "in Zone C" is unity.

Figure 6-3 illustrates the first four linguistic rules interpreted as fuzzy conditional propositions by FRESH. Each row represents one rule. These rules have up to three antecedent object connected with AND operators. All rules
are then connected with OR operators. The comparison with Appendix C describes the output of FRESH.

Table 6-2 illustrates the case of experimental design undertaken to investigate the effect of the input values of antecedent objects and their reasoning results. This experimental design contains twenty different combinations of antecedent objects. These are:

1) Current Storage
   (1) just on the lower control curve and (2) in Zone C.

2) Simulated Future Minimum Storage
   (1) 7.5 % and (2) 2.5 %.

3) Condition of Lake District (storage falling rate per week)
   (1) 0 %, (2) 1.75 %, (3) 2.5 %, (4) 4 %, and (5) 10 %.

   Figure 6-4 illustrates reasoning results for each fuzzy conditional proposition by FRESH under the case (1)-(2)-(4) (Current Storage is just on the lower control curve, Simulated Future Minimum Storage is 2.5 % of net capacity, and Condition of Lake District is 4 % storage falling rate per week) in Table 6-2.
Figure 6-3. The First Four Fuzzy Conditional Propositions by FRESH
Table 6-2  Experimental Design Cases  
and Matching Degrees

<table>
<thead>
<tr>
<th>Current Storage</th>
<th>Simulated Future Minimum Storage</th>
<th>Condition of Lake District</th>
<th>Normal Operation Draw from Lake District</th>
<th>Consider Bans</th>
<th>Need Bans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Control Curve</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5 %</td>
<td>0 %</td>
<td>10 %</td>
<td>40 %</td>
<td>10 %</td>
<td>0 %</td>
</tr>
<tr>
<td>1.75 %</td>
<td>8 %</td>
<td>34 %</td>
<td>34 %</td>
<td>8 %</td>
<td></td>
</tr>
<tr>
<td>2.5 %</td>
<td>0 %</td>
<td>10 %</td>
<td>40 %</td>
<td>10 %</td>
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<tr>
<td>4 %</td>
<td>0 %</td>
<td>8 %</td>
<td>34 %</td>
<td>34 %</td>
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<tr>
<td>10 %</td>
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<td>11 %</td>
<td>50 %</td>
<td>11 %</td>
<td>0 %</td>
</tr>
<tr>
<td>1.75 %</td>
<td>9 %</td>
<td>39 %</td>
<td>39 %</td>
<td>9 %</td>
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<tr>
<td>2.5 %</td>
<td>0 %</td>
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<tr>
<td>4 %</td>
<td>0 %</td>
<td>9 %</td>
<td>39 %</td>
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<tr>
<td>10 %</td>
<td>0 %</td>
<td>0 %</td>
<td>11 %</td>
<td>50 %</td>
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<tr>
<td>Zone C</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7.5 %</td>
<td>0 %</td>
<td>13 %</td>
<td>80 %</td>
<td>13 %</td>
<td>0 %</td>
</tr>
<tr>
<td>1.75 %</td>
<td>8 %</td>
<td>34 %</td>
<td>34 %</td>
<td>8 %</td>
<td></td>
</tr>
<tr>
<td>2.5 %</td>
<td>0 %</td>
<td>13 %</td>
<td>80 %</td>
<td>13 %</td>
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<tr>
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<td>0 %</td>
<td>0 %</td>
<td>11 %</td>
<td>50 %</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6-4 (1). Reasoning Results for Each Fuzzy Conditional Propositions by FRESH
Figure 6-4 (2). Reasoning Results for Each Fuzzy Conditional Propositions by FRESH
Figure 6-4 (3). Reasoning Results for Each Fuzzy Conditional Propositions by FRESH
Figure 6-4 (4). Reasoning Results for Each Fuzzy Conditional Propositions by FRESH
For instance, the result of the second rule (Rule_Normal_2) is that the possibility of "Current Storage is in Zone B" is 0.5, and the possibility of "Simulated Future Minimum Storage is More Than 15% of net capacity" is zero; thus, the possibility of "Operating Policy is Normal Operation" is zero because the AND operator is interpreted as a minimum operation.

The seventh rule (Rule_Concern_1) can be interpreted as: The minimum possibility of antecedent objects is 0.5. Thus, the possibility of consequence may also be 0.5. However, the truth value of this rule is 0.3. Therefore, the possibility of consequence is 0.15 (= 0.5 * 0.3). The filled portion of consequence membership function represents the possibility distribution of reasoning result under given condition.

Figures 6-5 and 6-6 illustrate the matching results and a summary of fuzzy reasoning by FRESH. These reasoning results imply that under the given condition, it is difficult to justify only one consequence. Instead, FRESH suggests that 1) the best relevant consequence is Consider Bans with 39% matching degree, 2) the second best relevant consequence is Need Bans with also 39% matching degree, and 3) the third best relevant consequence is Draw from Lake
MATCHING RESULT OF FUZZY REASONING

Operating_policy (THEN Part)

Reasoning Result:

Matching Degree for Each Consequence:
- 0% matching degree for Normal_Operation
- 9% matching degree for Draw_from_Lake_Dist.
- 39% matching degree for Consider_Bans
- 39% matching degree for Need_Bans

Horizontal Centroid of Reasoning Result
0.7

(Press any key to go to next screen)

Figure 6-5. Matching Result of Fuzzy Reasoning by FRESH
Summary of Fuzzy Reasoning

=== User Input ===
Storage_in_June(MI) : 3092    Sim_Result(% of Cap) : 2.5    Lake_Dist(% of NL) : 4

=== Reasoning Results ===
The number of total propositions : 14
The number of active propositions : 8

The best three matched consequences:
Consider Bans with 39 % matching degree.
Need Bans with 39 % matching degree.
Draw from Lake Dist. with 9 % matching degree.

Horizontal centroid of reasoning result : 0.7

(Press any key to go to next screen)

Figure 6-6. Summary of Fuzzy Reasoning by FRESH
District with 9 % matching degree. In this example, one of the first two pieces of advice might be used in practical operation because their matching degrees are the same.

Figure 6-7 illustrates the sensitivity analysis results regarding matching degrees of each consequence based on the different conditions shown in Table 6-2. These results suggest that if the given conditions are on the fuzzy boundaries, such as just on the control curves, crisp consequence may not be obtained. For the other cases, however, the best relevant consequence is obvious.

Summary

This chapter has presented the results of an application of FRESH to the NWWA water resources management in England. The application illustrates that FRESH can produce results that are considered to approximate the reasoning of an expert.

The final chapter discusses the conclusions of this research and recommendations for future direction.
Figure 6-7. Sensitivity Analysis Results of Matching Degrees
CHAPTER SEVEN

Conclusions and Recommendations

Conclusions

This research discusses the use of fuzzy sets in propagating uncertainty in expert system knowledge bases. Also, the research describes software that has an interactive knowledge acquisition environment and a fuzzy logic based inference engine. This software is denoted as FRESH (Fuzzy Reasoning Shell).

FRESH integrates a menu-driven system, user input screens, graphics, and a fuzzy logic based inference engine. With these features and a syntax error evaluation system, an expert can create his/her knowledge base without extensive computing skills. Much of uncertainty in the decision making process contains elements of possibility; therefore, FRESH can be a useful tool to develop knowledge-based systems.

Approximate reasoning using linguistic synthesis by FRESH is applied to the North West Water Authority in England. The example also illustrates an effective application of fuzzy propositional logic.
It is important to note that fuzzy propositional logic allows one to simplify complex decision making systems using linguistic rules. Because of this, it is easier for a programmer to synthesize and implement approximate reasoning in linguistic terms. This fact implies that development of the set of linguistic rules can be concise (only 14 rules for this example).

The fuzziness occupies the important role in the human decision making process. Thus, it is critical that knowledge-based systems have the capability to quantify the uncertainty and trace its influence on decision making process. Since FRESH allows one to incorporate all expertise in a system, a novice user can use this system and obtain an expert's approximate reasoning results without being familiar with the full complexity of the decision making process.

The proposed operational rule base used in the linguistic decision making system has a very simple structure. It can be characterized as a collection of conditional statements, all of the type "IF A AND B AND C THEN D". However, this structure can be extended to more complex settings including forward/backward chaining in the future.
Recommendations

A major limitation of FRESH is that it can only deal with one-layer reasoning (no forward/backward chaining). In Chapter Four, the example of multi-layer reasoning by fuzzy logic is described. However, fuzzy logic is an active area of research, and the definitions of intersection (the logical AND or minimum operator) and union (the logical OR or minimum operator) are still an arena of active debate for the propagation of uncertainty in multi-layer reasoning (Dubois and Prade, 1989). It is hoped that further theoretical and experimental work will be done on this problem.

Another limitation of FRESH in the application phase is that a programmer has the responsibility of defining the shape of membership functions and propositions based on a trial and error method in order to build successful knowledge bases. Although FRESH provides an interactive knowledge acquisition environment, this process remains a difficult task in the development of knowledge base. Therefore, induction or the machine learning approach should be considered in further research.
The facts, relations, and judgments of human experts contain degree of impression and uncertainty. Thus, the management of possibility in the design of expert systems is a key factor for the successful modeling of reasoning process. The utility of fuzzy set theory and fuzzy logic for this purpose has been and continues to be extensively studied.
BIBLIOGRAPHY


APPENDIX A

Mathematical Example of Fuzzy Reasoning

This appendix illustrates that both single and multi-layer fuzzy reasoning satisfy modus ponens. In the single-layer reasoning example, the logical-product (Mamdani, 1977) and the algebraic-product (for example, Dubois and Prade, 1980) of fuzzy sets are examined, and fuzzy reasoning under given facts is illustrated. In the multi-layer reasoning example, only the logical-product is examined.

Background of Fuzzy Sets and Fuzzy Logic

Let $A$ be a subset of the universe of discourse $X$. The characteristic function $M_A(x)$ of classical subset $A$ takes its values in the two-element set $\{0, 1\}$, and is such that $M_A(x) = 1$ if $x \in A$ and $M_A(x) = 0$ otherwise. A fuzzy set $A$ has a characteristic function $M_A(x)$ taking its values in the interval $[0, 1]$, with the grade 1 and 0 representing full membership and nonmembership, respectively. Thus, $M_A(x)$ is called the membership function of $A$. Since $M_A(x)$ is the grade of membership of $x \in A$, it is also possible to interpret $M_A(x)$ as the degree of possibility (Zadeh, 1983) that $x$ is the value of a parameter fuzzily restricted by $A$. 
A is symbolically denoted $[(x, M_A(x)) | x \in A]$.

Several fundamental structures can be defined on the interval $[0, 1]$, such as fuzzy set union, intersection, and complement operator. These functions coincide with those used in classical set theory. Zadeh (1965) first used the operator $([0, 1], \max, \min, \text{not})$ that are the most frequently employed, i.e.:

- **Union**: $M_{A\cup B}(x) = \max(M_A(x), M_B(x))$
- **Intersection**: $M_{A\cap B}(x) = \min(M_A(x), M_B(x))$
- **Complement**: $\Gamma M_A(x) = 1 - M_A(x)$

Fuzzy logic is a natural expansion of classical logic that every proposition is either true or false. Zadeh (1973) proposed a fuzzy logic in which the antecedent involves a fuzzy conditional proposition. For instance, "If $x$ is $A$ then $y$ is $B$", with $A$ and $B$ being fuzzy concepts.

**Logic** is the study of the methods and principles of reasoning in all its possible form. One area of classical logic is propositional logic that requires an object to be either true (unity) or false (zero). These values are termed truth values. Propositional logic deals with combinations of objects (or variables) that represent arbitrary propositions. As each object represents a hypothetical proposition, it may assume either of the two
truth values. Various forms of reasoning rules (or inference rules) can be used for making deductive reasoning.

*Modus ponens* is one of the most frequently used reasoning rules, i.e.:

\[(A \land (A \Rightarrow B)) \Rightarrow B\]

where \( \land \) is and-function (or conjunction) and \( \Rightarrow \) is implication. *Modus ponens* states that given two true propositions \( A \) and \( A \Rightarrow B \) (the antecedents), the truth of the proposition \( B \) (the consequence) may be inferred.

**Single-Layer Reasoning**

First, we shall consider the following simple form of approximate reasoning that includes a fuzzy conditional proposition:

**Ant**: IF \( x \) is \( A \)

THEN \( y \) is \( B \) WITH TV.

**Fact**: \( x \) is \( A \).

-------------------------------
cons: \( y \) is \( B' \).

where \( x \) and \( y \) are the names of objects; \( A, A' \) ("similar to" \( A \)), \( B, \) and \( B' \) ("similar to" \( B \)) are the labels of fuzzy subsets in the universes of discourse \( X, X, Y, \) and \( Y, \) respectively; and \( TV \) is the truth value of the antecedent.
This reasoning scheme is called generalized modes ponens, and has two major differences from classical modes ponens (Bellman and Zadeh, 1977). First, \( A' \) is not required to be identical with \( A \), as it is in the classical case. Second, \( A, A', \) and \( B \) are not required to be crisp.

The implied relation between the two objects \( x \) and \( y \) is expressed in terms of the cartesian product of the two fuzzy subsets \( A \) and \( B \). The cartesian product, characterized by a function \( M_R(x, y) \) by which each pair \((x, y)\) is assigned a number in \([0, 1]\) indicating the extent to which the relation \( R \) is true for \((x, y)\), of two fuzzy subset \( A \) and \( B \) is denoted as \( A*B \). The following two equations illustrate two different definitions of cartesian product, the logical-product and the algebraic-product.

**Logical-Product**

\[
A*B = \Sigma_i \Sigma_j \min \{M_A(x_i), M_B(y_j)\}
\]

**Algebraic-Product**

\[
A*B = \Sigma_i \Sigma_j (M_A(x_i) * M_B(y_j))
\]

where \( \Sigma \) stands for union; \( x_i \) and \( y_j \) are generic elements of the universes of discourse \( X \) and \( Y \), respectively; \( i = 1, 2, \ldots, m; \) and \( j = 1, 2, \ldots, n. \)

These equations can be used to infer the fuzzy subset of the second object given a membership function, or a
value, for the first. Suppose we denote the relationship between two objects by $R_{AB}$ ($A \Rightarrow B$). $A'$ is given membership function, then, the fuzzy subset $y$ is inferred by:

$$B' = A' \circ R_{AB}$$

where $\circ$ is composition. Composition is performed by the maximum-product of $A'$ and $R_{AB}$. For example, let:

$$M_A(x_i) = [0.0 \ 0.5 \ 1.0 \ 0.5 \ 0.0]$$

$$M_B(y_j) = [0.6 \ 0.8 \ 0.4 \ 0.0]$$

Figure A-1 illustrates the membership functions of $A$ and $B$. The conditional inference of this form is:

"If $x$ is nearly equal 3 then $y$ is nearly equal 2."

![Diagram](https://via.placeholder.com/150)

Figure A-1. Membership Functions of $A$ and $B
Logical-Product:

\[ R_{AB}(i, j) = \min_{i, j} (M_A(x_i), M_B(y_j)) \]

\[
\begin{array}{cccc}
0.0 & 0.0 & 0.0 & 0.0 \\
0.5 & 0.5 & 0.4 & 0.0 \\
0.6 & 0.8 & 0.4 & 0.0 \\
0.5 & 0.5 & 0.4 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{array}
\]

If \( A' \) is equal to \( A \), \( B' \) must be equal to \( B \). This relationship is *modus ponens*.

\[ M_{B'}(y_j) = \max_i \left\{ \min_j (M_A(x_i), R_{AB}(i, j)) \right\} \]

\[
\begin{array}{cccc}
0.0 & 0.0 & 0.0 & 0.0 \\
0.5 & 0.5 & 0.4 & 0.0 \\
0.6 & 0.8 & 0.4 & 0.0 \\
0.5 & 0.5 & 0.4 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{array}
\]

\[ = \max_i \begin{bmatrix} 0.6 & 0.8 & 0.4 & 0.0 \end{bmatrix} \]

\[ = M_B(y_j) \]

Thus, the logical-product satisfies *modus ponens*.

In the FRESH application, \( A' \) and \( B' \) were chosen to be nonfuzzy vectors and with only one element equal to unity and the remaining equal to zero. For example, let:

\[ M_{A'}(x_i) = [0.0 \ 1.0 \ 0.0 \ 0.0 \ 0.0] \]

Then, the consequence \( M_{B'}(y_j) \) will be:
\[ M_B'(y_j) = \max_i \left[ \min_j \left( M_A'(x_i), R_{AB}(i, j) \right) \right] \]

\[
= \max_i \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.5 & 0.5 & 0.4 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

\[ = [0.5 \ 0.5 \ 0.4 \ 0.0] \]

This result implies that even though the given fact is not exactly identical with the condition of the relationship, fuzzy logic infers something according to the relationship.

**Algebraic-Product:**

\[ R_{AB}(i, j) = (M_A(x_i) \ast M_B(y_j)) \]

\[
= \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.6 & 0.8 & 0.4 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

If \( A' \) is equal to \( A \):

\[ M_B'(y_j) = \max_i \left[ \left( M_A(x_i) \ast R_{AB}(i, j) \right) \right] \]

\[
= \max_i \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 \\
0.15 & 0.20 & 0.10 & 0.00 \\
0.60 & 0.80 & 0.40 & 0.00 \\
0.15 & 0.20 & 0.10 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \\
\end{bmatrix}
\]

\[ = [0.6 \ 0.8 \ 0.4 \ 0.0] \]

\[ = M_B(y_j) \]
Thus, the algebraic-product also satisfies modus ponens.

It should be noted that the maximum possibility of antecedent object must be equal to unity to satisfy modus ponens; however, the maximum possibility of a consequence object can assume a value less than unity. This fact implies that the management of truth value (TV) can be dealt under the same framework of fuzzy logic.

Under the same condition as logical-product, let:

\[ M'_A(x_i) = [0.0 \ 1.0 \ 0.0 \ 0.0 \ 0.0] \]

Then, the consequence \( M'_B(y_j) \) will be:

\[ M'_B(y_j) = \max_i [M'_A(x_i) \ast R_{AB}(i, j)] \]

\[
= \max_i \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

\[= [0.3 \ 0.4 \ 0.2 \ 0.0] \]

Even though the same fact is used, the reasoning result of the algebraic-product is different from the result of the logical-product. The result by the logical-product shows that the upper portion of the consequence membership function is truncated, and the shape of membership function is changed. By contrast, the result by the algebraic
product illustrates that the consequence membership function is algebraically multiplied by the grade of the antecedent membership function associated with the given fact, and the original shape of membership is maintained.

Sugeno (1984) suggests that using the algebraic-product can improve reasoning results because of its property that maintain the original shape of the consequence membership function. Therefore, algebraic-product is adopted as a fuzzy reasoning operator for this research.

More complex rules are developed by use of two fuzzy antecedent objects, \( x_1 \) and \( x_2 \), that are connected by AND.

\[
\text{Ant} : \text{IF } x_1 \text{ is } A \text{ AND } x_2 \text{ is } B \\
\text{THEN } y \text{ is } C.
\]

\[
\text{Fact1: } x_1 \text{ is } A'.
\]

\[
\text{Fact2: } x_2 \text{ is } B'.
\]

\[
\text{Cons : } y \text{ is } C'.
\]

In this example, the algebraic-product and composition can be defined as:

\[
A \ast C \text{ AND } B \ast C = \Sigma_i \Sigma_k (M_A(x_{1i}) \ast M_C(y_k)) \\
\cap \Sigma_j \Sigma_k (M_B(x_{2j}) \ast M_C(y_k))
\]

\[
C' = (A' \circ R_{AC}) \cap (B' \circ R_{BC})
\]

For example, let:
\[ M_A(x_{1_i}) = \begin{bmatrix} 0.0 & 0.5 & 1.0 & 0.5 & 0.0 \end{bmatrix} \]
\[ M_B(x_{2_j}) = \begin{bmatrix} 1.0 & 0.6 & 0.0 \end{bmatrix} \]
\[ M_C(Y_k) = \begin{bmatrix} 0.6 & 0.8 & 0.4 & 0.0 \end{bmatrix} \]

The AND operator can be interpreted as the minimum (or intersection) operation. Thus,

\[ R_{AC}(i, k) = (M_A(x_{1_i}) \ast M_C(Y_k)) \]
\[ = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.6 & 0.8 & 0.4 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \]

\[ R_{BC}(j, k) = (M_B(x_{2_j}) \ast M_C(Y_k)) \]
\[ = \begin{bmatrix} 0.60 & 0.80 & 0.40 & 0.00 \\
0.36 & 0.48 & 0.24 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \]

If \( A' \) and \( B' \) are equal to \( A \) and \( B \),

\[ M_C'(Y_k) = \max_i \left\{ \left( M_A(x_{1_i}) \ast R_{AC}(i, k) \right) \right\} \]
\[ \cap \max_j \left\{ \left( M_B(x_{2_j}) \ast R_{BC}(j, k) \right) \right\} \]

\[ = \max_i \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\
0.15 & 0.20 & 0.10 & 0.00 \\
0.60 & 0.80 & 0.40 & 0.00 \\
0.15 & 0.20 & 0.10 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \]
\[ \cap \max_j \begin{bmatrix} 0.60 & 0.80 & 0.40 & 0.00 \\
0.22 & 0.28 & 0.14 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \]
\[
\begin{bmatrix}
0.6 & 0.8 & 0.4 & 0.0 \\
0.6 & 0.8 & 0.4 & 0.0 \\
\end{bmatrix}
= M_C(y_k)
\]

Thus, also in this case, algebraic-product satisfies modus ponens.

If \( A' \) and \( B' \) are not equal to \( A \) and \( B \), let:

\[
M'_{A}(x_{1i}) = \begin{bmatrix}
0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]
\[
M'_{B}(x_{2j}) = \begin{bmatrix}
0.0 & 1.0 & 0.0 \end{bmatrix}
\]

Then, the consequence \( M'_C(y_k) \) will be:

\[
M'_C(y_k) = \max_i \left[ \left\{ M'_{A}(x_{1i}), R_{AC}(i, k) \right\} \right] \\
\cap \max_j \left[ \left\{ M'_{B}(x_{2j}), R_{BC}(j, k) \right\} \right]
\]

\[
= \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.3 & 0.4 & 0.2 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

\[
\cap \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 \\
0.36 & 0.48 & 0.24 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.3 & 0.4 & 0.2 & 0.0 \\
\end{bmatrix}
\]

\[
\cap \begin{bmatrix}
0.36 & 0.48 & 0.24 & 0.0 \\
\end{bmatrix}
\]
Finally, the OR operator is considered with the AND operator.

\[
= \begin{bmatrix}
0.3 & 0.4 & 0.2 & 0.0
\end{bmatrix}
\]

\[
\text{Ant} : \quad \text{IF } (x_1 \text{ is } A \text{ AND } x_2 \text{ is } B) \text{ OR } (x_3 \text{ is } C) \\
\quad \text{THEN } y \text{ is } D.
\]

Fact1: \( x_1 \) is \( A' \).
Fact2: \( x_2 \) is \( B' \).
Fact3: \( x_3 \) is \( C' \).

Cons : \( y \) is \( D' \).

This form of fuzzy reasoning can be divided into the following two sets of fuzzy conditional propositions.

\[
\text{Ant 1: IF } x_1 \text{ is } A \text{ AND } x_2 \text{ is } B \\
\quad \text{THEN } y \text{ is } D.
\]

Fact1: \( x_1 \) is \( A' \).
Fact2: \( x_2 \) is \( B' \).

Cons1: \( y \) is \( D' \).

\[
\text{OR}
\]

\[
\text{Ant 2: IF } x_3 \text{ is } C \\
\quad \text{THEN } y \text{ is } D.
\]

Fact3: \( x_3 \) is \( C' \).

Cons2: \( y \) is \( D' \).
Thus,

Cons: Cons1 OR Cons2.

The algebraic-product and composition is defined as:

\[(A*D \text{ AND } B*D) \text{ OR } C*D\]
\[= \left[ \sum_i \sum_l \left( M_A(x_{1l}) * M_D(y_{1l}) \right) \cap \sum_j \sum_l \left( M_B(x_{2j}) * M_D(y_{1l}) \right) \right] \]
\[\cup \left[ \sum_k \sum_l \left( M_C(x_{3k}) * M_D(y_{1l}) \right) \right]\]

\[D' = \left[ (A' \circ R_{AD}) \cap (B' \circ R_{BD}) \right] \cup \left[ (C' \circ R_{CD}) \right] \]

Composition and the AND operator are performed in same manner. Then, the OR operator is interpreted as the maximum (or union) operation, and connects these two sets of fuzzy conditional propositions.

For example, let:

\[M_A(x_{1i}) = \begin{bmatrix} 0.0 & 0.5 & 1.0 & 0.5 & 0.0 \end{bmatrix} \]
\[M_B(x_{2j}) = \begin{bmatrix} 1.0 & 0.6 & 0.0 \end{bmatrix} \]
\[M_C(y_k) = \begin{bmatrix} 0.6 & 0.8 & 0.4 & 0.0 \end{bmatrix} \]

\[R_{AD}(i, l) = (M_A(x_{1i}) \ast M_D(y_{1l})\]
\[ R_{BD}(j, l) = (M_B(x2j) \ast M_D(y_1)) \]
\[ = \begin{array}{cccc}
0.60 & 0.80 & 0.40 & 0.00 \\
0.36 & 0.48 & 0.24 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{array} \]

\[ R_{CD}(k, l) = (M_C(x3k) \ast M_D(y_1)) \]
\[ = \begin{array}{cccc}
0.60 & 0.80 & 0.40 & 0.00 \\
0.12 & 0.16 & 0.08 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{array} \]

If \( A', B' \) and \( C' \) are equal to \( A, B \) and \( C \),

\[ M_D'(y_1) = [ \max_i ( M_A(x1i) \ast R_{AD}(i, l) ) ] \]
\[ \cap \max_j ( M_B(x2j) \ast R_{BD}(j, l) ) ] \]
\[ U [ \max_k ( M_C(x3k) \ast R_{CD}(k, l) ) ] \]

\[ = [ 0.6 \ 0.8 \ 0.4 \ 0.0 ] \text{(See previous example)} \]

\[ U \max_j \begin{array}{cccc}
0.60 & 0.80 & 0.40 & 0.00 \\
0.02 & 0.03 & 0.02 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{array} \]

\[ = [ 0.6 \ 0.8 \ 0.4 \ 0.0 ] \]
\[ U [ 0.6 \ 0.8 \ 0.4 \ 0.0 ] \]

\[ = [ 0.6 \ 0.8 \ 0.4 \ 0.0 ] \]

\[ = M_D(y_1) \]

Thus, algebraic-product satisfies modus ponens in this case also.
If $A', B', \text{ and } C'$ are not equal to $A, B$ and $C$, let:

$$M_{A'}(x_1) = [0.0 \ 1.0 \ 0.0 \ 0.0 \ 0.0]$$
$$M_{B'}(x_2) = [0.0 \ 1.0 \ 0.0]$$
$$M_{C'}(x_3) = [0.0 \ 1.0 \ 0.0]$$

Then, the consequence $M_{D'}(y_1)$ will be:

$$M_{D'}(y_1) = \max_i \left( M_{A'}(x_1i) \times R_{AD}(i, l) \right)$$
$$\cap \max_j \left( M_{B'}(x_2j) \times R_{BD}(j, l) \right)$$
$$U \left[ \max_k \left( M_{C'}(x_3k) \times R_{CD}(k, l) \right) \right]$$

$$= [0.3 \ 0.4 \ 0.2 \ 0.0] \text{ (See previous example)}$$

$$U \max_j \left| \begin{array}{cccc}
0.00 & 0.00 & 0.00 & 0.00 \\
0.12 & 0.16 & 0.08 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{array} \right|$$

$$= [0.3 \ 0.4 \ 0.2 \ 0.0]$$
$$U [0.12 \ 0.16 \ 0.08 \ 0.00]$$

$$= [0.3 \ 0.4 \ 0.2 \ 0.0]$$

**Multi-Layer Reasoning**

Multi-layer reasoning by fuzzy logic is an active area of research and controversial topic. Therefore, this
appendix illustrates the simplest two-layer reasoning using
the logical-product. Let,

\[
\begin{align*}
\text{value:} & \quad 1 \quad 2 \quad 3 \quad \quad \text{value:} \quad 2 \quad 4 \quad 6 \\
M_{A1} &= [ 0.2 \quad 0.5 \quad 1.0 ], \quad M_{B1} = [ 0.1 \quad 0.3 \quad 1.0 ] \\
R_{AB1} &= \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.3 \\ 0.2 & 0.5 & 1.0 \end{bmatrix} \\
M_{C1} &= [ 0.8 \quad 1.0 \quad 0.1 ] \\
R_{BC1} &= \begin{bmatrix} 0.1 & 0.3 & 0.8 \\ 0.1 & 0.3 & 1.0 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}
\end{align*}
\]

The example of this reasoning is:

If rainfall (A) is about 3 then inflow (B) is about 6.
If inflow (B) is about 6 then release (C) is about 6.

We can also have the other rules, like:

\[
\begin{align*}
\text{value:} & \quad 1 \quad 2 \quad 3 \quad \quad \text{value:} \quad 2 \quad 4 \quad 6 \\
M_{A2} &= [ 0.6 \quad 1.0 \quad 0.8 ], \quad M_{B2} = [ 0.4 \quad 1.0 \quad 0.6 ] \\
R_{AB2} &= \begin{bmatrix} 0.4 & 0.4 & 0.4 \\ 0.6 & 1.0 & 0.8 \\ 0.6 & 1.0 & 0.6 \end{bmatrix} \\
M_{C2} &= [ 1.0 \quad 0.5 \quad 0.2 ] \\
R_{BC2} &= \begin{bmatrix} 0.4 & 1.0 & 0.6 \\ 0.4 & 0.5 & 0.5 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}
\end{align*}
\]

The example of this reasoning is:
If rainfall (A) is about 2 then inflow (B) is about 4.
If inflow (B) is about 4 then release (C) is about 3.

Figure A-2 illustrates these membership functions.

Figure A-2. Membership Functions of A1, B1, C1, A2, B2, and C2
If the rainfall is about 3 \((A' = A1)\), then the consequence membership function of inflow \((M_{B1}')\) must be equal to "inflow is about 6" \((M_{B1})\).

\[
M_{B1}' = M_A' \circ R_{AB1}
\]

\[
= \begin{bmatrix} 0.2 & 0.5 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.3 \\ 0.2 & 0.5 & 1.0 \end{bmatrix}
\]

\[
= \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.3 \\ 0.2 & 0.5 & 1.0 \end{bmatrix} \quad \text{Minimum Operator}
\]

\[
= \begin{bmatrix} 0.1 \\ 0.3 \\ 1.0 \end{bmatrix} \quad \text{Maximum Operator}
\]

\[
= M_{B1}
\]

where \(\circ\) is composition. Thus, modus Ponens is satisfied.

At the same time:

\[
M_{B2}' = M_A' \circ R_{AB2}
\]

\[
= \begin{bmatrix} 0.2 & 0.5 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 0.4 & 0.4 & 0.4 \\ 0.6 & 1.0 & 0.8 \\ 0.6 & 1.0 & 0.6 \end{bmatrix}
\]

\[
= \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.2 & 0.5 & 0.8 \\ 0.2 & 0.5 & 0.6 \end{bmatrix} \quad \text{Minimum Operator}
\]

\[
= \begin{bmatrix} 0.4 \\ 0.8 \\ 0.6 \end{bmatrix} \quad \text{Maximum Operator}
\]

Therefore,
\( M_{B'} = M_{B1'} \text{ OR } M_{B2'} \)

\[
= [0.1 \ 0.3 \ 1.0] \land [0.4 \ 0.8 \ 0.6]
\]

\[
= [0.4 \ 0.8 \ 1.0]
\]

This is not equal to \( M_{B1} \) because OR operator takes maximum operation of \( M_{B1'} \) and \( M_{B2'} \).

The next layer's reasoning result will be:

\( M_{C1'} = M_{B'} \circ R_{BC1} \)

\[
= [0.4 \ 0.8 \ 1.0] \circ \\
\begin{array}{ccc}
0.1 & 0.3 & 0.8 \\
0.1 & 0.3 & 1.0 \\
0.1 & 0.1 & 0.1 \\
\end{array}
\]

\[
= \begin{array}{ccc}
0.1 & 0.3 & 0.8 \\
0.1 & 0.3 & 1.0 \\
0.1 & 0.1 & 0.1 \\
\end{array} \text{ Minimum Operator}
\]

\[
= \begin{array}{c}
0.8 \\
1.0 \\
0.1 \\
\end{array} \text{ Maximum Operator}
\]

\[
= M_{C1}
\]

Thus, modus Ponens is also satisfied.

At the same time:

\( M_{C2'} = M_{B'} \circ R_{BC2} \)

\[
= [0.4 \ 0.8 \ 1.0] \circ \\
\begin{array}{ccc}
0.4 & 1.0 & 0.6 \\
0.4 & 0.5 & 0.5 \\
0.2 & 0.2 & 0.2 \\
\end{array}
\]

\[
= \begin{array}{ccc}
0.4 & 0.8 & 0.6 \\
0.4 & 0.5 & 0.5 \\
0.2 & 0.2 & 0.2 \\
\end{array} \text{ Minimum Operator}
\]
\[
\begin{array}{c|c}
\text{Maximum Operator} \\
0.8 & 0.5 \\
0.5 & 0.2 \\
\end{array}
\]

Therefore,

\[M'_{c1} = M'_{c2}\]

\[
= [0.8 \ 1.0 \ 0.1] \cap [0.8 \ 0.5 \ 0.2]
\]

\[
= [0.8 \ 1.0 \ 0.2]
\]

This is not equal to \(M_{c1}\) because OR operator takes maximum operation of \(M'_{c1}\) and \(M'_{c2}\).

This example illustrates that modes ponens is also satisfied in multi-layer reasoning using the logical-product.
APPENDIX B

Installation of FRESH

FRESH runs on the IBM PC or any IBM compatible computer that has a color EGA or VGA monitor, a hard disk drive, 640 K RAM, and PC-DOS/MS-DOS operating system.

Installation of FRESH is very straightforward, and requires a minimal understanding of DOS to perform. FRESH files must be copied to a hard disk. There must be over 600 K of storage space remaining on the hard disk.

After the computer has turned-on, create a 'FRESH' directory in the C (hard disk) drive to contain FRESH files. This is accomplished by typing MD FRESH at the C: prompt and pressing the <ENTER> key. This will create the FRESH subdirectory on the C drive. Next, move to the new directory by typing:

C:\> CD FRESH

followed by <ENTER> key. At this point the user is ready to copy FRESH files to the hard disk.

Place the FRESH program disk in the A drive. Enter at the DOS prompt:

C:\FRESH> COPY A:*.*
then press <ENTER> key. The screen will display the name of each file as it is copied to the hard disk.

After installation of FRESH, enter at the DOS prompt:

C:\FRESH> FRESH

then press <ENTER> key. Learning to use FRESH is a very simple process. Since FRESH adopts a menu-driven system, the user can immediately begin using the sample knowledge base provided that is explained in Chapter Five.
APPENDIX C

Listing of a Sample Knowledge Base by FRESH

===============================================
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>List of Data File</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
===============================================

File Name : C:\FRESH\NWFW_JUN.DAT
Number of Subintervals : 20
Number of Objects : 4
Number of Propositions : 14

===== Object Data =====

Object Name : Operating_policy
Minimum Boundary : 0.00
Maximum Boundary : 1.00
Number of Membership Functions : 4

Membership Function Name : Normal_Operation
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
0.00    0.20    0.20    0.40

Membership Function Name : Draw_from_Lake_Dist.
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
0.20    0.40    0.40    0.60

Membership Function Name : Consider_Bans
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
0.40    0.60    0.60    0.80

Membership Function Name : Need_Bans
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
0.60    0.80    0.80    1.00

Object Name : Storage_in_June(W1)
Minimum Boundary : 0.00
Maximum Boundary : 4355.00
Number of Membership Functions : 3

Membership Function Name : Zone_C
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
0.00    0.00    2892.00    3292.00

Membership Function Name : Zone_B
X-Axis 1  X-Axis 2  X-Axis 3  X-Axis 4
2892.00    3292.00    3635.00    4035.00
Membership Function Name: Zone_A

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3635.00</td>
<td>4035.00</td>
<td>4355.00</td>
<td>4355.00</td>
</tr>
</tbody>
</table>

Object Name: Sim_Result(% of Cap)

Minimum Boundary: 0.00
Maximum Boundary: 30.00
Number of Membership Functions: 3

Membership Function Name: Failure

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Membership Function Name: Less_than_15%

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>5.00</td>
<td>10.00</td>
<td>20.00</td>
</tr>
</tbody>
</table>

Membership Function Name: More_than_15%

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>20.00</td>
<td>30.00</td>
<td>30.00</td>
</tr>
</tbody>
</table>

Object Name: Lake_Dist(%fall/wk)

Minimum Boundary: 0.00
Maximum Boundary: 10.00
Number of Membership Functions: 3

Membership Function Name: Healthy

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>1.50</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Membership Function Name: Less_than_4%/week

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>2.00</td>
<td>3.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Membership Function Name: More_than_4%/week

<table>
<thead>
<tr>
<th>X-Axis 1</th>
<th>X-Axis 2</th>
<th>X-Axis 3</th>
<th>X-Axis 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>5.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>
==== Proposition Data ====

Proposition Name: Rule_Normal_1
IF Storage_in_June(M1) IS Zone_A
AND Operating_policy IS Normal_Operation
THEN TV = 1.00

Proposition Name: Rule_Normal_2
IF Storage_in_June(M1) IS Zone_B
AND Sim_Resp(%) Of Cap IS More_than_15%
AND Operating_policy IS Normal_Operation
THEN TV = 1.00

Proposition Name: Rule_Draw_1
IF Storage_in_June(M1) IS Zone_B
AND Sim_Resp(%) Of Cap IS Less_than_15%
AND Lake_Dist(%) Fail/wk IS Healthy
THEN Operating_policy IS Draw_from_Lake_Dist.

Proposition Name: Rule_Draw_2
IF Storage_in_June(M1) IS Zone_B
AND Sim_Resp(%) Of Cap IS Failure
AND Lake_Dist(%) Fail/wk IS Healthy
THEN Operating_policy IS Draw_from_Lake_Dist.

Proposition Name: Rule_Draw_3
IF Storage_in_June(M1) IS Zone_C
AND Sim_Resp(%) Of Cap IS Less_than_15%
AND Lake_Dist(%) Fail/wk IS Healthy
THEN Operating_policy IS Draw_from_Lake_Dist.

Proposition Name: Rule_Draw_4
IF Storage_in_June(M1) IS Zone_C
AND Sim_Resp(%) Of Cap IS Failure
AND Lake_Dist(%) Fail/wk IS Healthy
THEN Operating_policy IS Draw_from_Lake_Dist.

Proposition Name: Rule_Concern_1
IF Storage_in_June(M1) IS Zone_B
AND Sim_Resp(%) Of Cap IS Less_than_15%
AND Lake_Dist(%) Fail/wk) IS Less_than_4%/week
THEN Operating_policy IS Consider_Bans

Proposition Name: Rule_Concern_2
IF Storage_in_June(M1) IS Zone_B
AND Sim_Resp(%) Of Cap IS Failure
AND Lake_Dist(%) Fail/wk IS Less_than_4%/week
THEN Operating_policy IS Consider_Bans

Proposition Name: Rule_Concern_3
IF Storage in June(M1) IS Zone C
AND Sim Result(% of Cap) IS Less than 15%
AND Lake Dist(%Alt/wk) IS Less than 4%/week
THEN Operating_policy IS Consider Bans WITH TV = 0.80

Proposition Name: Rule_Concern_4
IF Storage in June(M1) IS Zone C
AND Sim Result(% of Cap) IS Failure
AND Lake Dist(%Alt/wk) IS Less than 4%/week
THEN Operating_policy IS Consider Bans WITH TV = 1.00

Proposition Name: Rule_Need_1
IF Storage in June(M1) IS Zone B
AND Sim Result(% of Cap) IS Less than 15%
AND Lake Dist(%Alt/wk) IS More than 4%/week
THEN Operating_policy IS Need Bans WITH TV = 0.30

Proposition Name: Rule_Need_2
IF Storage in June(M1) IS Zone B
AND Sim Result(% of Cap) IS Failure
AND Lake Dist(%Alt/wk) IS More than 4%/week
THEN Operating_policy IS Need Bans WITH TV = 0.50

Proposition Name: Rule_Need_3
IF Storage in June(M1) IS Zone C
AND Sim Result(% of Cap) IS Less than 15%
AND Lake Dist(%Alt/wk) IS More than 4%/week
THEN Operating_policy IS Need Bans WITH TV = 0.80

Proposition Name: Rule_Need_4
IF Storage in June(M1) IS Zone C
AND Sim Result(% of Cap) IS Failure
AND Lake Dist(%Alt/wk) IS More than 4%/week
THEN Operating_policy IS Need Bans WITH TV = 1.00